Exploring the challenges for trainee teachers in using a Realistic Mathematics Education (RME) approach to the teaching of fractions

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We report on the second part of a study into the subject knowledge of Secondary Mathematics trainee teachers enrolled on a Subject Knowledge Enhancement (SKE) course prior to their PGCE. In the first part of the study, trainees revealed a predominantly procedural knowledge of fractions. Most used the procedure as the authority over their answers, and few were able to make sense of the fractions or represent the fractions pictorially. The trainees then studied the teaching of fractions, examining alternative learning trajectories based on Realistic Mathematics Education (RME), after which they taught the topic in schools. We focus here on the challenges faced by trainees attempting to adopt a classroom approach that did not concur with the nature of their knowledge of fractions or with their own experience of learning fractions. Many trainees were able to adopt some of the features of RME, including appreciating the value of visualising fractions and the important role of discussion. However, the need for the trainee to have knowledge of a learning trajectory through fractions that is not dominated by procedures, and a belief in this trajectory, emerged as critical features.

Keywords: fractions, understanding, secondary, trainee teachers

Introduction

In the first part of this study, we examined the nature of the knowledge of a group of SKE trainees at Manchester Metropolitan University (MMU) with respect to fractions (Dickinson & Hough, 2012). The trainees demonstrated a procedural fluency operating with fractions, but when asked to describe how they knew they were right many revealed it was only their belief in the procedure, that enabled them to know. In other words, their knowledge of fractions was predominantly 'instrumental' rather than 'relational' (Skemp, 1976; Empson, Levi and Carpenter, 2011).

Realistic Mathematics Education (RME) offers approaches to teaching and learning fractions which focus heavily on developing relational understanding (Streefland, 1991). This is in contrast to our findings about trainees' subject knowledge. Consequently, the study reported here examines the following research question: What are the issues related to trainee teachers adopting a more conceptual approach to the teaching of fractions in terms of their classroom practice and in relation to their beliefs about teaching?

Methodology

The nine-month SKE course at MMU includes working on RME as part of an Education module. The trainees then complete a two-week placement in school, where they deliver lessons based on RME. For the purpose of this study, we delivered the RME sessions in the context of fractions.

During the intervention phase, we attempted to develop the trainees' conceptual knowledge of fractions. We chose a series of lessons which have been extensively researched (Streefland, 1997; Fosnot & Dolk, 2002), and that we have used many times previously. The first lesson is based on 'Fair Share' problems where learners are asked to consider how much sandwich they would get in each of four scenarios. (See Figure 1)

Figure 1 Fair sharing sandwiches
Source: Mathematics in Context.

As well as observing the whole group, we tracked three particular trainees. We observed them teaching the fractions lesson during their two-week placement and later carried out semi-structured interviews after they had taught fractions during the first teaching practice of their PGCE course. Finally we videoed each trainee teaching the fractions lesson again towards the end of their second PGCE placement and interviewed them straight afterwards.

Findings and Discussion

The use of context and models

In RME, realistic contextual problems are not only the starting point, but the source for the mathematics and the domain in which students can bring their own ideas and strategies. Contexts are chosen carefully because they will naturally lead students to bring strategies and models which are helpful to the mathematizing process (van den Heuvel-Panhuizen, 2003).

The three observed trainees made a point of engaging their students in the sandwiches context and asked supplementary questions such as "Can anyone eat a whole Subway sandwich?" One teacher talked about her experience of living in America and confirmed the idea that on school trips groups of children are given giant sandwiches to share between them. It would appear that the trainees were able to enact the use of context to good effect and post-lesson interviews confirmed this.

When the trainees drew their own versions of the sandwiches, either they drew 'sandwich like' versions (with rounded ends) or they drew rectangles. Both of these representations are classed as 'models of' the sandwich (van den Heuvel Panhuizen, 2003); one looks very much like the context, the other has better potential as a model. One trainee handled the sensitivity of this issue well, acknowledging to her class the advantages and disadvantages of each representation. Another trainee encouraged debate around the unfairness of receiving one of the crusty end pieces. A third trainee seemed unaware of the importance of this issue.

