Mathematics at university: practices, values and participation

Paul Hernandez-Martinez and Merrilyn Goos

Loughborough University, UK and The University of Queensland, Australia

In this paper we report on a developmental design research study in an engineering mathematics undergraduate course, where previous attempts to increase participation had failed. We take ideas from socio-political theories to frame the teaching (re-)design and use a socio-cultural theoretical framework – where learning is seen as participation - to evaluate its impact. We collected data from students' written feedback and used peer observation to reflect on and refine the teaching strategy and to analyse the students' learning. Results showed a positive participation (although not all students engaged or liked the approach) and we discuss the implications of our results for Mathematics Education.

Keywords: Higher Education mathematics, values, participation, socio-political theories

Background

Mathematics education research at university level is still scarce in comparison to other levels of education. In terms of practice, the majority of instruction has remained unchanged for quite a long time, where "chalk-and-talk" practices are the norm. In this context, the research reported here was born from a university mathematics teaching innovation project. The first author lectures a second year engineering mathematics undergraduate module where students are typically unmotivated by the subject (most of them consider it useless in relation to their engineering course) and highly strategic when it comes to engaging with the module's learning activities.

In order to improve achievement the first author, whose research area is in Mathematics Education, began developing teaching resources and incorporating changes in his teaching practice, following a developmental design research approach (Gravemeijer & Cobb, 2006). Given the vast literature on mathematical modelling and the positive results reported in it (Blum, Galbraith, Henn & Niss, 2007), a teaching approach of this type was implemented. But after two years of developing resources, implementing and refining the modelling approach, the results were discouragingly limited. Most students still did not engage with the "realistic" modelling tasks, attendance rates remained poor, students continued to be very strategic about assessment and there was no real evidence of substantial progress in conceptual understanding in the majority of them. A different viewpoint was required, a re-thinking of the design was needed. The aim of this paper, therefore, is to describe this re-design – which adopted a more critical approach to Mathematics Education – and to analyse the impact that it had on students' participation in mathematically meaningful activity.

Socio-political theories in Mathematics Education

It has only been very recently that Mathematics Education has turned to socio-political theories to gain a wider understanding of teaching and learning (Gutierrez, 2010). However, as Pais (2013, p.19) points out, “by emphasising issues of power and identity, socio-political theories disavow a broader comprehension of *schools as places of economic production*” (emphasis in original). In his critique of the use-value of mathematics, he suggests that the discourse of mathematics as important for everyday use conceals its real importance as “a testing and grading device. What is seen as direct property of object mathematics – its utility – is indeed the result of the place mathematics occupies within the structure of capitalist economics” (p.17). In view of this critique, the “solution” to failing students would not be in better research and subsequent “better” practice because a degree of failure is necessary so that mathematics can maintain its status in our capitalist societies. Therefore, research in Pais’ view, should take these “unwritten rules” and investigate them as “fundamental conditions of today’s schooling” (p.31).

We concur with Pais’ point of view but we also believe that school (or university) mathematics learning is not entirely devoid of some sort of utility. In his attempt to merge Cultural Historical Activity Theory (CHAT) with Bourdieu’s sociology, Williams (2012, p.63) argues that “the purpose of schooling for the adolescent is to lead their development through engagement in new, more culturally advanced, collective activity, engaging with new more developed, social motives that transform school actions into more socially and culturally meaningful activity”. Hence, “if mathematics enhances intellectual labour power, it may also enhance the capacity of labourers to be critical” of the system (p.65). Furthermore, “if we are to understand how the education system gives cultural value to mathematics, we have to understand its cultural ‘capital’” (p.66). In other words, school mathematics is more than just a “test-taking game” to obtain a certificate than can be exchanged at a later time, for example, for a good job. Mathematics education has a use-value in the “cultural development of the mind”, providing a “tool for the critical, scientific examination of society” (p.70).

Consistent with this theoretical perspective, the re-design of the teaching strategy needed to reconcile *in practice* the contradiction between mathematics as exchange value of capital in the market and as use-value in the development of the mind. In evaluating the impact of this teaching strategy, this paper addresses two research questions: (1) What were the students’ perceptions of the value of the teaching strategy? (2) What was the impact of the strategy on students’ participation in mathematically meaningful activity?

Methodology

The teaching approach

The re-design of the teaching approach had to take into account the type of value that students give to mathematics: a rather useless but unavoidable and potentially convenient-to-have subject – a view in line with its value in an economic capitalist educational system, as described by Pais (2013). However, the aim of the teaching design was to try to counterbalance this value of mathematics with a more utilitarian value (solve the “contradiction”), that is, with something that might in some way prove intellectually useful and that might appeal to these students’ future aspirations. Thus the aim was to convince students that engaging with the module’s activities could prove useful for them in some way, a view more in line with CHAT perspectives of the aims of education.

