Exploring the generalisation strategies children apply to visual spatial patterns

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The purpose of my research is to explore to what extent children in Irish primary schools are developing skills in algebraic reasoning. Generalisation has been identified as a highly significant component of algebraic reasoning (Carpenter & Levi, 2000; Kaput, Blanton & Moreno, 2008). Patterning plays a key role in supporting children's developing skills in generalisation and internationally visual spatial patterns tasks have been utilised in many research projects to investigate children's success in generalising (Radford, 2011; Rivera & Becker, 2011; Warren, 2005; among others). In this paper I discuss preliminary findings of research into the algebraic reasoning of children in Irish primary schools, focusing on generalisation strategies in response to visual spatial patterns. The research instrument I am using is the clinical interview. In assessing children's ability to generalise, I am interested in exploring the reasons behind responses they proffer and the clinical interview is a research instrument which offers this facility. Through the method of clinical interviewing I investigate the participants' responses to the patterns and ask them to consider near and far generalisations of the patterns. In this way I aim to develop "an interpretation or a series of interpretations" that may provide a new perspective on Irish children's algebraic reasoning and in particular their thinking about generalisation (Ginsburg, 1997, p. 114).

Keywords: algebra, algebraic reasoning, patterning, generalisation, primary school, clinical interview, assessment.

Introduction

> Algebraic thinking does not appear in ontogeny, nor does it appear as the necessary consequence of cognitive maturation. To make algebraic thinking appear, and to make it accessible to the students, some pedagogical conditions need to be created (Radford, 2011, p. 308).

Radford (2011) asserts that algebraic thinking does not develop without intervention yet there is evidence to indicate that algebraic thinking is present in infants and that it may precede arithmetical competence (Britt & Irwin, 2011; Mason, 2008). Vygotsky (1962) discusses the difference between the application of scientific concepts and that of spontaneous concepts, where scientific concepts are those acquired in an educational setting and spontaneous concepts are acquired in informal settings. In a formal setting such as responding to a mathematical task, children find it easier to apply the concepts acquired formally, and achieve less success applying the concepts developed informally. Thus, while preschool children may demonstrate an ability to generalise in their learning of language, they may struggle to apply this ability to a mathematics task (Hewitt, 2009). It may be said, therefore, that educational opportunities to generalise may support children in generalising in mathematics tasks. Also, children who have not been facilitated in generalising in a mathematical context may find such thinking both novel and challenging.

This paper discusses findings from a clinical interview wherein children were asked to generalise from visual spatial patterns. Within the Irish Primary Mathematics Curriculum (Government of Ireland, 1999) generalisation is not mentioned in the content objectives of the algebra strand. While patterning is present within the curriculum, it consists predominantly of repeating patterns in infant classes and numerical pattern identification as an aid to computation in First to Fourth class[1] (corresponding to children aged from 6 to 11 years). It is unlikely that many children attending Irish primary schools would have experience with extending visual spatial patterns and it is possible that many children would not have been asked to generalise in any context. Similarly in 2009 a National Assessment of Mathematics and English Reading found that there was insufficient attention paid to problem-solving in Irish primary school mathematics classes and it is probable that some children in engaging with the clinical interview may have experienced challenge due to the multi-step nature of the tasks (Eivers et al., 2010). The tasks presented in the clinical interview are designed largely in adherence to the principles of good task design as developed by Ronda (2004), which recommend that "tasks should be formulated in such a way that what made tasks problematic would be the mathematics rather than aspects of the situation" (p. 71). It was necessary however to incorporate multi-step problems to afford an opportunity for children to explicitly identify, predict and generalise from a pattern.

Method

Data collection consisted of individual task-based clinical interviews. Most of the interviews were videotaped. Some children did not give consent for videotaping and their interviews were audiotaped. In analysing transcripts from the clinical interviews I adopted a 'generative' approach, whereby I interpreted episodes of each interview, rather than coding and analysing each clause of the transcript (Clement, 2000). Clement suggests that a generative approach allows a researcher to form a model of student thinking that "is grounded in protocol data in order to explain important, but poorly understood, behaviours" (p. 588).

