Planning for active participation in mathematics: promoting democratic practices in mathematics classrooms.

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We believe students' participation in school activities should be democratic and that this can best be achieved by planning for 'authentic' mathematical activity, which is characterised by the way in which students and their teachers work together mathematically. Students have the opportunity "to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs" and to engage in mathematics as "art of explanation" (Lockhart, 2008, p.5). We make use of 'Big Ideas' as a tool for shifting the object of activity in the mathematics classroom to participation in authentic mathematical activity. This report draws on data from an EU sponsored research project, 'Awareness of Big Ideas in Mathematics Classrooms', and a small scale follow-up project with Secondary Mathematics PCGE students at London South Bank University.

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Participation in all school activities should be democratic

We would like all schools to be microcosms of the kind of democratic society about which Dewey wrote, that is a society characterised by:

> the necessity for the participation of every mature human being in formation of the values that regulate the living of men together: which is necessary from the standpoint of both the general social welfare and the full development of human beings as individuals. (Dewey, 1937, no page number)

If schools were such places, what might the mathematical activity we'd find within them look like? More immediately, as teachers of mathematics whose modest ambition can extend only to affecting our own classrooms, if the activity that takes place in our classrooms is to stand as an example for how such schools should be, what should that activity look like? We answer that it would be no bad thing were it to look like what we call authentic mathematical activity.

Participation in 'authentic' mathematical activity

Acknowledging an apparent element of circularity, we characterise authentic mathematical activity, in part, as entailing a commitment to democratic participation in that activity, and this brings us back to Dewey; we echo his call for "a society that [has] a type of education which gives individuals a personal interest in social relationships and control, and the habits of mind which secure social changes without introducing disorder" (Dewey, 2008, no page number). We are convinced that most of what goes on currently in mathematics classrooms is not democratic. Authentic mathematical activity requires: that those engaged in it explain, and so listen to each other's explanations; that the classroom in which it takes place is one where all participate on equal terms; a classroom that models the type of society desired by Dewey.

Our position is that mathematics is a specialised, esoteric[3] and powerful language, access to which can be made available to all school children through democratic participation in classroom mathematics practices. Whilst this does not exclude critical or 'real life' mathematics, we seek to promote access to an authentic mathematical activity that is, or can become, no less a real part of students' lives than any other form of school activity. Lockhart best represents what we understand by authentic mathematical activity. We set out to work with students in ways which, in Lockhart's words, give them the opportunity "to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs" (Lockhart, 2008, p. 5) in short to engage in mathematics as what Lockhart calls the "art of explanation". Notice that we can and do talk of mathematical activity in terms that do not refer to specific content; we characterise authentic mathematical activity not by what mathematics is done, but by the way in which teachers and students do it. We no more want to specify the content of authentic mathematical activity for it to be authentic, than we would want to specify the content to be addressed by a poem as people seek to write poetry.

Such is the power of the dominant educational discourse that we feel we need to talk about aligning students to the authentic mathematical practices we seek to encourage, seeking to produce within their classrooms powerful identity-changing communities of practice (Lave, 1996; Winbourne, 2008). This means familiarising students with the idea that what, initially, they see of mathematics will inevitably be only the externals, and that this is 'OK'; we must encourage them somehow to apprehend that which lies beyond these externals. Of course, what we *see* as 'that which is to be learned' will depend on our sense of where we might be headed, but it will, inevitably, be most strongly framed by what our teacher shows us.

So, assuming democratic participation in authentic mathematical activity, the 'way' of that participation, to which there are necessarily limited external, visible pointers, must be sensed more subtly; whatever it is that lies beyond those pointers, and by definition cannot be articulated, must be 'lived'.

From this position we are led to say that the *way* young people engage in mathematical activity is more important than the mathematical content they encounter. We want young people to have a sense of a direction of travel, trajectory, (close to what Lave, 1996, calls 'telos'), that encourages them to see beyond the externals, the content stuff, that presents most obviously to the novice; to that which, for the mathematician, is signified by those externals and includes, most importantly, a way of thinking about and doing mathematics, indeed, a way of 'being'. (We think Lockhart is talking about the *way* rather than the *what*.)

