Linking dragging strategies to levels of geometrical reasoning in a dynamic geometry environment.

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Students working in Dynamic Geometry Environments interact with geometric figures by dragging constituent objects on the computer screen. A number of researchers have described different dragging modalities and linked them to cognitive activity. This paper draws on data from recordings of students working with a dynamic figure based on fixed length perpendicular diagonals. The diagonals can be dragged in the figure thus generating a number of quadrilaterals and triangles. Two new dragging strategies have been observed in use by students within the context of working with the dynamic figure. Refinement dragging is used when students check and review side and angle properties of shapes they have generated. Dragging maintaining symmetry is used when students drag so that one diagonal bisects the other generating what could be termed a 'dragging family' of shapes. This paper describes these dragging strategies and relates them to the Van Hiele levels of reasoning. The students' innate sense of symmetry also emerged as an important aspect of how they conceptualise 2D shapes.

Keywords: dynamic figure, dragging, cognitive, Van Hiele

Introduction

This paper describes a study, taking a Design Based Research approach with iterative development of tasks, which aims to explore students' geometrical reasoning and how this developed as they worked on a task. The task is based on a figure which is constructed around two perpendicular line segments of fixed length which are referred to as 'bars'. The bars cross each other and line segments join their ends to complete the figure as shown in figure 1. Dragging the bars generates different quadrilaterals and triangles.

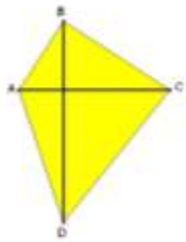
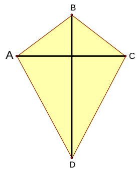
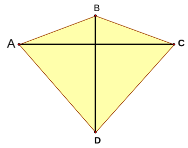


Figure 1 The figure based on perpendicular bars of unequal length

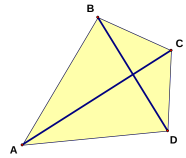
If the bars are positioned so that the end of one bar touches the other then triangles are generated. If the bars are positioned so that one intersects the other at its mid-point then kites, a rhombus, isosceles triangles and arrowheads (concave kites) are generated. It is considered that humans have a preference for a horizontal and vertical frame of reference (Piaget and Inhelder, 1956) and that a vertical axis of symmetry is preferred (Palmer, 1985). Therefore in later iterations of the study the bars were oriented at an angle to ascertain whether students would drag the figure in the same or different ways as students who worked with horizontal and vertical bars.



vertical bar is longer



horizontal bar is longer



bars oriented at an angle

Figure 2 Figures with the bars in different orientations

Dragging and measuring strategies in DGS

Geometry provides students with the opportunity to develop skills in mathematical reasoning which can be built up by working on mental imagery and visualisation, (Qualification and Curriculum Authority (QCA), 2004). Dynamic Geometry Software (DGS) provides the means to investigate figures which can be manipulated by dragging on the computer screen allowing students to view many different representations of the figure. Measurements of side lengths and angles of the figure are displayed on the computer screen and these continuously update as the figure is dragged, which may help students to observe properties of the figure and to investigate whether these properties remain constant for all examples of the figure (Olivero & Robutti, 2007).

Arzarello et al. (2002) describe a number of dragging strategies and suggest these are aligned to cognitive activity. Among these are *wandering dragging* (randomly dragging objects to discover properties of a figure) and *guided dragging* (purposefully dragging objects to give a figure a basic shape). A further dragging strategy identified is '*dummy locus dragging*' which entails dragging an object along a path in order to maintain certain properties. The path is not visible hence its name 'dummy locus'. Arzarello et al (2002) describe dummy locus dragging as "wandering dragging that has founds its path" (ibid, p.68).

Another category of dragging strategy, described by Baccaglini-Frank and Mariotti (2010) is '*maintaining dragging*' which is "dragging a base point so that the [DGS figure] maintains a certain property" (p. 230). Maintaining dragging has similarities with dummy locus dragging but the student has the specific intention to keep a certain property constant. Dragging strategies are also used with the measuring facility of the software to allow students to move back and forth between practical experimental geometry and theoretical geometry (Olivero & Robutti, 2007).

