Calculating: What can Year 8 children do?

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This paper reports on the findings from a sample of 985 Year 8 pupils who were given age-appropriate calculation questions, and examines the range of strategies they used. The introduction of the National Numeracy Strategy (NNS) Framework (DfEE, 1999) brought together research and recommendations which sought to improve primary age children's calculation strategies (e.g. Plunkett, 1979; Thompson, 1999). The Year 7, 8, 9 Framework (DfEE, 2001) for Key Stage 3 was the conduit for sharing this with secondary schools. Fourteen years later, just as the mathematics curriculum in the UK has been revised again, this study considers how pupils in Y8 respond to questions for each of the four operations. While over the last decade a range of strategies have been promoted, and recent research (e.g. Nunes, Bryant and Watson, 2009) has focused in particular on methods of calculation, our study suggests that Y8 pupils still struggle to select an efficient and effective strategy.

Keywords: calculations, strategies, mathematics, Year 8, algorithms

Introduction

As pupils move from primary to secondary education there is an expectation that they have developed an understanding of number and indeed 'made sense of the operations' (Anghileri, 2007) by this stage. By Year 8 the use of an effective, efficient and accurate strategy, whether this is an algorithm or an informal written method, for each of the four operations is expected. According to Plunkett (1979), standard algorithms are symbolic, automatic, contracted, efficient, analytic and generalisable. They are almost always recorded vertically and the numbers involved in the calculation do not have a quantitative value but rather a digit value (Thompson, 2000) and are much more abstract in nature. In contrast, and for clarity within this research, informal written methods are taken to be those that develop from concrete representations, mental methods and the use of jottings, and include strategies such as partitioning, the use of number lines and the grid method.

Thompson (2001) states "there are many articles on the teaching of written algorithms but the majority appear to be based on 'reflection' rather than 'research'' (p.17). The purpose of this research is to explore the range of strategies that are the outcomes of that teaching. It would be expected that pupils' responses represent a range of different points along the continuum between informal written methods and standard algorithms, because mathematical maturity, attainment and ability to deal with abstract concepts varies. Kilpatrick, Swafford & Findell (2001) describe calculation fluency, sometimes called computational fluency, as the ability to quickly and accurately perform arithmetic problems. Through examination of the outcomes, the intention is to be able to at least comment on the suitability of particular calculation strategies that are taught and reflected in proportions of pupils who are successful.

The pupils involved in this study are all ‘products’ of the NNS (DfEE, 1999). By 2004, when these Y8 students were beginning their Reception year, the Strategy was well-established in English primary schools. By the time they began their secondary education in 2011, the likelihood is that they had been exposed to strategies such as number lines, grid methods, chunking and partitioning. This paper considers how far pupils’ ability to calculate has moved forward since the inception of the National Numeracy Strategies (DfEE, 1999; 2001). We explore the range of strategies they are using.

Methodology and context

The research reported here is a study carried out at the end of the Autumn Term of 2012. Data was collected from questions completed by Year 8 pupils from 7 secondary schools from a range of contexts throughout Norfolk. Responses to four questions, taken from the Qualifications and Curriculum Development Agency (QCDA) optional tests, from each of 985 pupils, were analysed for their calculation strategies. One question each for addition, subtraction, multiplication and division, presented horizontally, was used. The four questions selected were chosen as they had no context, and required pupils to perform a calculation (as opposed to less abstract problems that involve some interpretation before a calculation can be carried out).

Findings

The categories for strategies that pupils used are based on previous Year 5 research carried out in Norfolk primary schools (Borthwick & Harcourt-Heath, 2007; 2010; 2012). The Year 5 research used strategies based on the NNS (DfEE, 1999), and although the Year 8 research in this paper does not offer any comparisons, future studies may do so. Where the strategy used could not be determined, where it fell outside the categories or where no response was provided, these have been included in the sub-category ‘other’. However, while we acknowledge that since the first piece of Year 5 research in 2006 calculation strategies have evolved, we do not wish to change the categories at this stage.

Addition

Table 1: Results from 985 children for addition question.

