‘Understanding mathematics in depth’: an investigation into the conceptions of secondary mathematics teachers on two UK subject knowledge enhancement courses

Mary Stevenson
Liverpool Hope University

This report is of an investigation into conceptions of ‘understanding mathematics in depth’, as articulated by two specific groups of novice secondary mathematics teachers in England. Most participants in the sample interviewed have completed one of two government funded mathematics subject knowledge enhancement courses, which were devised with an aim of strengthening students’ understanding of fundamental mathematics. Qualitative data was drawn from semi-structured interviews with 21 subjects. The data reveals some key themes common to both groups, and also some clear differences. The data also brings to light some new emergent theory which is relevant in novice teachers’ contexts. To provide background context to this study, quantitative data on pre-service mathematics Postgraduate Certificate in Education (PGCE) students is also presented, and it is shown that, at the university in the study, there is no relationship between degree classification on entry to PGCE, and effectiveness as a teacher as measured on exit from the course. The data also show that there are no significant differences in subject knowledge and overall performance on exit from PGCE, between students who have previously followed a subject knowledge enhancement course, and those who have followed more traditional degree routes.

Keywords: SKE courses, PGCE courses, mathematical knowledge for teaching, understanding mathematics in depth, pedagogical content knowledge

Background

The Mathematics Enhancement Course (MEC) forms one strand of the initiatives developed by the UK government in recent years to enhance subject knowledge preparation for entry to secondary teaching. The MEC sits within the wider framework of Subject Knowledge Enhancement (SKE) courses which have also included programmes in other subjects. The MEC is aimed at graduates who wish to train as secondary mathematics teachers, whose mathematics background is insufficient for entry to Postgraduate Certificate in Education (PGCE) or other routes to Qualified Teacher Status, but who otherwise are suitable candidates for initial teacher education programmes. The MEC has a strong focus upon the development of subject knowledge, and there is also in many courses implicit reference to pedagogical content knowledge (PCK). A significant proportion of secondary mathematics (PGCE) students now enter the course from a MEC/SKE background.

The Mathematics Development Programme for Teachers (MDPT) is a part-time course for serving teachers. It arose following the recommendations of the Smith Report (2004) that significant opportunities for professional development should be

made available to serving mathematics teachers, in particular those who are non-subject specialists. Thus, it is aimed at teachers who are already teaching mathematics at secondary level, but who did not originally qualify in the subject. It is also primarily a subject knowledge enhancement course, but, as in the MEC, there is an overlap with PCK. Since 2011, the MDPT has fallen within the Subject Knowledge Enhancement (SKE) category, so that the government now funds what are termed pre-ITE and post-ITE SKE courses.

Location in the field

There is a growing literature concerning mathematics in and for teaching. Categorisation models such as those developed by Shulman (1986) and Ball, Thames & Phelps (2008) provide a useful framework and vocabulary for discourse about different forms of teacher knowledge and how they overlap and interact, in particular drawing distinctions between subject matter knowledge (SMK) and PCK. Work by Hill et al. (2008), Baumert et al. (2010), and others in developing ways of measuring mathematical knowledge for teaching, show links between teachers' mathematical knowledge for teaching and their students' achievements.

Models such as the 'Knowledge Quartet' developed by Rowland, Huckstep & Thwaites (2005) give an insight into the dynamic nature of knowledge for teaching. Ruthven (2011) comments on the importance for teachers of reconstructing existing knowledge as against acquiring new knowledge. The idea of mathematics as a process, and mathematical knowledge as a way of being or acting, is espoused by Watson & Barton (2011)). From this perspective, growth of knowledge takes place through being involved in doing mathematics and 'being mathematical'. These perspectives also highlight the importance of the mathematics teaching community of practice. This links to Hodgen's (2011) comments on knowledge for teaching as situated in the classroom context, which in turn echoes Rowland, Turner, Thwaites & Huckstep (2009, p. 24) assertions that aspects of knowledge come together "in the teaching moment". Beswick's (2005) position on knowledge as a single construct, Davis's (2011) view of mathematics knowledge as a "learnable disposition" (p. 1507) and Askew's (2008) call for a "mathematical sensibility" (p. 22) all advance the debate along similar lines.

Ma's (1999) work has developed key theory about the importance for teachers of 'profound understanding of fundamental mathematics' (PUFM). Ma offers clear recommendations for mathematics teacher education courses in terms of the integration of subject and pedagogical knowledge. Adler explores a similar theme in a different way - understanding mathematics in depth (UMID) - in both the South African (Adler & Davis, 2011) and UK (Adler, Hossain, Stevenson & Clarke, 2013) contexts.

This study focuses on the voices of the teachers themselves. Mathematics education tutors place a high value on understanding mathematics in depth, and so it is relevant to investigate how course participants, especially those on programmes with an explicit aim of developing deep understanding of mathematics, view this idea.

Contextual quantitative data

Data was explored to investigate (1) the nature of the relationship (if any) between degree classification and success at PGCE at one university, and (2) whether there are any differences in subject knowledge and in overall achievement upon completion of PGCE, between former Mathematics Enhancement Course (MEC) students and

others. Data was collected from three cohorts of mathematics PGCE students at one university, between 2006 and 2009. All cohorts comprised students from both MEC and degree mathematics backgrounds.

Comparison of PGCE entry and exit scores

Paired data scores for entry and exit for 95 PGCE mathematics students were analysed. These 95 students were from the 2006-7, 2007-8 and 2008-9 cohorts at the university in this study. The relationship between entry and exit scores was investigated using Kendall's tau test (non-parametric data). Analysis shows that there was no discernible relationship between students' level of academic qualification on entry to the PGCE course, and their level of success on the PGCE course, as measured by QTS score ($r = 0.129$, $n = 95$, $p = 0.1$). Therefore, from this data sample, there is no evidence to suggest that a high scoring degree classification may be a predictor of success in initial teacher education. This is in line with the findings of Tennant (2006), and is supported by Monk (1994) and others. There are some important limitations in the use of this data relating to objectivity; however in the absence of other available measures, it is practical to use the data available and to proceed, albeit with caution.