Task Design

Algebraic reasoning skills include the ability to identify a pattern, to predict near and far terms, to consider a general term and to communicate a rule for the pattern (Threlfall, 1999; Owen, 1995). In identifying tasks for inclusion in the clinical interview, I aimed therefore to choose items which would facilitate children in demonstrating the skills of identification, prediction, generalisation and communication of a rule. The task which I will discuss in this section of my report is taken from research conducted by Warren (2005), where she conducted a teaching experiment and utilised pre and post-tests. Warren's research was focused on children's ability to generalise a pattern rule from a growing pattern and the participants had an average age of nine years and six months. Rivera and Becker (2011) also presented children with this pattern in their research into generalisation strategies of children in 3rd grade in California. In this task, participants are required

[1] In Ireland children commence primary school at 4 or 5 years of age and complete 8 classes before proceeding to secondary school. The 8 classes are entitled *Junior Infants, Senior Infants* and from *First* to *Sixth Class*. Children attending First to Fourth Class would be aged between 6 and 11 years.

to generalise from a visual spatial pattern, when presented with the first four terms. The terms presented to the participants are shown as Figure 1.

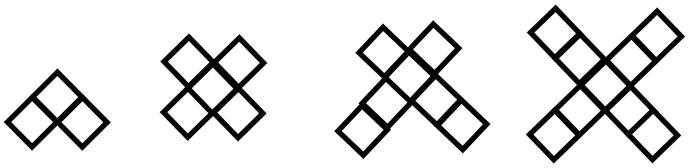


Figure 1: Diamonds pattern

On presentation of the pattern, I asked each child what he/she could tell me about it and also to describe the pattern, asking them to imagine that they needed to describe it to somebody who couldn't see it. Through asking open-ended questions, I aimed to allow the children freedom in their responses and thus elicit "the expression of personal ways of thinking" (Ginsburg, 1997, p. 126). I then proceeded through warm up exercises, asking children to extend the pattern to the 5^{th} and the 6^{th} terms, by drawing. If a child had no difficulty extending the pattern, I asked her/him to describe the 10^{th} term, the 11^{th}, the 100^{th} and the 101^{st}. I chose the 10^{th} as being a near generalisation and the 100^{th} as a far generalisation. In this task, children could solve the near generalisation by counting on from the 6^{th} term but solution of the far generalisation would require the application of a general rule. In this context, 100 is sufficiently large to play the part of a general number (Stacey, 1989).

Participants

The participants included in this analysis are four girls from Fourth class (with a mean age of 10.08 years) who took part in the clinical interview and to whom I assigned the pseudonyms of Bella, Nikki, Tara and Natasha. I asked each girl during a preliminary information meeting to choose a pseudonym and the names used are those which the girls chose. All four girls attend a co-educational primary school in a small town.

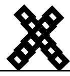
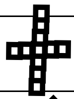
Findings and Discussion

As out-lined above, my first question to each participant upon presentation of the pattern was "what can you tell me about this?" After inviting each child to describe the pattern, I asked her to draw the next term, the 5^{th} in the pattern. The 5^{th} term should include a central diamond, two legs of length two diamonds pointing upwards and two legs of length three diamonds pointing down. Table 1 contains the participants' responses to the opening questions and Table 2 contains a description of their extension of the pattern.

Table 1: Initial participant responses to Diamonds task.

Participant	Initial verbal response
Natasha	They're white boxes. That's a half a box, and then that's a whole box but there's no square there, but on the top of them. That's a.. if you take that away then there'll be a box there. If you take them two away and that one and then you have a top.
Bella	They're all starting off like that and then the top makes an x so it's like a puzzle
Tara	No, not really. Ehm, it's kinda like an x. I don't really know.
Nikki	I can't really describe it.

Table 2: Responses of participants to request for extension of pattern.

