Calculating: How have Year 5 children's strategies changed over time?

Alison Borthwick & Micky Harcourt-Heath
Norfolk Children's Services

This paper reports on longitudinal research into the calculation strategies used by Year 5 children from 2006 to 2014. It considers not only what proportions of children are successful in these calculations but also examines the range of strategies used and whether these have changed over time. Each of the 5 data sets across the 9 years has included approximately 1000 children from the same group of 22 schools spread across one large UK County.

Key words: calculations; written strategies; mathematics; primary; Year 5; algorithms

Introduction

In spite of much educational reform over the last few decades, calculation strategies are still provoking discussion. This study is one of a suite of research papers that discuss and track children's written calculation strategies since 2006. One of the outcomes of the educational reform movement suggests that mathematics teaching should not only focus on children's selection of standard algorithms, but that it should nurture the development of their abilities to solve mathematical problems 'efficiently, creatively and flexibly (or adaptively) with a diverse of meaningfully acquired strategies' (Tobeyns & Verschaffel, 2013, p.129). The position and importance of standard written algorithms in the mathematics curriculum have varied over time and are well documented (e.g. Anghileri, 2001a). Over the years a number of mathematics educators have focused on particular calculation strategies. For example, Verschaffel, Greer & Torbeyns (2007) and Kilpatrick, Swafford and Findell (2001) carried out studies on subtraction, while Anghileri (2001b) investigated children's approaches to division questions.

While the ability to calculate mentally is recognized as being an important part of solving calculation questions (Harries and Spooner 2006) our studies have only focused on written methods. Again there is a wealth of research that suggests that many of the difficulties children encounter are associated with being introduced to formal written algorithms too early and often as a replacement for mental strategies. Our previous studies have also supported this.

The purpose of this paper is to examine the changes over time seen in our research in the proportions of children successfully answering age-related calculation questions and to explore the strategies employed. Previous articles relating to our research have concentrated on each individual year's set of data but the focus for the 2014 study is to look at the trends over time whilst also presenting this year's data. To date we have discovered that those children who show a commitment to processes built on understanding, rather than rote learning, are more successful than those who do not.

Methodology and context

As for previous data collections in 2006, 2008, 2010 and 2012, in 2014 data was once again collected from test papers completed by Year 5 children from the same 22 schools throughout Norfolk. Using the same categories as previously (Borthwick & Harcourt-Heath, 2012), responses to four questions from each of the papers were analysed for their calculation strategies. One question each for addition, subtraction, multiplication and division was used.

Findings and Discussion

Rather than reporting solely on data collected for 2014 (which previous research has documented (Borthwick & Harcourt-Heath, 2007; 2010; 2012)), this paper considers the data sets longitudinally in order to identify any emerging patterns or trends over time. This Year 5 research used calculation strategies based on the National Numeracy Strategy (NNS) (DfEE, 1999), and it seems from the outcomes of this research that schools continue to promote those strategies even though the NNS ceased to exist in 2011. The tables that follow show the overall total proportions correct and incorrect for each year's data in bold figures at the top. Underneath this the breakdown for either correct or incorrect shows the spread across the strategies and each column therefore totals 100%.

Addition

546+ 423	2006 ✓	2006 ×	2008 ✓	2008 ×	2010 ✓	2010 ×	2012 ✓	2012 ×	2014 ✓	2014 ×
Overall Totals	***90***	**10**	***89***	**11**	***90***	**10**	***94***	**6**	***92***	**8**
Not attempted **	-	-	-	6	-	12	-	0	-	3
Standard Algorithm	*28*	13	*30*	21	*31*	8	*46*	16	*51*	31
Number line	*1*	7	*6*	13	*3*	12	*3*	11	*3*	17
Partitioning	*31*	22	*34*	18	*29*	21	*19*	16	*21*	18
Expanded vertical	*6*	4	*9*	<1	*17*	8	*18*	10	*15*	11
Answer only **	-	-	*20*	34	*19*	32	*12*	35	*9*	16
Other	*34*	54	*1*	7	*1*	7	*2*	12	*1*	4

Table 1: Results from 1021 children for addition question (546 + 423).
** Answer only and not attempted were introduced as categories from 2008

Overall for addition, the proportions of children gaining a correct response have remained relatively high. The nature of the question is such that no bridging is required and therefore the addition of either place value or digits both lead to a correct response, irrespective of understanding. Responses were therefore recorded as 'standard algorithm' where pupils had re-written the calculation vertically and the answer appeared in a single horizontal row under this.