THE VAN HIELE MODEL FOR GEOMETRICAL REASONING

I will use the Van Hiele (VH) levels to analysing the sophistication of students' geometrical reasoning (Van Hiele, 1986). Although Van Hiele considered students' progress through the levels as discrete and hierarchical, Battista (2007) and others take the view that students make progress in small incremental stages through any particular level. I have adopted this view and consider it useful to categorise the Van Hiele model as different types of thinking in which students may engage in depending on the task and its complexity (Papademetri-Kachrimani, 2012).
Briefly the first three levels (pertinent to this study) are:
Level 1 - visual holistic; where children recognise and name shapes.
Level 2 - descriptive; where children can describe properties of shapes.

Level 3 - structural; where children are able to make connections between different shape properties, identify the minimum required definition and classify shapes hierarchically.

The students in the study already showed evidence of geometrical reasoning at level 2 and the task was designed to support their development of reasoning at level 3.

Methods

The students in the study were 12-13 years of age and achieving at levels 5/6 of the national curriculum for England and Wales. As such they are expected to be able to solve geometrical problems using side and angle properties of triangles and quadrilaterals, explaining reasoning with diagrams and text (Qualifications and Curriculum Development Agency (QCDA), 2010). Those who participated in the study were chosen by their regular class teacher as being comfortable working with an adult (the researcher) who they had not met previously. Pairs of students working with one computer which was loaded with the Geometers Sketchpad version 4 (Jackiw, 2001), were asked to investigate which shapes they could make by dragging the bars inside the figure. This activity does not require the students to know much about the software itself. They only need to know how to use a mouse and to learn to carry out a small number of actions such as using the Measures menu.

Dialogue between the students, and with me, was recorded along with on-screen activity. The research sessions took the form of a mathematics lesson with me acting as teacher of two students. As the teacher / researcher my role was to introduce the software and the task and facilitate the students' development of geometrical reasoning by giving them appropriate problems to solve and questions to answer.

In analysing the data, first the dialogue was transcribed. Since this entailed replaying the on screen activity, it served as an initial familiarisation with the data. Next the on-screen activity was analysed. Most of this activity comprised of dragging the bars and some pointing with the cursor and choosing objects when using the measures menu. The various dragging strategies used by the students were identified and analysed separately. The third phase of the analysis was to connect the dialogue to the on-screen activity in a narrative account of the sessions. Phase four entailed the identification of themes which ran through the session narratives and presenting the data as providing evidence for these themes.

Findings: the dragging strategies used by the students

The students were observed to use four dragging strategies. *Wandering dragging* was observed when students randomly dragged to see which shapes they could generate, and *guided dragging* was used when the students had more experience of working in the file and could move the bars purposefully into position (Arzarello et al, 2002).

Refinement dragging

After they had dragged the bars to generate a specific shape, the students used the Measures menu of the software to check properties of sides and angles. Usually the side and angle measurements which should be equal in order to mirror the shape properties were not exactly equal and so the students used small movements of the bars to get the required measurements as close as possible. I have called this third strategy *refinement dragging* (Forsythe, 2011) because that describes the way in which the students refined the positions of the bars to get the measurements to be right. *Refinement dragging* also helped the students to check and review the properties of the shapes they generated and thus supported them in moving from the field of practical experimental geometry to theoretical geometry.

Often the students were observed to use a *guided dragging* strategy in placing the bars to generate a specific shape and then move straight into *refinement dragging* in order to get the shape to be more accurate. This would suggest that first they used a holistic sense of the shape and were focusing on how to place the bars to generate that shape. Next the students used their conception of the properties of the shape whilst attending to the measures of sides and angles using the *refinement dragging* strategy.

Dragging maintaining symmetry

The fourth dragging strategy I have identified is perhaps a special case of maintaining dragging (Baccaglini-Frank & Mariotti, 2010) and I have named this *dragging maintaining symmetry.* The students appeared to be dragging in a purposeful manner to keep symmetry a constant (Forsythe, 2011) as they moved one bar so that it continued to be the perpendicular bisector of the other bar in order to move between kites, rhombus and equilateral triangles. (In this dynamic figure a triangle can be a special case of a kite).

This is demonstrated in the following excerpt (Figure 3). Tilly said that they would make a kite by first making an isosceles triangle, ensuring that they put the vertical bar with its lower end on the mid-point of the horizontal bar (C and A lined up). When she said 'move it down from there' she referred to the vertical bar BD. This can be seen from the screen shots in figure 3 which shows the cursor touching the bar BD. Tilly explained that if the bars were put into the correct position (lined up) then the measurements of sides which should be equal would be the same.