In order to do this, the objectives of the module, as presented to students, were to develop useful *employability skills*, amongst which are mathematical skills for problem solving and communication. To achieve these aims, students would have to engage in activities such as discussing, explaining to others, presenting, developing methods to solve problems, constructing logical arguments, writing reports in a clear way, et cetera. These skills – they were told – would enable them to become better professionals and might, in the future, prove invaluable in securing a good job in an ever increasingly competitive world. In order to convince students of the merits of this approach, a successful professional engineer was invited as a guest speaker to tell students how much employers value mathematical skills. Also, students were continuously presented throughout the module with information about employers, the type of skills these look for in potential employees and how they could show evidence of these skills in, for example, an interview or a job application.

To enable sufficient time for these skills-development activities, the tutorial sessions – which normally consist of students working on textbook type exercises – were "merged" with the lectures so that all weekly class sessions contained a combination of lecturer presenting new material, students working individually or collaboratively on problems and exercises, and students discussing and presenting their (sometimes incomplete) solutions.

Developmental research and collegial peer observation

A key characteristic of developmental design research is the interweaving of research and development of practice (Gravemeijer & Cobb, 2006). However, pedagogical practices at university are typically very individualistic and isolated – one lecturer, usually a professional mathematician, is in charge of his/her module and has a certain amount of academic freedom to conduct it as he/she sees fit (usually it follows the model of "how I was taught"). It is therefore, not surprising that practices have changed very little throughout the history of university education, remaining on the whole "transmissionist" or teacher-centred (Pampaka, Williams, Pepin & Sikko, 2013).

In this paper, we want to advance the concept of collegial peer observation as a way to influence change in pedagogical practices at university (Goos & Hernandez-Martinez, 2014). This method of observing a peer's practice, discussing and reflecting what was observed and implementing developmental changes is beginning to prove useful in schools (Goos, 2013) but as far as we know it is uncommon in university. This method is also useful as a way to research the impact that a design study has on student learning. The second author, a visiting Mathematics Education researcher, served as an "external" observer and someone with whom the first author could reflect and discuss his practice. We conducted the analysis of the data in a qualitative fashion, where both authors agreed the meaning of the various pieces of information. We took a socio-cultural perspective of learning as change in participation in socially situated practices (Lerman, 2001) to inform our analysis and conclusions.

Our data came from written student feedback in weeks 4 and 11 of a 12 week semester and from written records of classroom observations throughout the module, with the two authors having conversations before and after the lectures were observed. The group of students consisted of 40 second year undergraduate engineers.

Results and discussion

Student perceptions of the value of the teaching strategy

Students were asked for written feedback at the beginning of week 4. They were asked to answer three questions: 1) What have I learned until now? 2) What do I like about this module? and, 3) What would I change about this module? During that lecture, 30 out of the 40 students attended and returned feedback.

To the question "What have I learned until now?", 18 students responded with a content-related answer, such as "How to solve a differential equation", 9 of them made specific reference to mathematical modelling and learning how to create models of real world situations, and 3 students referred to the learning of skills such as teamwork skills or how transfer knowledge to new contexts. For example, one student wrote: "I have learnt how to solve inhomogeneous second order ODEs and have improved my team work skills", and another one wrote: "I have learnt how to properly apply models to solve situations. Although my understanding needs improving I can apply myself better and draw from other areas".

To the question "What do I like about this module", 16 students referred to aspects of the teaching approach (e.g. worked examples on the board while they work through them themselves, interactivity of classes, group work, thorough explanations), 5 students mentioned something related to the content, and 3 students mentioned the feedback provided (e.g. solutions to worksheets, online tests).

To the question "What would I change?" 10 students mentioned the need for more time to work on problems and examples, 6 of them mentioned print outs of slides or more time to copy these from the projector, 4 students requested more explanations of solutions to examples, 3 students mentioned "Nothing", 2 said they did not like group work, 1 wanted more background on why they were learning specific topics and 1 asked to ensure that everyone in each group had to present their solutions.

These answers reflect the co-existence of the two values of mathematics: on the one hand, a substantial number of students focused on aspects of the teaching approach such as explanations, clarity, more worked examples, et cetera, but a lot of them were conscious of their learning and the development of useful skills, being able to transfer this knowledge, interacting with others in group, et cetera. Only a few did not like the new approach or made negative comments.

Students also provided written feedback as part of the module's formal evaluation questionnaire during week 11. The questionnaire (standard questions for all modules in the degree) contained two open ended questions: 1) What did you like about this module? and, 2) How could the module be improved? Again, there were 30 questionnaires returned although not all students commented on the open questions.

To the first question, 11 students referred to aspects of the teaching approach (e.g. well structured, worked examples, revision of previous topics, one student did not like it), 2 of them referred to modelling and its usefulness in the future, 4 students mentioned particular topics and 2 referred to the lecturer (knowledgeable, enthusiastic). To the second question, 9 students asked for more time and explanations of harder questions, 11 mentioned more worked examples, notes on important equations and handouts of notes, 2 asked for past exam papers and printed solutions of in-class exercises and 1 student mentioned "Nothing".

