Prospective Secondary Mathematics Teachers' Interpretations of Students' Thinking

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Teachers' understanding and interpretation of students' mathematical thinking are among the important components of knowledge for teaching as often stressed by the mathematics education community. Thus, teachers should acquire and enhance their knowledge and skills for understanding and interpreting students' thinking even before they begin their professions. In teacher preparation programs, using documentation of instructional practices such as students' written works and video records of classroom lessons would provide prospective teachers with opportunities for in-depth exploration of students' thinking. Thus, the purpose of this study was to investigate to what extent prospective secondary mathematics teachers enhanced their interpretation of students' thinking when they first worked on non-routine tasks themselves as students and then examined actual solutions produced by high school students. Twenty-five prospective mathematics teachers were the participants of the study. The data sources consisted of individual reflection papers, focus group interviews and notes of prospective teachers while working on students' work and field notes. The results showed that as a result of investigation of students' thinking manifested in the students' written works and video cases, prospective teachers started to question and tried to examine the details of students' thinking and to understand students' ways of thinking in depth.

Key words: mathematical thinking, prospective teachers' knowledge, video records, non-routine tasks

Introduction

This study investigated the extent to which prospective secondary mathematics teachers in Turkey enhanced their interpretation of students' thinking when they worked first on non-routine tasks themselves and then examined actual work produced by high school students.

Teachers' listening, attending, understanding, and interpreting students' mathematical thinking are among the fundamental components of knowledge for teaching mathematics, and the necessity of teachers having knowledge of student' thinking has often been stressed by the mathematics education community in the last two decades (e.g., Ball, 1988; Ball & Cohen, 1999). We see teacher preparation programs as the best places for forming the knowledge and skills for teaching mathematics: in such courses teachers would acquire and enhance their knowledge and skills for understanding and interpreting students' thinking even before starting their professions. However, conventional teacher preparation courses are criticized by several teacher educators (e.g., Feiman-Nemser, 2001; Lampert & Ball, 1998) since they do not require teacher candidates to engage deeply with theory in practice, and their curricula are so superficial and divide theory and practice both physically and conceptually. Similar to these critiques about conventional teacher preparation programs, teacher education courses in Turkey provide quite limited opportunities to pre-service teachers to work with K-12 students and to learn about students as well as students' thinking (e.g., Eraslan, 2009).

Accordingly, many studies find that both elementary and secondary prospective teachers are quite unprepared to listen to, understand and interpret students' thinking (e.g., Ball, 1988; Tirosh, 2000). In addition, these studies suggest that pre-service teachers do not have sufficient knowledge of common misconceptions, conceptions, errors and confusions of students and they are not ready to interpret students' thinking beyond its correctness. Moreover, they have difficulty to see mathematics through the eyes of students; instead, when they think about students, they mainly apply their own experience; namely, they prominently display an "egocentric perspective" (Ball, 1988, p. 277).

On the other hand, before starting the profession it is strongly suggested that prospective teachers should work with documentation of instructional practices derived from real classroom settings. Examples of students' written work, video records of classroom lessons or students' oral, written explanations and drawings on the board provide prospective teachers with opportunities for in-depth exploration of students' thinking (Ball, 1997; Ball & Cohen, 1999; Smith, 2001). The research emphasised that for instance, use of class discussion video-tape on a mathematical task could help pre-service teachers to make sense of what students are saying (Ball, 1997; Smith, 2001). Moreover, Smith (2001) stressed that when teacher candidates analyse students' written work, they might learn students' ways of interpreting mathematical concepts and develop their ability to interpret students' solution strategies. The findings from instructional materials drawn from practice displayed that using such materials provided an opportunity for pre-service teachers to construct their knowledge in the context of practice. To illustrate, in their research Lampert and Ball (1998) used records of students' own practices such as written notes (e.g., homework, quiz, daily notes), interviews, journals kept by teachers, video excerpts from lessons to provide pre-service teachers with more access to the complexities of the classrooms. They concluded that, in this way, pre-service teachers obtained opportunities to know more about children and their mathematical thinking as well as to understand teaching and learning in real classrooms.

Similarly, Masingila and Doerr (2002) used multimedia case studies which included a video overview of the school setting, the teachers' lesson plans, videos of class lessons, students' written work etc. to investigate how they could help pre-service teachers to make meaning of complex classroom experiences and to support them in developing strategies and rationales. The findings displayed that the pre-service teachers' analysis of the case study materials focused on more complex issues of classroom rather than discussing only the usual concerns with classroom management. They suggest this wider focus was afforded by the pre-service teachers' shared experiences of observing and interpreting teacher practices.