Talk of democratic participation brings to mind – or should – the literature of critical mathematics. Whilst our project does fit in with Skovsmose's ideas of mathemacy (Skovsmose, 2005), what we are doing is different from that: we are not looking to contexts other than the 'real mathematical life' of the classroom in which to set up democratic participation in authentic mathematical activity. We share Frankenstein's belief that "knowledge of basic mathematics and statistics is an

[3] esoteric in that sense that it represents a deliberate "rupture in the continuity of spontaneous, everyday knowing and learning, and so cultivation of the 'intellectualisation' of cognitive processes" (Štech, 2008)

important part of gaining real popular democratic control over the economic, political and social structures of our society" (Frankenstein, 1983, p. 315), but what this doesn't include is the kind of 'authentic mathematical activity' which Lockhart writes about so well (Though we would like to see the work of Alf Cole and his colleagues (Cole, Barwell, Cotton, Winter, & Brown, 2013) as providing a bridge between critical mathematics and authentic mathematical activity.)

Likewise, talk of democratic participation suggests that activity is experienced as relevant. As we see it, this becomes a function of activity within the classroom, where the teacher can seek to prompt authentic (in Lockhart's sense) mathematical activity, perhaps setting out to locate this within what Mason and Johnston-Wilder would call their students' ZPR:

> …every learner has the power to imagine, and through exercise of their mental imagery can become interested and intrigued in questions that are not immediately practical. The term zone of proximal relevance [ZPR] is useful in order to refer to settings and tasks that, although not already of immediate relevance to learners as they see things, can become 'real' for them through the use of their power to imagine. (Mason & Johnston-Wilder, 2004, p. 110).

'Big Ideas': Tools for planning for democratic participation in authentic mathematical activity.

We have been working with a set of tools that we call 'Big Ideas' to shift the object of activity in classrooms – our own and those of our students - from narrow, pre-specified mathematical objectives, to the *way* of authentic mathematical activity that we have described above. We believe that 'Big Ideas' is a tool which, carefully and critically deployed, can help us to plan for democratic participation in authentic mathematical activity.

The activity we describe here comes out of two courses with which we have been involved: *Teaching Advanced Level Mathematics* (TAM) and a Secondary Mathematics PGCE course at London South Bank University (LSBU). The TAM course is designed to enable teachers who are competent to teach students up to GCSE to develop the depth of knowledge, skills and understanding of mathematics and its pedagogy to teach A-level mathematics.

This table (Table 1) shows the particular set of Big Ideas with which we have been working. Note that we distinguish between domains of 'Big Ideas' in ways suggested by Shulman's distinction between pedagogical content knowledge (PCK) and subject matter knowledge (SMK) (Shulman, 1986a; Shulman, 1986b). 'Big Ideas', distinguished in this way, has provided an explicit framework for the professional development and initial teacher education work we have been doing. The decision to make these all gerunds has been quite deliberate and reflects, in part through the process of thinking about 'Big Ideas', what we see as our essential principles. These principles suggest strongly to us that, where we have any say in the matter, we want to specify any mathematics curriculum in terms of activity, doing things, doing mathematics.

Big Ideas – a health warning

We must also acknowledge the possibility that, as in any pedagogical relationship, the same talk may itself position some teachers as disempowered as they struggle (by force of habit?) to make sense of what they might experience as a strongly framed discourse (of 'Big Ideas'). We can easily appear to be giving them more 'stuff to learn'.

Subject Matter Knowledge (SMK)	Pedagogic Content Knowledge (PCK)
Proving/ arguing	Using argumentation
Using multiple representations	Using multiple representations/ perspective change
Doing/undoing, (inverting)	Doing/undoing
Specialising/Generalising	Going beyond / extending the domain / asking 'what if?'
Modelling (approximation, linearisation, structuring)	
Using functional dependency	
Using ideas of randomness/inference	
Dealing with infinity (including limits/continuity)	
Using recursion	
	Exploring students' understanding through questioning
	Using misconceptions and errors for learning
	Teachers using their awareness of existence of multiple strategies
	Making connections within mathematics
	Using analogies

Table 1: Showing the set of 'Big Ideas' we used in our research.

Having listed the particular 'Big Ideas' in the toolkit with which we have been working, we note the dangers of any taxonomy and ponder the extent to which we might hope to be aware of these dangers in our research and our teaching. For the purposes of our research, initially with our European partners, we decided upon a set of 'Big Ideas' with which to work. At the same time, we were clear that it was the *biggest* idea – working with 'Big Ideas' – that was most important to us. We neither need, nor make any ontological commitment to a realm of 'Big Ideas' awaiting 'exploration' or 'discovery' in order to get on and use them in our activity.