It seems that students' perceptions of the course remained stable: the teaching approach was something that most of them liked and valued while suggestions were made for more explanations and worked examples as improvements to the module. For instance, one student wrote: “Well organised, (I) liked the ideas of modelling real life situations”. However, it is presumed that there were a few students that did not find value in the course. One of these students wrote: “Not much (that I liked) as I don’t particularly like maths and I find it hard”.

Impact of the teaching strategy on student participation

The second author attended lectures in 5 of the 12 weeks of the course, starting on week 3. She made field notes focusing on the mathematical content, the teacher’s explanations and questions, and the students’ responses, including conversations they had with each other while working on problems. This allowed her to identify evidence of students’ understanding and misunderstanding that were shared with the teacher after each observation. Some of these observations gave way to small changes in subsequent activities. On a couple of other occasions the teacher and observer had more reflective, general discussions about the progress of the module. The consensus between the authors was that most students seemed to participate willingly in all activities; it appeared that they felt confident enough to present their work either by standing up and explaining a concept to the class (after having some minutes on their own or with a neighbour to work/discuss an exercise/question) or by coming to the front and writing an answer to an exercise on the whiteboard.

Throughout the module, students were asked to work in groups outside of lectures to solve some modelling problems and to hand in an individual report of their solution. Most of them handed in well-structured reports even though these did not carry any marks towards the module’s assessment. One student wrote about how working in a group allowed him to speak his ideas aloud and engage in “healthy debate”. In one occasion, students gave written feedback to each other on their reports and most of these were thoughtful and supportive. On a task where students had to prepare a group presentation, 6 out of 10 groups (24 students) gave well-thought, well structured presentations even though these did not count for the final grade.

The final results of the module were very encouraging. The average mark was 60 (while in the previous two years had been 45 and 51, respectively). The attendance rate remained high throughout the semester (75% on average) while in previous years had been <50% on average. We are careful not to take attendance as the only evidence of participation: there were students who attended most lectures but obtained a low mark in the module. However, there was not a single student who had low attendance and obtained a good mark at the end. Therefore, participation is much more than just attending, but for those who involved themselves fully in most activities the results were good.

Conclusion and implications

Observations throughout the module evidenced a substantial participation of most of the students in the different activities that were designed to enhance their learning, in spite of some of these not counting towards the assessment of the module. Students’ written feedback and observations of their willing engagement in group work, problem-solving and modelling tasks, presenting, discussing, writing reports, et cetera, shows that most of them believed that these tasks could be useful to their future and that developing these skills was worth investing time and effort.

Certainly, the exam-driven mentality was still very much present in these students but the data show that it can co-exist with a genuine sense that mathematics can somehow be useful. Its value goes beyond the final exam or the exchange of a degree certificate for a good job; their participation in culturally valued practices shows that these students believe it can be useful, for example, in a job interview or when they "go out" into the real world of engineering – at some point, these skills are going to be put to the test (engineers still have to build bridges, for example, and they need mathematics to do so) and what they learned might become handy. They have, in this sense, engaged in the cultural development of their minds, and hopefully, for some of them this enhanced labour power can result in a "critical, scientific examination of society".

As for those students who did not engage, there might be many reasons why they chose not to participate. Maybe, in their own perspective of learning there is no space for the kind of activities that were on offer or they might not think this is the way to acquire the skills to which they attribute value. Further development is then necessary to understand these reasons and see if there is any possibility to attract in some ways these type of students into participating in mathematics learning.

As we see it, the implications of this study for mathematics teachers and researchers is not to ignore that learning is shaped in fundamental ways by the laws and values of our capitalist societies – and that students will behave according to these – but to create opportunities in practice for them to become individuals that can fully participate in society in an informed, critical and responsible way.

References

Blum, W., Galbraith, P., Henn, H.-W. & Niss, M. (2007). *Modelling and applications in mathematics education*. New York: Springer.

Goos, M. (2013). Researcher–teacher relationships and models for teaching development in mathematics education. *ZDM*, Online First, 1–12.

Goos, M., & Hernandez-Martinez, P. (2014). Collegial peer observation as a means of influencing change in university mathematics teaching. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia*, (p. 755). Sydney: MERGA.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. Van Den Akker, K. Gravemeijer, S. McKenney & N. Nieveen (Eds.), *Educational design research* (pp. 17-51). London: Routledge.

Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education,* 41, 1–32.

Lerman, S. (2001). A review of research perspectives on mathematics teacher education. In F. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 33–52). Dordrecht: Kluwer.

Pais, A. (2013). An Ideology Critique of the Use-Value of Mathematics. *Educational Studies in Mathematics*, 84, 15–34.

Pampaka, M., Williams, J., Pepin, B. & Sikko, S.A. (2013). Teaching and learning practices in mathematics: students' perception of teaching practices at different contexts and educational levels. In *Proceedings of the European Conference on Educational Research (EERA Network: 24. Mathematics Education Research)*. Istanbul, Turkey: EERA.

Williams, J. (2012). Use and exchange value in mathematics education: contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics*, 80(1&2), 57–72.