Tilly: Erm, put it in a triangle first.

Alice: Cos then you can see what, how you go from that.

Int: So when you say put them in a triangle.

Tilly: Erm, that (indicating the triangle on the screen).

Int: Make it a triangle shape?

Tilly: Yeah, make a triangle shape. Then if you've got the line C and A lined up, you move it down then.

Alice: Yeah, that was it.

Tilly: Then you might be able to get it right. Cos these two are lined up and then you'll have the same measurements.

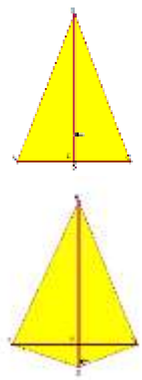


Figure 3 An excerpt from the data using vertical and horizontal bars

Students who worked with the figure whose bars were oriented at an angle commented on the orientation. For example one student referred to the shape in figure 4 as "a bit of an angled kite".

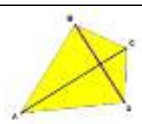


Figure 4

However the students working with the oriented figure still used the same dragging strategies as the students working with the vertical figure. Eric used the same reasoning to describe how he dragged the bar to move between the kite and the isosceles triangle.

Int: OK. How did you move from the kite position to the isosceles triangle?

Eric: Well we had to move it down, so near the solid line and I made sure it was in the middle so that it made the equal side lengths

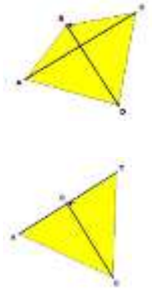


Figure 5: An excerpt from the data using bars oriented at an angle.

When Eric said they moved it 'down' he referred to point B (which pulled the bar BD) and this can be seen by the position of the cursor on the two screen shots. Of course he was not dragging B vertically down the screen but down the slope of the perpendicular bisector of AC, *dragging to maintain symmetry*. Perhaps Eric had mentally rotated the figure so that BD was visualised as being vertical which would support Pinker (1997) in his assertion that human beings mentally rotate images which are drawn at an angle to the vertical. Several students when working with the bars oriented at an angle actually turned their heads to one side.

The students spent a significant amount of time during the sessions engaged in *dragging maintaining symmetry* and could articulate that this resulted in the generation of kites, arrowheads, rhombus and isosceles triangles. They could describe that what these shapes had in common was that one bar crossed over the other at its mid-point. However when I suggested that the shapes might be in a dragging family the students were not happy with considering that the rhombus might be a special case of a kite.

Discussion

The students appeared to have an innate sense of symmetry which they employed when positioning the bars inside the shapes they generated and this appeared to be true as much for when the shape was oriented at an angle as when it was oriented to the vertical axis. This suggests that the students' sense of the shape was holistic, which could be categorised as VH level 1 reasoning. One conclusion from this observation is that it is more intuitive to learn the properties of shapes by beginning with their symmetry and using this to generate properties of equal sides and angles rather than the other way about, as is generally the case in most school curricula.

Orientation was clearly involved in how the students saw the shape and even though vertical orientation was preferred they were able to separate orientation from the properties of the shape.

Attending to the measures and using *refinement dragging* to ensure that they had generated a valid shape helped the students to check and review their knowledge of shape properties. This indicates reasoning at Van Hiele level 2 and it also shows how DGS supports students to move from practical experimental geometry to theoretical geometry.

The *dragging maintaining symmetry* strategy generates a family of shapes whose default member is the kite (both convex and concave kites) and specific examples are the rhombus and isosceles triangles. This task has the potential to support the move towards VH level 3 reasoning if the students can conceptualise a family of shapes with a common property ("one bar crosses the other in the middle") whilst dragging maintaining symmetry. However, most students in the study found it difficult to accept that the rhombus could be a special case of a kite. I postulate that the problems arise because the students view the shapes as generated at discrete positions along the *dragging maintaining symmetry* locus rather than as examples of a continuously changing figure. Mamon Erz and Yerushalmy (2007) suggest that students who classify shapes according to a partitional classification (indicative of Van Hiele level 2 reasoning (De Villiers, 1998)) tend to visualise a figure under dragging as changing from one discrete shape to another. In order for students to develop an appreciation of a hierarchical classification of shapes there needs to be a modification of the task. It may be that an animation of the figure under dragging maintaining symmetry could help students to see the figure continuously morphing through many kites with the discrete positions of the rhombus and isosceles triangles.

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