‘Number sense’ through three theoretical lenses

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The national numeracy strategy (NNS) (DfEE, 1999) in England promoted mental calculation skills, built on a strong ‘number sense’, developed throughout primary schooling. The new national curriculum (DfE, 2013) places emphasis on formal algorithms for fluency in calculation. At a time of transition, this paper explores three contrasting theoretical perspectives on number sense: cognitive psychology, situated cognition and Bourdieusian social theory. It is suggested that cognitive theories dominate the teaching literature, while limited attention has been paid to social perspectives in this area. From this position, it is proposed that number sense acts as a gatekeeper to wider mathematical opportunity.

Keywords: arithmetic fluency, number sense, cognitive psychology, Bourdieu, situated cognition, subitising, mathematical identity

Introduction

Curriculum reform in primary mathematics in England is positioned against a background of the necessity of school mathematics, e.g. “...the more you practice, the better you get and the more you will earn in your future career” (Truss, 2014). This demonstrates how mathematics acts as a gatekeeper to life’s opportunities; higher paid jobs are open to those who have higher mathematical qualifications. However, success in school mathematics does not ensure mathematically literacy – the capacity to understand the mathematical world in which we live (Venkat, 2014). Central to mathematical literacy is the concept of number sense which, for the purposes of this paper, I take as a *flexibility and confidence with number*. For some, this might be seen as the core aim of the primary mathematics curriculum and perhaps the legacy of the national numeracy strategy (NNS) (DfEE, 1999). This contrasts with the renewed focus on formal algorithm in the new primary national curriculum (DfE, 2013).

Given the increasing importance of such skills, it is timely to consider children’s opportunities to develop number sense within and beyond arithmetic and mathematics. In this paper I consider three differing theoretical perspectives of number sense: cognitive neuropsychology, situated cognition and Bourdieusian social theory. For each in turn, I offer a definition of number sense before exploring the relevance of each perspective for primary mathematics.

Cognitive Neuropsychology

When considered from a cognitive perspective, number sense is usually restricted to the recognition of numerosity. There is a wealth of research into the development of such number sense, based on the speed and accuracy of enumerating quantities. Cognitive psychology recognises two distinct systems of processing numerosity, subitising and approximation (Dehane, 1992), but their role in the development of symbolic arithmetic is the subject of ongoing research (Hyde, Khanum & Spelke, 2014). Each of these are discussed before the impact of cognitive psychology on educational practice is explored through mathematical difficulties and broader policy.

Subitising allows the accurate enumeration of small quantities, usually up to 4 (Butterworth, 1999). As this skill is present across species an innate mechanism for tracking specific quantities is proposed. It is identified in young infants who also show awareness of core mathematical principles such as addition and subtraction within this limited range. Whether subitising is a unique numerosity skill or simply a visual processing system allowing tracking of a small number of items is still debated. This aside, the capacity to enumerate such quantities rapidly does appear to be persistent (e.g. Dehaene, 1992).

The approximate number system (ANS) allows discrimination of large quantities with a low level of accuracy (Halberda, Mazzocco & Feigenson, 2008). Thus, adults can accurately identify the larger of two numerosities without the opportunity to count them. The ANS is a general numerosity perception system which improves in acuity at an advanced rate throughout the first year of life and then more modestly through childhood and adolescence (Halberda et al., 2008) until roughly the age of 30 (Feigenson, Libertus & Halberda, 2013).

Number representation in the ANS is non-linear; discrimination between quantities relies on their ratio rather than their absolute difference. This non-linear representation is observed in young children, when representing numbers beyond their range of confidence, and in adults, who retain non-linear representation of large numbers (Giaquinto, 2007). Importantly for education, ANS performance correlates with mathematics achievement (Halberda et al., 2008) and recent studies suggest a link between ANS and formal calculation with training in non-symbolic arithmetic, enhancing symbolic arithmetic, (Hyde, Khanum, & Spelke, 2014).

Mathematical Difficulties

Significant difficulties in learning mathematics are thought to be experienced by 6-10% of the population (Butterworth, 2010) and the impact of low numeracy can be significant for individuals and society (Every Child a Chance Trust, 2009). Some children experiencing difficulties in learning mathematics have a significantly less accurate ANS compared to individuals with similar cognitive measures but higher mathematical achievement. Similarly, some children who experience mathematical difficulties lack subitising skills (Butterworth, 1999). It is suggested that a limited ability to subitise could provide a mechanism for identification of children at risk of mathematical difficulties and subsequent intervention. The possibility to train the subitising skill and indeed durability of training in this area are under-researched but potentially offer a route to supporting individuals who are at risk (Hyde et al., 2014).