Using Big Ideas

We have some evidence that 'Big Ideas' helps us to talk about the *way* we might want young people to engage in authentic mathematical activity. The responses we elicited from teachers and student teachers afford us insights into beliefs (often tacit) about the learning and teaching of mathematics.

The following note was submitted by a group of PGCE students after they had been doing some teaching using 'Big Ideas' as a tool:

> We discussed how even in our attempt to be radical we stayed within prescribed frameworks of the curriculum. We didn't challenge what we actually teach the children, just how we introduce particular concepts. We should have thought about the role children play in identifying what they study and how we need to challenge our power relations with them. (S1)

These students show recognition both of potential and tensions/contradictions; note, too, that they are prepared to get on and use 'Big Ideas'. We think this comment[4]

[4] Our data include many other comments, to which we hope to refer in greater detail in a RME paper

shows the potential of 'Big Ideas' for securing democratic participation; the two comments that follow, also from PGCE students, reflect the constraints upon the developing practice of beginning teachers when they think about using 'Big Ideas' in this way. The first reflects the prevailing assumption that passing maths exams requires doing mainly exam questions:

> [Big Ideas] develops pupils' mathematical understanding but it won't necessarily make pupils better at passing exams and at the end of the day that is what the school really cares about (S2);

and the second suggests, sadly, that this student believes 'democratic participation' must be sanctioned by more powerful people in the school:

> Big Ideas should be fed through from KS3, introducing them at KS5 makes independent thought an alien concept. There is a place for this sort of teaching but Lockhart's views were too extreme. Is this a conversation for trainee teachers, or more suitable for people who have the influence to put these into action as widespread techniques? (S3)

We conclude this report with some illustrations of uses of particular 'Big Ideas'.

Doing and Undoing

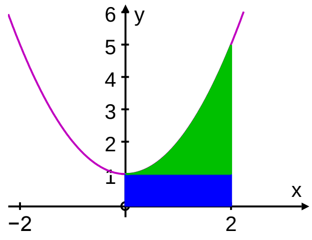
The availability of 'doing and undoing' to this student (S4) could, potentially, significantly change the quality of student and teacher experience (and it relates back to the ideas of authentic mathematical activity and democratic participation):

> The starter had involved simplifying fractions and the pupils had gone on to simplify algebraic expressions and use the correct notation for algebra. For the plenary I decided to reverse this process and get the girls to complicate a simple term. This made them think about undoing what they had been doing earlier and also, since there are an infinite number of correct answers, it was possible for all the girls to contribute something'. (S4)

Using multiple representations

For TAM tutors, this 'Big Idea' became a tool for shaping more careful lesson feedback focused on (authentic?) mathematical activity:

> $\int_0^2 x^2 + 1\,\mathrm{d}x = \left[\frac{1}{3}x^3 + x\right]_0^2 = \left(\frac{8}{3} + 2\right) - 0$. Focussing on this you asked where the final 2 came from in the calculation. One student explained that it was from substituting 2 for x but you said you wanted a geometrical reason. When you showed them where the area of 2 appeared in the graph their response showed that this made perfect sense to them and reminded them of exactly what they were doing. (Tutor 1)



It also afforded TAM teachers powerful mathematical insights:

> I really liked the work we did on the first day about radians - I had not thought about approaching the formulas from a geometrical perspective before and this seemed so straightforward and also made sense as to why we use radians (Teacher 1);

Exploring students' understanding through questioning

This 'Big Idea' mediated discussion of observed TAM lessons, enabling explicit and accurate reference to this particular aspect of PCK.

> Your questioning here, and throughout the lesson, was excellent; you continually probed to find out what they understood and moved their thinking on. One student said that as the width of the strips got smaller "it gets more accurate". "What gets more accurate?" you asked – you looked for precision in their explanations and didn't assume they knew anything they didn't clearly explain. (Tutor 2)

> You used questioning briefly and carefully to introduce students to the activity. An early question drew an incorrect response where one student offered a zero of the function as a stationary point; through your question in response you invited the student to review her answer, and she saw her mistake. (Tutor 1)

We would like to think that such attention to questioning can be seen as working towards democratic participation.

Conclusion

Our experience and our evidence tell us that, if our aim as teachers of mathematics is to set up our classrooms so that all participate democratically in authentic mathematical activity, then we would do well to look to the *way* - the *how*, rather than the *what* - of doing mathematics; we offer the 'Big Idea' of using 'Big Idea as a way of pointing to what we mean.

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