The Interactional Treatment of Mathematical Mistakes

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In this paper we will explore the role of errors in both the teaching and the learning of mathematics. Analysis of classroom interactions show that mathematics teachers are implicitly treating errors as something to avoid despite commenting on the positive role they have in the learning of mathematics. Similarly, the students themselves treat errors as something to be avoided. This leads us to consider, and therefore explore, what possible roles errors may have in the learning and teaching of mathematics.

Keywords: classroom interaction, conversation analysis, errors

Introduction

There have been recommendations recently both in the research literature and education policy that have encouraged mathematics teachers to use mistakes in their teaching (DfE, undated; Swan, 2005). This is often advocated as an alternative to avoiding mistakes (Swan, 2005). The arguments offered are that encouraging the exploration of errors can result in deeper, longer lasting learning compared to approaches that focus on avoiding errors (Bell, 1993). In this paper we initially report our research findings that examine how errors are handled in whole class interactions by both teachers and students. This then leads to a theoretical exploration of what it means to use errors in mathematics teaching and the role that errors may have in the learning of mathematics.

The original data for the study described in this paper consisted of video recordings of 17 mathematics lessons with four different teachers from different schools. The students were either aged 12-13 years old or 13-14 years old. The focus was on interactions that involved the whole class, and all these occasions were transcribed.

The analysis of the classroom interactions uses a conversation analytic approach based in ethnomethodology. One key principle of this approach that influences the analysis discussed in this paper is that it is how the participants themselves treat different turns at talk that forms the basis of analysis. The resulting analysis is an in-depth description of what the teachers and students do in their turns and how they do this, and no inference of intent or thought is made. This is most evident in this paper in how we define an error. An extract is identified as containing an error if the teacher or the students treat it as such, either explicitly or implicitly, in the interactions that occur. This is in contrast to most research into errors in mathematics where errors are defined in terms of their mathematical content or relevance from the perspective of the researcher. The other noticeable difference in the presentation of our findings is that details such as the teacher's gender, experience, nature of the school, nature of the class and so forth are omitted as they are only seen as relevant to the interactions if the teacher or the students themselves make them relevant. As such, for ease of reading, all teachers are referred to using masculine pronouns and all students are referred to using feminine pronouns but these do not represent the gender of the participants.

Our initial research) shows that both teachers and students predominantly treat errors as dispreferred (see Schegloff (2007) for details of how the term dispreferred is used in conversation analysis). In other words, errors are treated as something to be avoided by both teachers and students. In fact, teachers and students do a great deal of interactional work, using a variety of strategies to avoid directly negatively evaluating errors (see Ingram (2012) for some examples of these strategies). This leads to an apparent paradox. On the one hand, we would argue that errors are an essential part of learning mathematics but at the same time those involved in the learning and teaching of mathematics are treating errors as something to be avoided.

There are two occasions where errors are not treated as something to be avoided and these both occur in the lessons of one of the teachers in the study. The first type of interaction is where the teacher poses a question that receives multiple and contradictory responses from his students. For example, in one extract he asks 'what is the smallest prime number' and the responses include zero, one and two. The teacher in these interactions does not treat any of the responses as either correct or incorrect, instead he uses his turns to prompt and maintain the point of contention (Gellert, submitted). The students who participate in the interaction, on the other hand, begin to justify and explain their responses and baldly negatively evaluate (saying "no" with no pauses, hesitations or other discursive markings) the contradictory responses offered by their peers. In the example offered, the students offering the answer of 'one' and the students offering the answer of 'two' both defend their answers, offering reasons and explanations for why their response is right.

The second occasion occurs when the errors relate to the use of procedures or calculations that are not directly relevant to the topic of the interaction. This is the only time in the full data set where the teacher directly and immediately negatively evaluates the answer and he continues to ask more students the same question until he accepts an answer as correct. Whilst there are other occasions in the data set where the errors relate to arithmetic or procedures, this is the only occasion where the teacher does not treat the error as something to be avoided. The other errors that occur in this teacher's lessons are treated as dispreferred.

Subsequently, our research has examined extracts where teachers are using errors explicitly in their teaching. Again, in these extracts, the teachers and students predominantly treat the errors, through the structure of the interactions, as something to be avoided. They use a range of additional, more explicit strategies to do this. For example, distancing themselves from error by attributing the error to an unidentified third party, emphasising in the content of the interactions that the errors are being discussed so that they are not made by the students and apologising for making errors.

This led us to explore further the role of errors in the teaching and learning of mathematics and whether the message that errors are to be avoided does in fact contradict this role. Several conjectures arose, particularly in response to the different interactional strategies used by the different teachers. Firstly, the role of errors in the teaching and learning of mathematics could relate to the nature of mathematics in that particular mathematics classroom. Secondly, the role of errors relates to the nature of the error as inferred by the teacher and demonstrated through the ways in which the error is handled in the interactions. Finally, the role of errors may differ depending upon whether they are being used explicitly and deliberately by the teacher or when they occur naturally during an interaction.