Considering changes over time, the expanded vertical strategy has increased in terms of the proportions of children successfully selecting it, perhaps indicating that teachers are sharing this strategy as a precursor to the standard algorithm because it maintains quantitative value as opposed to digit value (Thompson, 2000). The other notable change has been in the proportion of children recording only an answer, decreasing from 20% in 2008 to 9% in 2014. This could suggest that children are increasingly choosing to record working or that they are more confident in using a written calculation strategy as opposed to working out the answer entirely mentally. This is positive as when the questions become more difficult the children will already have a written strategy on which to draw.

Subtraction

317 – 180	2006 ✓	2006 ×	2008 ✓	2008 ×	2010 ✓	2010 ×	2012 ✓	2012 ×	2014 ✓	2014 ×
Overall Totals	***42***	**58**	***54***	**46**	***58***	**42**	***69***	**31**	***75***	**25**
Not attempted **	-	-	-	5	-	7	-	4	-	5
Standard Algorithm – decomp *	*23*	16	*23*	16	*12*	13	*15*	23	*19*	20
Standard algorithm – equal add	-	-	*4*	1	*<1*	0	*0*	<1	*0*	0
Number line	*30*	3	*49*	10	*64*	18	*70*	23	*70*	28
Negative number	*4*	2	*<1*	<1	*<1*	0	*2*	2	*<1*	<1
Counting up ***	*13*	1	*6*	21	*4*	<1	*3*	21	*5*	6
Counting back	-	-	*2*	2	*3*	6	*2*	<1	*2*	<1
Answer only**	-	-	*9*	8	*8*	25	*4*	3	*2*	11
Other	*31*	78	*6*	36	*7*	31	*3*	24	*2*	29

Table 2: Results from 1021 children for subtraction question (317-180).
* Standard algorithm was split between decomposition and equal addition from 2008
** Answer only and not attempted were introduced as categories from 2008
*** Categories revised 2008 onwards - counting on was then split between counting up and counting back

In 2014, the data shows that 345 more children gained a correct response than in 2006. Over the nine year period of this research, the proportion of children able to answer an age-related subtraction question has almost doubled. The most significant changes over time within the subtraction data show that while the use of a standard algorithm has decreased, the use of the number line has increased and is now the strategy selected by the greatest proportion of children answering correctly (from 30% in 2006 to 70% in 2014). It should also be acknowledged that the proportion of children selecting the number line and answering incorrectly has increased over time. The number line offers children a transparent method which is built on mental calculation strategies. This method enables children to forge the link mathematically between working from concrete, through pictorial and eventually to abstract thinking.

Although the proportion of children achieving a correct answer overall has increased, the number of children answering incorrectly, and whose methods are attributed to ‘other’, suggests that they either do not have a strategy on which to draw or the one they are selecting is not reliable.

Multiplication

56 x 24	2006 ✓	2006 ×	2008 ✓	2008 ×	2010 ✓	2010 ×	2012 ✓	2012 ×	2014 ✓	2014 ×
Overall Totals	***22***	**78**	***29***	**71**	***36***	**64**	***42***	**58**	***51***	**49**
Not attempted*		-		18		20		19	-	18
Standard Algorithm	*9*	7	*<1*	3	*1*	3	*1*	3	*14*	2
Grid method	*70*	11	*87*	22	*94*	33	*95*	49	*82*	50
Expanded vertical	*11*	7	*10*	2	*1*	1	*1*	1	*2*	2
Two partial products*	-	-	-	28	-	20	-	12	-	14
Answer only *	-	-	*1*	6	*<1*	6	*0*	2	*<1*	3
Other	*10*	75	*1*	21	*3*	17	*3*	14	*2*	11

Table 3: Results from 1021 children for multiplication question (56 x 24).
* Answer only, not attempted and two partial produces were introduced as categories from 2008

Overall, the proportion of children answering the multiplication question correctly has increased by 29% between 2006 and 2014. The grid method remains both the most selected and most successful strategy over time but also continues to produce the highest proportion of incorrect responses. Looking at the children’s work, these errors seem to arise from basic calculation mistakes with both multiplication and addition. For example, children appear to understand the model of the grid and the need for four partial products. However, these individual calculations are either not completed correctly, for example, 50 x 20 giving a product of 200 rather than 2000, or errors in addition then follow.

Dropping the model of the grid seems equally problematic, contributing to the high proportion of children recording only two partial products and suggesting that the concept of multiplication is still not well understood; in 2014 this represented 140 children. Over time the proportion of children who do not attempt this question has remained stable with almost one in five children each year falling into this category. Whilst success for children attempting the question has increased, there are still significant numbers of children who seem not to have a strategy to employ.

