Teacher visualisation loss mid explanation: an issue when teaching geometry

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From several years of teaching an in-service masters course 'Learning geometry for teaching', a set of teacher self-report data has been accumulated that records incidences of teachers not being able to 'see' a theorem or geometrical relationship that they were in the middle of explaining or discussing. This paper uses neuroscientific understanding of the self-oriented (egocentric) and other-orientated (allocentric) processing pathways in the brain as a theoretical lens to start to understand this phenomenon. It will be argued that 'visualisation loss mid explanation' needs not be due to lack of teacher mathematical knowledge. The related issue of teacher defence against the discomfort of loss of geometrical insight is also raised and the question of whether a consequence of this defence might be avoiding geometrical practice in the classroom is discussed.

Keywords: Visualisation, teaching geometry, communication.

Orientation

The diagram in Figure 1. represents the theorem *Triangles AMF and CDM are of equal area*. This can be seen – in the sense of 'visualised' – by rotating either triangle around M through a right angle (in either sense). The result of such a rotation is that equal-lengthed sides of the triangles are in a straight line, the triangles have two common vertices. The common vertex that is not on the bases' line defines the height of both triangles. Hence areas are equal.

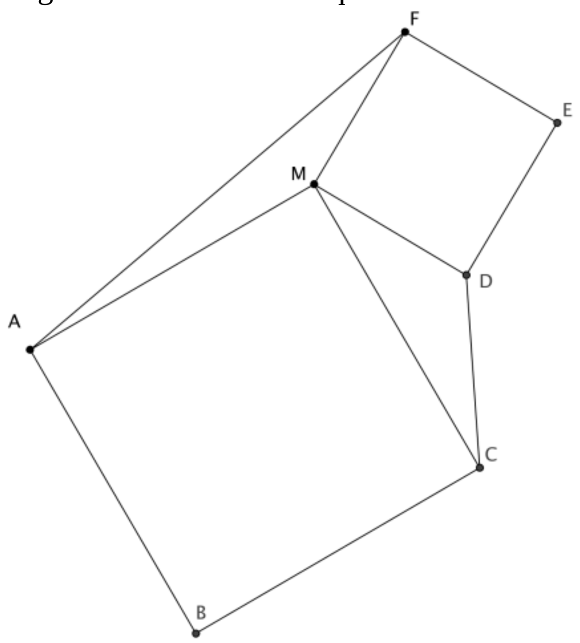

Figure 1. Theorem: the triangles are equal in area.

Area of a Euclidean triangle? I know about that! Furthermore, I am also familiar with the Figure 1 diagram together with the transformation (rotation) that provides a convincing way of seeing the triangle area equality. However, when explaining this transformation to students I have experienced turning from the students back to look at the Figure 1 diagram on the board and not being able to 'see' that the triangles' areas are equal; I know why they are equal, but in the moment I fail to *see* that they are equal.

This paper is concerned with why, when teaching, we are vulnerable to losing 'sight' of a geometrical theorem, such as this one. The outline of the paper is as follows: a short presentation of some literature on visualisation and geometry teaching is given. Then the 'tools for thought' used to analyse the phenomenon are introduced and applied together with presentation of data and discussion (for further discussion see http://geometryforteaching.weebly.com/bsrlm-16-nov-2013.html).

Some relevant literature

Reports from students and fellow teachers about similar experiences are frequently offered, yet reports in the literature are scarce. One relevant report 'To see or not to see' (Gal & Linchevski, 2010) employed various perceptual/cognitive theories to analyse problems lower secondary students had in geometrical reasoning in order that teachers should have theoretical, rather than craft, knowledge in their interventions. Their results and hypotheses are consonant with those of this paper – in particular they attend to brain science results on the difference between visual and linguistic processing – although they do not integrate affective issues (e.g., emotions, moods or defences). There are a few reports that discuss how affect and reasoning interact. For example, in Rodd (2010) I reported on the interaction of visualisation capacity and affect. Gómez-Chacón (2000) catalogues the affective and cognitive responses of individuals to mathematics; while the theoretical terms used are different, Gómez-Chacón explains that the case study student Adrian likes practical work in geometry but is unable to produce symbolic reasoning. Another study, by Presmeg and Balderas-Cañas (2001), describes participants creating visual images which are personal and affect-laden (though their work was not focussed on geometry). Barrantes and Blanco (2006) studied pre-service teachers and found that they considered geometry more difficult than other subjects (though these participants' notions of geometry was more like measurement than theorem proving). While there is acknowledgment that affect and cognition interact intimately, and curiosity about this interaction, there is no one approach to develop our better understanding.