Appreciating what is the 'whole one'

Issues relating to 'what is the whole?' in the teaching and learning of fractions are well documented. Streefland (1997) and Ball (1990) suggest the importance of offering students problems which encourage discussion of this. Fosnot and Dolk (2002) distinguish between particular cases relating to 'what is the whole?' In particular they note that when comparing two fractions the whole must be the same size and that when multiplying and dividing fractions there are two wholes to consider. E.g. Finding 1/3 of a 1/2 means finding 1/3 of the **whole** 1/2; which itself is 1/2 of the **whole** 1.These issues arose on several occasions during the lesson:

Example 1

A common strategy for sharing 2 sandwiches between 3 people (Figure 2) is to first halve both the sandwiches, give a half to each person and then split the remaining half into three parts. The dilemma here is what fraction name to give to each of those three parts. Experience suggests that many students will think each part is 1/3, an understandable idea given it has come from splitting one piece into three equal parts (indeed, some of the trainees also said it was worth 1/3!) To resolve the issue one trainee suggested chopping the whole of the right hand sandwich into pieces of that size (see Figure 1.) This helped to convince others that one of those sized pieces was 1/6 not 1/3. This relates to Fosnot and Dolk (2002) in that trainees are using two different sized pieces, each to represent the same whole - the whole sandwich seen on the left of Figure 1 and the whole half sandwich seen in the right hand sandwich. Also in that having split the half into three pieces, one is required to decide what a 1/3 of a 1/2 is worth. i.e find a relation of a relation.

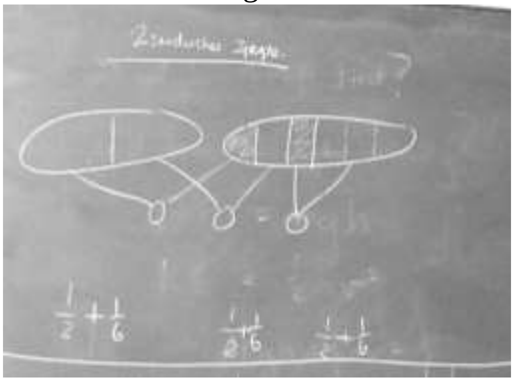

Figure 2: two sandwiches, 3 people

Example 2

One strategy for sharing three sandwiches between 6 people (Figure 3) is to cut each sandwich into 6 equal parts and give each person one part of each sandwich. In response to the question about how much does each person get, some trainees wrote down 1/2, others expressed the amount as 1/6 + 1/6 + 1/6. This is useful as it prompts discussion as to whether eating one sixth of one sandwich and one sixth of another sandwich and one sixth of another sandwich is the same as eating half a sandwich. Rather than attempting any formal procedures, students are able to use the pictorial 'model of' the sandwich as a means of comparing these quantities. A third trainee looked at the pictures and offered 3/18 as the amount eaten. Another trainee agreed: 'If you look at the diagrams it is 3 pieces out of 18'. This caused conflict, even anxiety, amongst the group and again highlights the need to consider exactly what 'the whole' refers to.

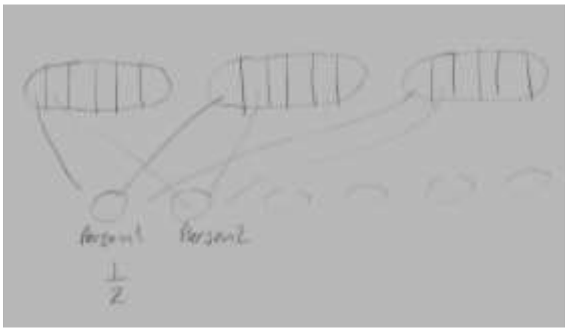

Figure 3: three sandwiches, six people

Developing a relational understanding of fractions

Fair share type questions are designed to elicit informal thinking (Streefland, 1997). For example, it is possible to argue that you would get less sandwich in group a (Figure 1) than you would in group b because both groups have the same amount of sandwiches, but there are more people to share the sandwiches in group a. When the trainees were asked to compare the groups, the majority gave responses indicative of more formal, procedural thinking:

"It's d because in d it's 3 sandwiches divided by 2 so that's 1½ each. In c it's 2 sandwiches divided by 3, so that's ⅔ each". Notice here reference to the operation 'divided by' and the immediate use of formal fraction notation. When questioned further, trainees who responded in this way described using a rule such as 'when you have to divide one number by another you can find the answer by writing the first number over the second number'.

Fair share problems also provide opportunities for learners to think 'relationally' about fractions (Empson et al., 2011). Consider the case of sharing 2 sandwiches between 3 people: If you use the strategy of sharing out the sandwiches one by one then you end up with 2 lots of 1/3 of a sandwich. Transferring each of the 1/3 pieces onto one bar also leads to seeing how 2 lots of 1/3 is equivalent to 2/3 of a whole bar. In other words you have a way of visualising and so understanding the relations contained within $2 \div 3 = 1/3 + 1/3 = 2 \times (1/3) = 2/3$. Empson et al. (2011) identify students who begin to anticipate that the exhaustive sharing of sandwiches one at a time has potential for completing the sharing fairly in all cases, as students who are beginning to think 'relationally' about fractions.