Cognitive psychology and educational Policy

These scenarios demonstrate the power of cognitive theories– the potential to identify and remedy a learning difficulty is very appealing. Of course, this subscribes to a ‘medical’ model of learning, that a deficiency in a skill such as subitising is a deficiency within an individual and can be ‘fixed’. It does not explain how these cognitive processes can be enhanced through education. Additionally, whilst the ANS appears to rely on a logarithmic number line (Feigenson et al., 2013), progression to arithmetic competence requires an exact linear representation of number. Thus children need concrete experience of number, in the range they are expected to calculate with, to represent those quantities in a linear fashion.

The relationship between the ANS and symbolic number may simply be an association created during a child's developing understanding of number (Hyde et al., 2014). Whether a causal link is established or not, it is clear that socialisation is important in this process. Children do not develop 'number sense' without manipulating concrete materials and linking numerosity to symbolic numbers.The development of the ANS throughout lifespans and across cultures is taken as evidence by psychologists of this being due to maturation (Piazza, Pica, Izzard, Spelke, Dehaene, 2013). However, maturation is not simply biological and understanding the impact of social factors on this process is vital for education, and is now considered.

Situated Cognition

Unlike the position above, situated cognition proposes that number sense is bound by the context in which it is being employed; that understanding numbers in the 'real-world' is different to 'school'. It attempts to explain how adults seem capable of solving problems in 'real-life' contexts, but when such problems are presented in a 'school' setting, they fail. Children do not learn and develop number sense in isolation. It happens from an early age, in a range of settings, through a variety of modes – nursery rhymes, food, toys and so on. Number sense development continues throughout primary school and beyond; those who work in specialised industries may have enhanced number sense relative to their work (Giaquinto, 2007).

The power of situated cognition is that it illustrates how mathematical behaviour in one setting can be distinctly different to another – why a 'level 4' pupil might miss a level 4 on a test paper. It draws a difference between 'school' mathematics as a specific site and type of mathematics, distinct from say 'work' mathematics. This is useful when we consider the continuous reference to young people who are ill-equipped for the world of work (PBE, 2014).

To illustrate the different uses of mathematics, Lave (1989) uses supermarket observations where 'just-plain-folks' (jpfs) made mathematical decisions in the context of needing to purchase products. When the jpfs could not fathom a mathematical reason for a purchase, other scenarios were employed as solutions, for example a preferred brand. This is a suitable decision to be made in terms of purchasing at the supermarket, but it is not necessarily a 'mathematical decision'. Supermarket shopping has as its motivation, the purchase of goods for consumption; not the accurate calculation of the cost of each item.

Mathematics employed in the supermarket is guidance to enable the shopping to be done. That jpfs exhibit greater success in this setting is not due to the fact they are able to undertake better mathematics in such situations, but that they are doing different mathematics in this situation. A KS2 test script is never going to require the answer 'I prefer this brand of beans' – it will require a numerical answer with mathematical reasoning. Whilst, as a teacher, I am not solely aiming to prepare children to pass tests, this is our current measurement. Removing the tests would remove this difficulty. This is not currently being proposed; on the contrary mathematics testing in a similar form is due to remain for some time (DfE, 2011).

What Lave (1989) illustrates is reliance on a core set of skills. Much of the discussion of the strategies employed by the jpfs in the research relied on a sound sense of number; the individuals have good mathematical literacy (Venkat, 2014). In the supermarket, the jpfs were able to make judgements based on simple scenarios, for example a small price increase for a large weight increase must make the larger option the 'better buy'. The participants did not calculate the item with the lower unit price; they inferred it from the information available. Thus they show application of calculation skills, skills which largely constitute 'number sense'. The mathematical literacy in this scenario is just what people need in daily life.