The first of these conjectures is explored elsewhere (Ingram, 2012; Ingram et al. 2011). The other two can only be explored to a limited extent through the analysis of the classroom interactions collected due to the rarity of these events. These conjectures therefore led to the design and implementation of a survey of teachers' beliefs about the origins of students' errors, their role in the learning of mathematics, and their role in the teaching of mathematics, with the intention of identifying contrasting cases for further exploration through observation and interviews.

How errors are used in the teaching of mathematics

In the data from this study, errors are most explicitly used as a pointer to students as to what the common errors are within a particular topic and these errors often relate to procedures and calculations. For example, in one lesson a student offers $3^2=6$ as a common error that 'other' students might make, and in another a student uses the frequencies to calculate the range of grouped data rather than the values needed. The emphasis throughout is on raising students' awareness of errors. The content of the interactions surrounding these errors focuses on remembering not to make these errors themselves or again. Hence, these instructional practices emphasise the avoidance of errors in mathematics.

In the literature, two main approaches to the use of errors commonly occur. The first is as a diagnostic tool and the second to generate a cognitive conflict. The literature on both of these focuses on the design of tasks or assessment items rather than the interactions that might accompany these. The research into the diagnosis of particular types of errors within certain topics in mathematics is based on the premise that these errors illustrate particular misconceptions that students might have (Smith, DiSessa & Roschelle, 1993) and that this information can then be used by teachers to inform how they will adapt their teaching to support these students (Bennett, 2011).

A great deal of this research has subsequently informed the design of tasks and activities that either direct students' attention (Mason, 1998) to these errors or misconceptions or their own understandings or conceptions, or to that have the potential to generate a cognitive conflict for the students that may hold particular misconceptions. This research not only focuses on the design of tasks but also the implementation of these tasks in the classroom. Swan (1983) emphasises the importance of tasks that can give rise to cognitive conflict and the value of the subsequent discussions the students are involved in. The value of discussion between peers surrounding the cognitive conflict is seen as useful in supporting students to become aware of the conflict and to feel the need to address the inconsistencies or incompleteness of their own conceptions. For example, Stylianides and Stylianides (2008) talk about the role of 'pivotal counter examples' in students' developing understanding of proof in mathematics and the use of 'conceptual awareness pillars', usually in the form of prompts by the teacher, to support students in developing their awareness of the cognitive conflict.

In our own data one teacher in particular regularly asks a question related to a common error in mathematics where his students offer multiple contradictory responses such as the prime number example offered above. He uses these questions to generate and argument or debate between his students as they justify or explain their responses to their contradicting peers. In these situations, the errors are not treated as something to be avoided but instead provide the need for the students to justify and explain their answers without prompting from the teacher.

On final use of errors that does not occur in our data is the use of errors to explore new areas of mathematics. Borasi (1994, 1996) talks about using errors as 'springboards for inquiry' and offers examples where activities that explicitly focused on the analysis of errors led to the students engaging in mathematical thinking, communication and problem solving as well as leading to a deeper understanding of the mathematical constructs being considered and consideration of wider mathematics not anticipated by the teacher. The students also showed creativity and mathematical intuition and took control over the mathematics. Borasi does also point out that students using errors as springboards for inquiry may prove to be quite challenging for the teacher, particularly when not anticipated.

Conclusion

The way that errors are handled interactionally can influence the nature of mathematics that students experience and also can have consequences on how students are viewed. For example, Bauersfeld (1992) talks about Jungwirth's study where boys used 'concealing strategies' to trivialise their errors. These strategies focused on how these errors were treated interactionally by the students and led to different perceptions of the competencies of boys and girls in the class.

Importantly, the way that teachers handle errors contributes towards the establishment of norms for mathematical behaviour in their classrooms. In particular, their behaviour delineates what is or is not appropriate both mathematically and in the way that mathematics is communicated in that particular classroom (Cobb et al. 1992; Sfard, 2007). Within this is a consideration of the nature of tasks and activities that students will undertake in the lessons, and whether they be emphasising skills and knowledge or problem solving and reasoning (Borasi 1994).

When teachers talk about using errors explicitly they could be referring to any of these pedagogic approaches, some of which emphasise the avoidance of errors, whilst others emphasise the positive role errors can have in the learning of mathematics. Only one of the teachers in this study explicitly used errors in a way that does not treat them as dispreferred, yet he also treated most of the errors that occurred in his classroom as dispreferred. This suggests that a combination of treating some errors as to be avoided and others as part of learning mathematics may be possible. Whilst the conjectures identified above still need further exploration, there is some evidence that the nature of the mathematics within the interactions can influence how errors are handled both implicitly and explicitly. More research is needed to explore the influence interactions surround errors may have on the teaching and learning of mathematics.

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