Methodology

Participants

This research was carried out in an undergraduate course offered in the secondary mathematics teacher education program of a state university in Ankara, Turkey. This course was designed for prospective secondary mathematics teachers to develop their own mathematical modelling competencies and to interpret students' mathematical thinking in the context of modelling activities. Twenty-five prospective secondary mathematics teachers enrolled, during the 3th, 4th, or 5th years of their degree. Data were collected over eight weeks during the second semester of the 2011-2012 academic year.

Design of the Study

Throughout the study, prospective teachers worked in groups of 3-4, shared their ideas and reflected on the assigned course materials. The design of the study comprised four two-week cycles. In the first week of each cycle, prospective teachers initially worked on a non-routine task in varied subject matters such as trigonometry, trigonometric functions, derivative, average and exponential functions and produced their own solutions to the tasks. Next, each group presented their solutions, other prospective teacher groups asked their questions and made comments related to their solutions, and discussed them as a class.

Then, in the second week of each cycle, prospective teachers were given high school students' solution papers and watched classroom videos of students' discussing the same task. The prospective teachers examined and collaboratively analysed students' thinking as manifested in their written works and the videos. In this process, they were asked to respond to given questions about students' solution strategies, strengths and challenges of their solutions, mathematical concepts used in their works and other things they noticed. Figure 1 represents the process as prospective teachers worked on students' works.

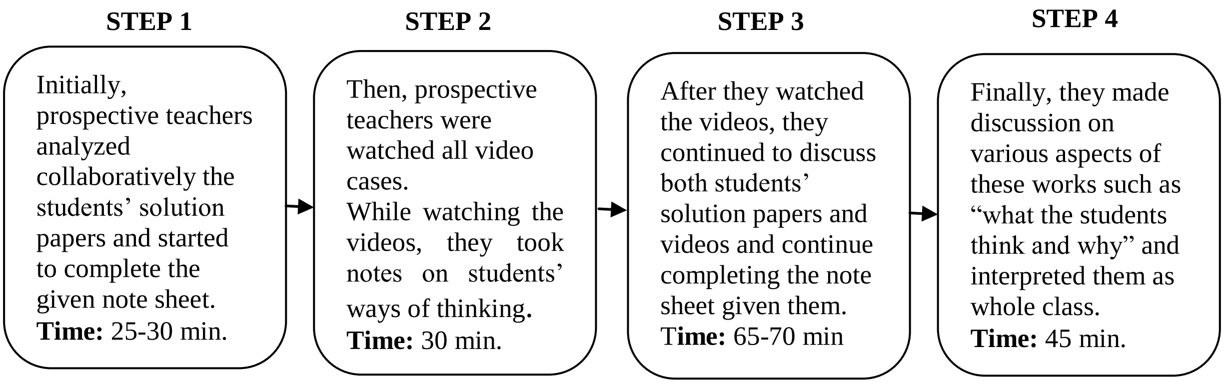

Figure1**.** The process as prospective teachers worked on students' works.

Non-routine Mathematical Task

The non-routine tasks used in the research were mathematical modelling tasks. In contrast to traditional routine school problems, these non-routine mathematical tasks were realistic tasks where students could make sense of problem situation. These tasks provide students with opportunities to describe, justify and represent their thinking process as well as document it. Besides, they allow students to produce different kinds of solution ways beyond short answers (Lesh, Kelly, Hoover, Post, & Hole, 2000)

Students' Work

In this course, students' work comprised students' solution papers and video-cases belonging to these solution papers. They were produced from classroom implementations with secondary students at grade level from 9 to 12 as a part of an ongoing three-year research project about mathematical modelling in secondary school (see acknowledgement). In these classrooms, students had attempted and discussed non-routine thought-revealing tasks in small groups of four to six, and then shared their solutions with other groups.

We selected papers to show different ways of thinking that emerged during their solution process, and video cases which involved students' explanations and discussions of their solution strategies, their understanding and misunderstanding about the issues being discussed and their interaction with each other and teacher during their solution process.

Data Sources and Analysis

The data sources consisted of individual reflection papers, focus group interviews and notes of prospective teachers while working on students' work and field notes.

Our analytic process began with the transcriptions of each videotaped focus group interview and the organization of data sources. The initial reading of transcripts was done and all data were examined line-by-line. As the unit of analysis, groups of sentences that maintain meaning together were selected. Two different approaches were drawn to create codes. The analysis of data started with open coding; that is, some of the codes were data grounded. On the other hand, several codes came from the conceptual framework of the research and previous research. This was followed by clustering codes and creating themes. As a result of analysis, the main findings were organized into four themes.