If the trainees are to appreciate the learning potential of Fair share problems then they have to resist their procedural responses and learn to think about fractions both informally in the context and relationally. Some of the trainees found this a challenge; they already had a method for answering the problem and did not see the need to develop other approaches.

Influence on trainees' beliefs about teaching

Ernest (1991) distinguishes three categories of teacher beliefs; beliefs about the nature of mathematics, the nature of mathematics teaching, and the process of learning mathematics. Swan (2006) comments on how central beliefs are often established young, firmly held onto, and incredibly difficult to change in adulthood. Given the relatively short period of time we worked with the trainees on RME, it is perhaps unrealistic to expect any change. However, from the sample of trainees who were observed and interviewed, there emerged a number of common threads, which appear to suggest some shifts in their beliefs about teaching and learning.

'Draw me a picture'

Using visualisations had been stressed in University sessions, and in interviews trainees provided vivid examples of where they had used this strategy. In each case they talked with genuine surprise and enthusiasm about the range of pictures their students had produced and how revealing these were in terms of gathering information about the students' perceptions of mathematical ideas. Several remarked that in one of their own mathematics sessions near the start of the course, a university tutor had asked them to 'draw a picture' to convince themselves that $(n^2 - 1)$ and $(n – 1)(n + 1)$ were always equal. They had thought the question ridiculous at the time but now they were not only using the strategy, but also showing an appreciation of its worth. This is one feature of RME that appears to have had significant influence on them as classroom practitioners.

A desire to promote discussion

Several trainees described ways in which they were trying to promote discussion. Stu described himself as asking questions such as “What do you think?”, “Who agrees with this?”, and (to a different student) “How do you see it?” These questions by their very nature imply that Stu is expecting a range of differing strategies and opinions. This feels different from asking the rather closed question “What answer did you get?” It would seem that their experience of being on the course has heightened their awareness of the possibilities of discussing mathematics problems in a productive and interactive way.

The pressure to conform

The perception of a number of trainees was that they felt a pressure to comply with the advice they were being given by class teachers and subject mentors. In some cases where trainees did deviate, they were often criticised and subsequently recommended to follow a more traditional format. School mentors can strongly influence the practice of a trainee and at times trainees reported feeling confused by what they saw as conflicting messages emanating from the school and the University.

Time constraints

Some of the trainees felt that using RME took a lot longer and that this was against the expectation of the school and the prescribed schemes of work. Indeed Romberg (1997) describes these very concerns expressed by experienced teachers when trialling ‘Mathematics in Context’ units based on RME. RME does cover the formal content of traditional mathematics courses, but not by instilling in students one technique at a time, and because of this many teachers new to RME feel it does not cover the topic content of a traditional curriculum.

Working with students’ responses in the classroom

RME involves exposing and working with students’ methods and conceptions of mathematics. This places considerable demands on the teacher- not only do they need to be able to process their student’s mathematical ideas ‘live’, but they also need to make instant decisions about how to proceed (Ball, 1990; Romberg, 1997). This can be particularly demanding for the novice teacher as they do not have a wealth of experience on which to draw.

The trainees in this study showed a definite commitment to exposing the way students think: they asked probing questions, made use of mini-whiteboards and encouraged students to write strategies on the board. Yet some were also, in the moment, unable to recognise their students’ line of thinking, and it was at these times that they were more likely to revert to their previously held beliefs and procedures.

Conclusion

This study has highlighted some of the obstacles related to adopting an approach based on RME. Firstly, it was apparent that the trainees did not themselves possess a conceptual knowledge of mathematics. If they are to teach in this way, then they need to develop their own subject knowledge in terms of conceptually understanding crucial elements of mathematics. Secondly, as was evidenced through the lesson observations, to use RME effectively requires the teacher to have the pedagogical skills required to create a classroom setting in which a student’s individual constructions and representations are brought to the fore (Ball, 1990). This requires sustained training and ongoing support to enable the developing teacher time to experiment, to reflect, and to re adjust their beliefs about teaching. Trainees showed a willingness to try out new ideas, and experienced some success while they could adhere to their pre-planned structure. But once a more flexible approach was required, for example in responding to students' own ideas, they quickly reverted to their prior beliefs and conceptions about teaching.

Thirdly, the pressures on teachers, to conform to the currently accepted norms should not be underestimated. The increasing focus on assessing pupil progress, the breaking of the curriculum into small units, the OFSTED expectation that pupils should demonstrate progress by levels within the course of each lesson, the practice of early and repeated entry at GCSE; all these factors potentially work against teachers who wish to spend time exploring and working with students' own perceptions of mathematics.

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