Tools for thought

If it is accepted that: when teaching we are vulnerable to losing 'sight' of a geometrical theorem, what are suitable tools of thought, or academic disciplines, with which to investigate the issue? Two different 'tools' are tried out here: neuroscience, which is a discipline that investigates how information is processed by living beings and psychoanalysis which is a discipline that investigates how unconscious processes manifest themselves in peoples' affects and behaviours. Both of these theoretical bases are enormous and reviews of these fields are not within the scope of this paper. Neuroscience develops explanations of how information is processed at a physical level, in the sense of what happens when attention switches rather than whether I noticed it switched. However, switches of attention can be turned unconsciously, hence the field of psychoanalysis is relevant.

The central query of this paper is: Why is it that geometrical visualisation and talking with students are difficult to do together? And the aforementioned disciplines are employed to answer it as follows. The key result from neuroscience used is that there are two distinct processing pathways in the brain – self-centred and other-centred; changing from visualising to communicating involves switching these attention-pathways; this switching of attention contributes to a visualisation getting lost. The key result from psychoanalysis used is that all affectively responsive people defend themselves against anxieties, much of this defending is done unconsciously and these defences contribute to the orientation of attention. Therefore, when the visualisation is lost, the teacher is defensive and looking 'outwards', towards the students, not 'inwards' towards re-gaining of the vision.

Neuroscience of attention

Information is processed via two anatomically distinct (yet intimately bound) processing pathways in the primate brain that correspond to different forms of attention characterized by 'concentration on' and 'receptivity to'. The first of these, concentration, is a self-willed, 'top-down' act. This sort of attention is called *egocentric*; the contents of concentration are subjective. In contrast, attention that processes the seemingly objective, external information from the 'bottom-up' is called *allocentric* (other-centred) (Austin, 2008). Information processing activity can be tracked and recorded by brain scans (Kravitz, Saleem, Baker, & Mishkin 2011). Doing geometry requires concentration on holding mental images and doing teaching requires attention to other people. Austin notes the receptive "processing stream is also poised to infuse *meaning and values* into whatever it perceives out there" (Austin, 2009, p.17 italics in original).

Recent results from the Lab for Affective Neuroscience (http://psyphz.psych.wisc.edu/) include the classroom-relevant result that "stress alters visual attention" (Shackman, Maxwell, McMenamin, Greischar & Davidson, 2011, p.4) so tasks that require spatial working memory (e.g., visualising a theorem) are hampered by stress as this disrupts the regulation of attention. 'Stress' in this context should not be read as a state to be denied, as some stress is part of teachers' experience of classroom life. Furthermore, Shackman's team found that anxiety disrupts spatial working memory more than it disrupts verbal working memory (Shackman et al., 2006).

This conception of information being processed in complementary ways may help explain the difficulty teachers of geometry can have with concurrently holding in one's own mind a 'vision of a theorem' while intending to explain that vision to others. This is because seeing the theorem is a first person perspective event (egocentric) but explaining it to others is a third person perspective event (allocentric). Switching attention 'from it (theorem) in relation to me' to 'it from the students' perspective' through communication requires a switch, in the brain-science model, of processing pathway. Why not switch and then switch back? Yes, this would be good. But, even with familiar problems, it is often not possible. Why?

The hypothesis offered here is that defence mechanisms are triggered, which makes that switching pathways more difficult. In a typical secondary school mathematics classroom where students' understandings are assessed and developed by verbal questioning and discussion, the teacher's allocentric pathway is more active and dominant (Shackman et al. 2006).

Defence mechanisms

Defence mechanisms were initially mooted by Sigmund Freud as unconscious responses – 'repression' for example – to deal with life's anxieties (Freud, 1896). Melanie Klein (e.g., Waddell, 1998) reformulated the idea to posit that not only do all of us defend against anxieties using defence mechanisms, whether we are aware or not, but that 'mechanisms' of psychological defence are integral to identity. Hence, defence mechanisms are essential for survival and not a priori 'negative'. They are called into play unconsciously for protection, for example, when a teacher is caught between a personal geometrical visualisation and a social and institutional need to communicate with students.

A classroom teacher will need to protect his/herself as part of the job, no matter how favourable their environment or his/her relationships with students and mathematics. In the context of this paper, the investigation concerns what happens when an aspect of the teacher's subject knowledge becomes suddenly unavailable: a theorem initially seen within a diagrammatic representation is 'not seen' just at the point at which the teacher wants to draw the students towards the theorem. In the previous section, it was argued that visualising a theorem employs the egocentric pathway and interacting with students employs the allocentric pathway. Furthermore, psychological defence mechanisms inhibit switching rapidly from one pathway to the other in such a classroom situation. Why? In a conversational classroom, relationships with students are central. The mathematics teacher's job is to draw students into mathematical practices, communities and knowledge domains. S/he does not do that by 'leaving the student' and the allocentric processing and return to the 'egocentric' close-range visualising. For that would exhibit not knowing the mathematics and also not be attending to the students. Experienced teachers, who have been in these situations before, may well have teaching gambits (e.g., giving out tracing paper, inviting small group discussion) that serve to protect them in the not-seeing-the-theorem moment yet position the students towards the theorem. That does not mean that it does not happen, just that their professional practice defends them.