Item 44.8 + 172.9	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		19 (2%)		
Standard algorithm	571	134	81%	19%
Number Line	0	1	0%	100%
Partitioning	80	56	59%	41%
Expanded vertical	60	16	79%	21%
Answer only	8	11	42%	58%
Other	12	17	41%	59%

Totals	731	254	74%	26%

Overall 74% correct / 26% incorrect

Responses were recorded as 'standard algorithm' where pupils had re-written the calculation vertically and the answer appeared in a single horizontal row under this. 70% of calculations were completed in this way with a success rate of 81%. An expanded vertical method was equally successful with 14% of pupils selecting this strategy. Partitioning was less successful with 59% achieving a correct answer from the 8% of the whole sample selecting this method. The lowest rates of success were seen where pupils provided an answer only or 'other' strategy.

Subtraction

Table 2: Results from 985 children for subtraction question.

Item 82 – 3.8	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		33 (3%)		
Standard Algorithm – decomposition	235	237	50%	50%
Standard Algorithm – equal addition	0	0	0%	0%
Number Line	104	30	78%	22%
Negative Number	0	0	0%	0%
Counting Up	17	1	94%	6%
Counting Back	136	41	77%	23%
Answer only	42	34	55%	45%
Other	29	46	39%	61%
Totals	563	422	57%	43%

Overall 57% correct / 43% incorrect

Higher proportions of correct responses fell into three categories. 14% of the whole sample of students selected a number line and 20% employed counting up or counting back with more than three quarters of those pupils gaining a correct response. By comparison, half of the pupils choosing to use the standard algorithm answered incorrectly (24% of the whole sample of pupils). This was less successful than where the answer only was recorded.

Multiplication

Table 3: Results from 985 children for multiplication question.

Item 23.4 x 4.5	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		77 (8%)		
Standard Algorithm	17	110	13%	87%
Grid Method	108	416	21%	79%
Expanded Vertical	3	7	30%	70%
Two partial products only		59	0%	100%
Answer Only	5	23	18%	82%
Other	82	78	51%	49%
Totals	215	770	22%	78%

Overall 22% correct / 78% incorrect

Almost 8% of pupils did not attempt this question. More than half of the pupils answering chose the grid method but 79% of these were not successful. Fewer selected the standard algorithm (13% of the whole sample), with 87% unsuccessful. The highest proportion of correct responses was recorded from 'other' strategies with half of the 16% of the whole sample of pupils in this position.

Division

Table 4: Results from 985 children for division question.

Item 91.8 ÷ 17	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		268 (28%)		
Standard Algorithm	164	230	42%	58%
Chunking Down	10	24	29%	71%
Chunking Up	19	66	22%	78%
Number Line	5	28	15%	85%
Answer Only	12	36	25%	75%
Other	16	107	13%	87%
Totals	226	759	23%	77%

Overall 23% correct / 77% incorrect

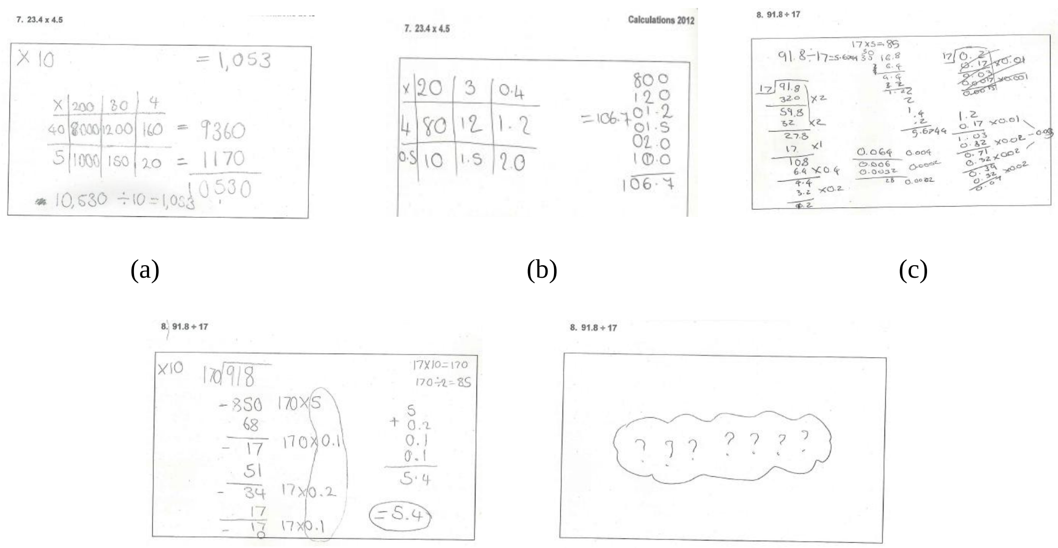
More than a quarter of pupils did not attempt this question. The highest number and proportion of correct responses came from the standard algorithm but this also left almost a quarter of the whole sample with an incorrect response. The spread of responses across the remaining strategies was fairly even, as was the proportion achieving a correct answer. However, this success represented only 6% of the whole sample.