Results

The analysis of prospective teachers' interpretations of students' works displayed different characteristics in nature over the four study cycles that fit into the following four categories.

Not to Understand Students' Thinking or Misinterpret Students' Thinking

The data revealed that during the investigations of students' thinking, on the one hand prospective teachers sometimes had great difficulty in understanding and interpreting students' thinking; on the other hand they sometimes misinterpreted students' ways of thinking. For example, in the first investigation of students' works, they did not only have difficulty in understanding students' mathematical operations used to solve the task but also did not obtain a common idea even when they discussed it as a whole class. Below is an excerpt from reflection papers of one of the prospective teachers numbered as eight to illustrate this lack of understanding students' operations.

> **PST8:** Because the solution ways are clear for each group, I did not have difficulty to understand that part. However, there are still several points that I didn't understand about the mathematical operations and equations formed in their way of solutions. For example, I still don't understand where the values 0.6 and 3.6 come from in the solution of the students in Group-3. Also, how come the length of the perpendicular is 3 in the second way of solution for the same group? These are still open questions for me. These points are either overlooked or they didn't explain [in their solution].

Describing and Evaluating General Features of Students' Thinking

Prospective teachers' initial interpretations were based on the general features of students' thinking. Prospective teachers did not pay attention to the details of students' thinking in their solutions at all and mainly described surface features of students' thinking. They used judgemental statements as "correct", "incorrect", "terrible", and "good" rather than questioning and describing students' ways of solutions. The following excerpt of the prospective teacher exemplifies this case.

> **PST16**: Group 5 is already a total *disaster*. They could not totally comprehend the problem. They thought the place of length "4.8 m" wrong. That is, they thought that the area for secure parking is a trapezoid.

As shown in the excerpt, this pre-service teacher just evaluates the error in the solution approach rather than trying to understand their reasoning behind it.

Questioning of Students' Thinking Ways to Understand in Depth

The findings indicated that prospective teachers tried to question students' thinking ways to understand them in depth while they were working on students' works for each task. They made broad discussions to understand what students would think.

> **PST20:** Students had written that the radius is 0.34.
>
> **Researcher**: Did anyone find what it is?
>
> **PST18**: We sought out; but no.
>
> **Researcher**: For example what were your thoughts?
>
> **PST 20:** Students also wrote 0.53 here. If we subtract 31.73 from the length 32.8 and then divide it by two, the answer is 0.53. Students found this value and then found 0.34 by using the arc length.
>
> **Researcher:** What is 0.53 in that context?
>
> **PST8:** I think it might be pi (π) times the radius (r)
>
> **PST4**: The arc length is 0.53. I mean when I watched the students in the videos, they told "0.34 was our assumption". So, I think 0.34 is their acceptance.
>
> **PST9:** Here, they accepted that the arc length is 100 m rather than horizontal displacement. The graph has nine quadrants and three descending paths, therefore, they wrote as 9x+3y=100. Then, by trial and error, they might have found the answer as 0.34. [...]

This excerpt from prospective teachers' classroom discussion in their second investigation of students' works exemplified that in order to understand the students' solution; prospective teachers questioned "how students calculated the radius of the circle as 0.34 meters and why?"

Focusing on Mathematical Details of Students' Thinking Ways

The data analysis revealed that prospective teachers became more sensitive to students' thinking ways and started to recognize the complexity of students' ways of thinking. They also started to interpret students' complex thinking by indicating "why students thought like that?" and also producing predictions students' possible thinking ways. For instance, as seen in the following excerpt, PST13 interpreted students' thought process reflected in their solution strategy in detail.

> **PST13:** [...] Students in the first group say that to stir up people excitement they made the length of the descending parts of the roller coaster track as maximum and the length of the ascending parts of the track as minimum. This shows that they give priority to the excitement factor in solving the problem. Also, sketching the bottom and top of the roller coaster tracks as semicircles shows that students may be thinking that calculating the arc length of the circle is easier. Their aim to

> use the arc length of the circle would be that they accept the given 100 meters horizontal distance as the length of roller coaster track [...]

Conclusion

In conclusion, similar to findings reported by Lampert and Ball (1998) and Masingila and Doerr (2002), the settings and experiences offered in this course provided prospective teachers a crucial experience to learn about students' thinking ways and to extend interpreting beyond their correctness and general features. These experiences were lacking in other aspects of their pre-service training.

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