Pedagogy, attention and defences

Professional practices change over time and culture. In the 'olden days' a teacher of geometry might well have written out a proof of a theorem onto a board in the classroom, undisturbed by queries from students. This 'non-interactive' teaching methodology allows the clean presentation of a new piece of mathematical knowledge through a step-by-step encounter. University lectures and academic schools would not have lasted as long as they have if this pedagogical method never worked! While a keen student may be able to use the lecture as a stimulus for study, this 'non-interactive' method does not work for all, arguably the majority, of potential learners of mathematics. Nowadays, the social dimension of learning is understood as a dimension of pedagogy and a teacher is charged with communicating purposefully with learners who are to be motivated to 'interactive' engagement with the content of the mathematics curriculum. This is the current route to increase attainment and participation in many countries.

On the level of the individual, a teacher of mathematics has a professional identity that includes their relationship with mathematical knowledge. So, when geometrical knowledge is challenged, as in the phenomenon of losing sight of a theorem (that is being discussed here), defences are evoked that protect this identity.

As Gal and Linchevski remark “conflicts between these two forms of information [visual and verbal] can produce difficulties in … communication between teacher and student.” (Gal & Linchevski 2010, p. 11). However, in some circumstances, the switch to allocentric processing may afford possibilities for discussion on the delicate nature of geometrical visualisation. In the following classroom incident, the teacher of an in-service course for mathematics teachers communicated with the participating teachers that he had ‘lost’ his vision of a theorem that he knows well.

A dynamic geometry diagram (DGD) is on the board which is being used in a novel proof of the circle theorem ‘angle at the centre is twice the angle at circumference’ (from Küchemann, 2003). Initially, the angle under consideration is set at 41° and how to visualise the theorem is pointed out to the students. The DGD is manipulated so that the angle is changed. And then the teacher could no longer visualise in the way he had just explained and he said to the students “I don’t know. There is a way; I can’t see it at the moment.” Then he returned the DGD to 41° and was able to visualise the theorem again and the students are told “don’t worry if you don’t see it. I have to work my way through every time. Momentarily I see it – which is lovely – then it fades.”

Sharing difficulties, such as this one, with students takes a certain courage and self-confidence. And yet, in order to better understand the delicate nature of geometrical visualisation, explaining the science of attention to students of geometry, may take a certain pressure off the need for performance.

Discussion and conclusion

Why does the phenomenon of seeing a theorem then losing the vision when talking happen? It happens because of the different ways the visual (ego-centric) and the communicative (allo-centric) are processed in the brain. Unconscious defence mechanisms militate against a teacher regaining his/her vision in an environment where there are stresses and demands for communication (like a typical secondary school mathematics classroom).

Is there a way to help teachers get back their vision of the theorem they were teaching? Not routinely. Yet if the teacher is able to return to processing ego-centrically the vision may pop back, as in example 2 where the DGD was manipulated back to the initial position, the visualisation of the theorem popped back.

In mathematics folklore, the mathematician G.H. Hardy is attributed to have said in a lecture “This is obvious.” Then paused and said “Hmm, is it really obvious?” After another pause he left the room to consider the point, returning 20 minutes later with the verdict: “Yes, I was right, it is obvious.” This illustrates what an uninhibited person might do to get back the egocentric processing.

Is there a way for this lack-of-teacher seeing to be a learning opportunity for students? Possibly. If the teacher is able to observe this lack of seeing in him/herself, and is able to reveal this lack of seeing to the students in the class and then explain that this phenomenon does occur and not to be too worried about it, as in example 2, perhaps the students too will be reassured that visualisation can be lost then re-found. The Hardy scenario has become a story because it is ‘not done’. Yet possibly, if an atmosphere of introspective concentration can be established within a classroom, the teacher, as well as students, may have the opportunity to regain the ‘obviousness’. This was possible with adult teachers of mathematics engaged in a Masters course. However, the challenge would be greater with adolescent students in compulsory education.

There is resistance to the idea that a person – teacher or student – can 'know' something but not 'perform' that knowledge at any point in time. This is because so much modern life is based on 'evidence' from assessment into which there is considerable investment. A consequence of accepting the central claim of this paper (that there are instances when a mathematical theorem is known but, a representation of that theorem is not 'seen'), is that 'understanding' mathematics is decoupled from 'performance' of mathematics.

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