Boaler (2001) employs this perspective to examine opportunities for children in secondary school mathematics. She demonstrates that children taught in 'mixed ability' groups using open ended problem solving achieve more highly in national testing. They also report greater engagement in mathematics, seeing the relevance of the subject in their wider lives. The evidence is compelling, however I hold two concerns. First, the curriculum covered is beyond the realm of what would be considered 'number sense'; does number sense need to be in place before such methods can be successful, or can number sense itself be developed in such a fashion? Second, Boaler (2001) notes that it is difficult to unpick whether the design of the curriculum encouraged deeper learning, or the style of teaching and learning promoted a "community of practice" which itself developed better learning, suggesting both could be at play.

I would suggest that mathematical reasoning or 'ability', particularly in the context of number sense is not situated, but that one's belief about being able to undertake mathematics may be situated – one's mathematical identity. With this in mind I turn to my last theoretical position, using the work of Bourdieu.

Boudieusian Social Theory

Bourdieu's central concern with how the education system reproduces social class is particularly relevant in the discussion of mathematics education. As noted earlier, academic success in mathematics acts as a gatekeeper to life's opportunities (Noyes, 2007). Importantly, such success may be cemented early in a child's education. Children who do not achieve 'level 2b' in mathematics at age 7 are most likely to be identified as having special educational needs (Every Child a Chance Trust, 2009).

In a Bourdieusian interpretation, number sense might be seen as an individual's 'taken-for-granted' response to a mathematically constructed situation. The way such a response is built through broader socialisation is now considered through exploring Bourdieu's main theoretical tools of field, capital and habitus.

Field

A useful definition of Bourdieu's concept of field is that of a site of "competing interests where there is struggle for recognition" (Green, 2013, p.145). In this context, I use'mathematics education in primary school' as the field. Mathematics education has a clear discipline which translates into classroom practices and expectations. I also propose that conceptualising the field as distinct from secondary mathematics reflects the split observed between primary and secondary schooling in pedagogy and expectation. Being 'secondary ready', having a *good* level 4 in primary school, is increasingly important to allow successful engagement with mathematics beyond this phase. With success in mathematics being so highly valued, it is worthwhile to examine the processes which allow an individual to succeed in this field.

Forms of capital

Capital is a set of social rules which mean that participation in society is not simply a game of chance, that there is not an equal playing field for all participants (Bourdieu, 1986). Bourdieu defines four key types of capital:

- economic capital - wealth, assets which can directly be converted into money;
- cultural capital - where Bourdieu sites the role of education in reproducing social class, the knowledge and educational advantages a person has.
- social capital - group membership mediated by the cultural capital of those within it;
- symbolic capital – a form of currency in a specific field, e.g. examinations

Habitus

Habitus, the ‘taken-for-granted’ way in which we act (Bourdieu, 1986) is shaped throughout one’s early socialisation and is well established on entry to education. It is an embodiment of a child’s early experiences and thus directly their social class. Walkerdine (1988) for example demonstrates how language use in the home is class dependent. Bryant (2013) also demonstrates that children from more prosperous homes demonstrate stronger mathematical reasoning skills. Habitus is more than simply accumulated experiences; it is informal skills and knowledge both informed by early socialisation and shaping the way a child acts in the world.

Where habitus does not align well with the field, children experience failure. They learn not just what mathematics is, but also how it is ‘not for them’ - that they cannot do mathematics. To accommodate the child, the curriculum may be changed - often greater rehearsal and practice of failed skills – number sense skills – producing a limited mathematical diet. Such an approach, whilst well meant and standard practice in primary schools, may be a form of symbolic violence (Bourdieu, 1986) enacted upon children from lower socio-economic backgrounds.Thus children from lower socio-economic groups are excluded from learning just the type of mathematics needed, to allow them access to the privileges mathematics provides.

In Summary

Drawing on Bourdieu’s concepts of habitus and field, I propose that number sense itself acts as a gatekeeper to mathematical success. As such, a child’s life opportunities are restricted from their earliest encounters with number sense. Does a reliance on cognitive psychology reinforce an approach to number sense which reifies a belief that some are ‘born with it’? Does trying to ‘fix’ number sense lead to an impoverished mathematical experience for some children? And how is learning mathematics as a discrete subject in the curriculum affecting children’s development of number sense? Are we situating their learning of school mathematics, or situating their identity of how they use mathematics?

Acknowledgements

I would like to thank the Department of Sport and Education at Brunel University who are funding my studentship, Dr Gwen Ineson and Dr Heather Mendick for their ongoing support and the participants at the BSRLM day conference in March 2014 for their comments and advice in constructing this paper.

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