Response	Participants
Drew 5th term correctly 	Bella, Tara
Extended top legs 	-
Rotated and extended opposite legs 	Nikki
Drew 6th term, conserved symmetry 	Natasha

Looking at Table 2, both Bella and Tara correctly extended the pattern to the next term. It may be that Nikki and Natasha experienced difficulty in distinguishing ‘what is changing’ from ‘what is staying the same’. Rivera and Becker (2011) discuss the role that perception plays in preparing students to generalise. Drawing on the findings from Mason, Graham and Johnston-Wilder (2005), Rivera and Becker state that perceiving some properties of terms in a pattern, and identifying those which are changing and those which are not are “necessary and fundamental in generalisation” (p. 329). Of the four children whose actions are discussed in this section, Bella and Tara appear to have succeeded in differentiating that which needed to change from that which didn’t. In this case, to extend the pattern, only the bottom legs of the diamond should change. All children extended the number of diamonds on legs of the x-shape but Natasha extended all four legs, and Nikki extended opposite legs. Interestingly, Natasha when asked how she knew what to draw commented from observing the 4th term “[t]here’s 2 there, 2 there, 2 there and 2 there and then I thought it’s 3 there and 3 there, 3 there and 3 there”. Even though she had described the pattern initially as a sequence of modified nets for cubes which were growing and altering in shape, she based her 5th term solely on the 4th term with no reference to previous terms.

Rivera and Becker (2011) discuss participants’ initial responses to patterns and how useful they may be to students when they are asked to generalise. They found that only 31% of the initial statements made by students could be adapted to construct a generalisation. Other statements focused too little on the structure and what was changing as the pattern progressed. Among the responses of my four participants to the initial request for an observation of the pattern, only two responded and of those neither commented on how the terms grew as the pattern progressed.

Generalisation

When each girl had extended the pattern to the next term, I asked her to consider the 10th term, aiming in this way to facilitate a consideration of a near generalisation. I then asked for a description of the 100th term in order to elicit a far generalisation of the pattern. Table 3 contains the responses of the children to these tasks, where in all cases the participants indicated the legs of the x-shape. Again Bella correctly identified the required term, as did Nikki on this occasion. While Natasha did not correctly identify the 10th term of this pattern, she did identify that the central diamond was constant and that the number of diamonds in each leg bore a relationship to the term number.

Participant	10th term	100th term
Possible correct description	An ‘x’ shape containing a central diamond and 5 diamonds on each leg.	An ‘x’ shape containing a central diamond and 50 diamonds on each leg.
Natasha	10 there, 10 there, 10 there and 10 there.	100 there, 100 there, 100 there and 100 there.
Bella	5 on all of them so there’ll be 5 there, 5 there, 5 there and 5 there	50 there, 50 there, 50 there and 50 there
Tara	Did not respond	Did not respond
Nikki	5 there and 5 there. There’d be 5 there and there as well.	There’s 50 going across there, and 50 there, and 50 there and 50 there.

Table 3: Descriptions of the 10th and 100th term.

Conclusion

From the initial request to describe the pattern, there was great disparity between the responses of the children. Bella was confident throughout – she described the pattern as a puzzle where “they’re all starting off like that and then the top makes an x”, and she proceeded to extend and construct both near and far generalisations successfully. While extending correctly, Tara seemed the least secure in her responses, indicating that the pattern is “kinda’ like an ‘x’, I don’t really know”, and indicating that she could not describe the 10th and 100th term. Natasha’s generalisations seem consistent in some way with her extension in that she extended all legs of the shape to construct the 5th term and doubled the number of diamonds in the 10th and 100th terms. Nikki seemed very insecure in her initial responses to the pattern, yet succeeded in constructing generalisations.

Further analysis is merited to explore how each child’s perception of the pattern supports construction of generalisations, given that Tara extended correctly but did not construct a generalisation whereas Nikki erred in extending the pattern yet constructed correct near and far generalisations (Rivera & Becker, 2011). In seeking to examine the children's algebraic thinking, I plan to explore the steps undertaken by each girl as she progresses from her initial response to the pattern, to the construction of a generalisation.

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