Discussion

Whilst addition was the most successful, with 74% gaining a correct answer, this still demonstrates that 1 in 4 children could not complete an age related addition calculation. The success of the standard algorithm could suggest that students have a good understanding of place value and are applying this effectively, but it could be masking the use of an algorithm without this level of understanding. The results from partitioning suggest that the understanding of place value is not so secure, although those students employing an expanded vertical method were more successful.

In terms of proportions of pupils gaining a correct answer, subtraction was the next most successful after addition. However, this only represented just over half of the pupils. Many pupils chose an inefficient strategy to solve what would be considered to be a 'taking away' question as opposed to a 'finding the difference' one. Based on the strategies and progression suggested by the Key Stage 1, 2 and 3 frameworks (NNS, 1999; 2001), the number line could have been an expected method. This research suggests that where it was employed pupils were more successful, whereas those selecting the decomposition standard algorithm appear not to have had the understanding to enable them to utilise this effectively.

It is surprising perhaps that the multiplication question was not answered any more successfully than the division, which is often perceived to be more difficult. Watson (2012) suggests that, "division, too often, divides those who are going to be able to achieve in mathematics from those who may not" (p.29). Although more than half of the pupils selected the grid method for multiplication, the lack of success with this seems to be attributable either to their inability to calculate partial products (particularly with the decimal numbers), or their lack of understanding that where they had altered decimal numbers to whole numbers in order to calculate, an appropriate adjustment to the answer would be required (Figure 1(a)). Incorrect calculation of the partial products is exemplified in Figure 1 (b) below, where the calculation of 0.5×0.4 has been incorrectly entered into the grid as 2.0 rather than 0.2, and additionally the error from 0.4×4 recorded as 1.2 rather than 1.6 which gives an overall incorrect answer.



(a) (b) (c)

(d) (e)

Figure 1: Examples of pupils' calculations

For division, the number of students not attempting this question and the spread across the range of strategies selected suggests that pupils were least likely to have a method on which to draw. The standard algorithm was most successful, although only 2 in 5 of these pupils recorded a correct response. Figure 1(c) is a typical example within this research of a response from a pupil who seems to have some knowledge of strategies for division without being able to follow these through to a successful conclusion. In Figure 1(d), the use of what the NNS (DfEE, 1999) referred to as 'chunking' demonstrates that this transparent method has the potential to provide an effective strategy. Contrast this with the response provided in Figure 1(e).

Conclusion

Although we acknowledge that the age-related questions were for Year 8 outcomes, and this research took place at the end of the Autumn Term, the proportions of children unable to gain a correct answer across each of the four calculations was surprisingly high. Whilst it is often assumed that standard algorithms are ultimately the most efficient calculation methods, this research suggests that overall, these Year 8 pupils were not able to utilise them successfully. As Anghileri (2000) notes, "all pupils are expected ultimately to use efficient written methods for calculating but the only way such methods can be meaningful is if they are developed progressively to support and extend mental strategies" (p.108). Efficiency could be either a standard algorithm or equally another written method that is used effectively (e.g. number line, grid method).

This research suggests that the pupils have either been moved too quickly away from mental and the more informal written methods, before they have sufficient maturity to work with the symbolic and abstract representations necessary to use the standard algorithms, or that the pupils did not have the underlying understanding that would enable them to access standard algorithms more successfully.

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