Division

222 ÷ 3	2006 ✓	2006 ×	2008 ✓	2008 ×	2010 ✓	2010 ×	2012 ✓	2012 ×	2014 ✓	2014 ×
Overall Totals	***21***	**79**	***28***	**72**	***33***	**67**	***38***	**62**	***47***	**53**
Not attempted*		-		28		29		25	-	20
Standard Algorithm	*13*	4	*7*	4	*8*	3	*12*	6	*10*	3
Chunking down	*21*	3	*21*	6	*13*	4	*15*	3	*19*	6
Chunking up	*34*	4	*33*	8	*31*	8	*22*	7	*29*	8
Number line	*10*	3	*22*	9	*28*	12	*40*	23	*38*	30
Answer only*	-	-	*9*	22	*7*	23	*5*	16	*1*	13
Other	*22*	86	*8*	23	*13*	21	*6*	20	*3*	20

Table 4: Results from 1021 children for division question (222 ÷ 3).
* Not attempted and answer only were introduced as categories from 2008

Even though the proportion of children gaining a correct response has more than doubled, increasing over time by 26%, this still represents over half of children in this research not being able to answer an age-related division question successfully.

Over time responses to this question have continued to result in a spread across the possible strategies, with no one method emerging as clearly favoured. Perhaps the most significant change is that a greater number of children are attempting the question. However 20% of children in 2014 still did not respond. In the most recent data set the number line is the most selected and successful strategy, although it should also be noted that it is also the one resulting in the highest proportion of incorrect responses. Our analysis of the children's work identified that the errors could perhaps be attributed to an inefficient use of the number line, with repeated small jumps rather than fewer larger 'chunks'.

Conclusion

Calculation	**Change over time 2006 – 2014**	**Children represented by proportional increase***
Addition	**2%** (90 – 92%)	20 extra (910 to 930)
Subtraction	**33%** (42 – 75%)	333 extra (424 to 757)
Multiplication	**29%** (22 – 51%)	293 extra (222 to 515)
Division	**26%** (21 – 47%)	263 extra (212 to 475)

Table 5: Proportional increases from 2006 to 2014
*Numbers of children are based on an average across the 5 sets of data

The table above shows that significantly higher proportions of children are more successfully answering age-related calculation questions in 2014 than they were in 2006 (with the exception of addition that already had high success rates for the particular question that formed part of this research). For all of the calculations it would appear that over time particular strategies have emerged in terms of greater proportions of children selecting them and also that these selections are more often resulting in a correct response. It is perhaps significant that the use of a number line for both subtraction and division and the selection of the grid method for multiplication are the emerging strategies following their promotion as mental-based strategies through the models adopted by the NNS work in the decade or so following its introduction in 1999. Whilst the formal input by the National Strategies finished in 2011, it could be suggested that its legacy in schools continues.

References

Anghileri, J. (2001a). What are we trying to achieve in teaching standard calculating procedures? Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education, 2: 9-16. Institute of Technology: Haifa, Israel.

Anghileri, J. (2001b). Principles and Practices in Arithmetic Teaching: Innovative Approaches for the Primary Classroom. Buckingham: Open University Press.

Borthwick, A., & Harcourt-Heath, M. (2007). Calculation Strategies used by Year 5 Children. Proceedings of the British Society for Research into Learning Mathematics 27(1): 12-17.

Borthwick, A., & Harcourt-Heath, M. (2010). Calculating: What can Year 5 Children do? Proceedings of the British Society for Research into Learning Mathematics, 30(3): 13-18.

Borthwick, A., & Harcourt-Heath, M. (2012). Calculating: What can Year 5 Children do now? Proceedings of the British Society for Research into Learning Mathematics, 32(3): 25-30.

DfEE (1999). Framework for Teaching Mathematics from Reception to Year 6. London: DfEE.

Harries, T., & Spooner, M. (2006). Mental Mathematics for the Numeracy Hour. London: David Fulton.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. Washington, D. C.: National Academy Press.

Torbeyns, J., & Verschaffel, L. (2013). Efficient and flexible strategy use on multi-digit sums: a choice/no choice study. Research in Mathematics Education, 15 (2), 129-140.

Thompson, I. (2000). Issues for classroom practices in England. In J. Anghileri (Ed.), Principles and Practices in Arithmetic Teaching (pp. 68-78). Buckingham: Open University Press.

Verschaffel, L., Greer, B., & Torbeyns, J. (2007). Whole number concepts and operations. In F. Lester (Ed.), Handbook of Research in Mathematics Teaching and Learning, 2nd edition, (pp. 557-628). New York: Macmillan.