Comparison of QTS scores on exit from PGCE: MEC and non-MEC students

Analysis of overall QTS grades

The difference between the scores for former MEC students ($M = 1.77$, $SD = 0.541$) and non-MEC students ($M = 1.68$, $SD = 0.479$) was not statistically significant; $t(104) = 0.937$, $p = 0.35$ (two-tailed independent-samples t-test).

Analysis of subject knowledge grades

The difference between the scores for former MEC students ($M = 1.71$, $SD = 0.638$) and non-MEC students ($M = 1.55$, $SD = 0.544$) was not statistically significant; $t(104) = 1.400$, $p = 0.165$ (two-tailed independent-samples t-test).

This evidence suggests that the differences between MEC students and others, in terms of their outcomes upon completion of PGCE as measured by QTS scores, are sufficiently small to be disregarded. This indicates that MEC students and degree mathematics students at the start of their teaching careers are more or less equivalent in terms of their gradings against QTS standards.

Main study: research design and methods

The research questions addressed in this study are: (1) How is 'understanding mathematics in depth' conceptualised by two particular groups of novice mathematics teachers? (2) What are novice mathematics teachers' beliefs about how 'understanding mathematics in depth' is attained? (3)What other key themes are prominent in the teachers' discourse in relation to their own training and preparation for mathematics teaching, and their experience of mathematics teaching?

Sample

A sample of current and former PGCE mathematics students (12) and MDPT teachers (9) were interviewed, with participants drawn from each of the 2007-08, 2008-09 and 2009-10 cohorts. Both PGCE students who had formerly taken the MEC and those

who had degree mathematics were included. Interview questions probed, amongst other things: participants' experience of mathematics during their training/course and how this prepared them for their own teaching; topics which the respondent understands well and where this understanding came from; what 'understanding maths in depth' means to the participant.

Analysis

Four stages of analysis of interview transcripts were undertaken. At the first stage, comments that were seen as interesting / significant were identified. At the second stage, annotations relating to emerging codes were added. The third stage involved tabulation of responses to questions and cross-participant analysis. Finally the interviews were re-read holistically at the fourth stage of analysis, and key themes emerging from the interviews were identified.

Results

PGCE students' growth in pedagogical content knowledge (PCK)

Analysis of responses to items where respondents talk about topics that they have taught shows that former PGCE students who are new to teaching highlight the importance of thorough planning and preparation. They also cite some clear examples of their own learning in terms of PCK, e.g., asking appropriate questions, pacing lessons, understanding the needs of the learners. On the whole, their growth in knowledge seems to be more located in PCK, whereas their mathematical content knowledge is already secure. Note that they have all studied mathematics at degree level or MEC prior to commencing PGCE.

MDPT teachers' growth in subject matter knowledge (SMK) and discourse about identity

MDPT teachers focus upon a growing confidence in teaching mathematical material that they previously were not confident with. Thus we see a growth in mathematical content knowledge and specialised content knowledge more so than a growth in general teaching strategies. Note that most of this group were already experienced teachers. MDPT teachers' discourse has resonance with issues of identity, seeing themselves in a different way – as maths teachers – as a result of following the course. This discourse around identity is also noted in the findings of Crisan and Rodd (2011) in their study of MDPT teachers. Graven (2004) also comments upon the shift in teachers' perceived identities, from 'teacher of mathematics' to 'mathematics teacher'.

Understanding mathematics in depth (UMID) as 'knowing why' and 'being able to communicate'

The majority of respondents conceived 'understanding mathematics in depth' as 'knowing why' and also in terms of being able to communicate ideas to others. This is supported by work by Adler et al. (2013) whose study of MEC students reveals that their perceptions of 'understanding mathematics in depth' strongly feature mathematical reasoning or knowing why. Adler et al. also detect a positive

mathematical disposition threaded through much of the students' discourse, which is also evident in this study - in the case of both MEC and MDPT teachers.

Development of UMID through investigation of problems and spending time

Various comments were made relating to immersion in the subject, and spending time working on mathematics. This supports Watson and Barton's (2011) ideas of teachers enacting mathematics, being involved in the process of mathematics. Mathematics subject knowledge as a 'learnable disposition' (Davis, 2011) and Askew's (2008) 'mathematical sensibility' are also visible here.

Other key emerging themes

Additionally when reading the interviews holistically, some key overarching themes emerge.

1. Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them.
2. Style of teaching is different when the material is new to the teacher.
3. Teachers can develop deep understanding of a topic through preparing to teach it
4. Growth in knowledge and confidence can alter one's perceived identity
5. Reconstructing existing knowledge can result in transformation of teaching approach

Themes 1, 2 and 3 speak of the early stages of knowledge growth and development that occur when a teacher prepares new material to teach, perhaps for the first time, or early in their experience. Several respondents spoke about making an effort to prepare material that previously they had not felt comfortable with, and then emerging with a new confidence in those concepts, to the extent that they would now regard them as areas of strength, not weakness. Especially during the first few years of their careers, new mathematics teachers are engaging in this simultaneous development of their own knowledge (both SMK and PCK) and that of their students, as they prepare material to teach.

Themes 4 and 5 relate strongly to changes or transformation that can take place when an established teacher encounters new concepts, or familiar concepts presented in a new way, and is offered the opportunity to try out new ways of thinking and working. These themes, whilst present in the background to the PGCE respondents' discourses, were most prominent in the responses from the MDPT teachers.

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