Investigating secondary mathematics trainee teachers' knowledge of fractions

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At Manchester Metropolitan University, approximately eighty students each year qualify to become teachers of secondary mathematics. Of these, roughly half do not have a mathematics degree, but have studied on a Subject Knowledge Enhancement (SKE) course. This research study is concerned not with the pure mathematical knowledge of such trainees, but with the nature of their knowledge. Asking them relatively routine questions on fractions showed almost all trainees reaching for a known procedure to answer the questions. Furthermore, when asked how they knew they were correct, most trainees used the procedure as the authority for this. The trainees then studied the teaching of fractions, after which they taught the topic in school. This paper focusses on the first part of the study, which analyses the trainees' own knowledge of fractions. A later paper will report on the classroom work of the trainees.

Keywords: secondary; understanding; fractions; trainee teachers

Introduction

> A 52 year-old policewoman was asked how she would work out $\frac{1}{4}+\frac{1}{2}$. She smiled, wryly. 'What did you say, a quarter plus a half?' she says, writing the two fractions with her index finger on the empty table in front of her.
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> 'It's something to do with common denominator?' she asks (gesturing a horizontal line underneath her imaginary quarter and a half). 'Then is it something to do with cross multiplying' (again gesturing this with her index finger pen on the table).
>
> 'Do you know the answer?' I ask.
>
> 'Oh yeah, it's three-quarters.'

It would seem she knew this answer all along and yet her first preference was to attempt to re-call a procedure which she had probably not used for over 30 years. We have seen other evidence of this when working in classrooms, with trainee teachers, and with adults. Why is it that so many people seem to elect to use a formal method instead of their common sense intuitions?

Research focus

Although much has been written about trainee teachers' subject knowledge (e.g. Schulman 1986; Ball 1990; Brown et al. 1999; Goulding, Rowland and Barber 2002), much of it has focussed on primary teachers and/or on pedagogic subject knowledge. With the advent of so many trainees now coming from SKE courses, it seems pertinent to look at the nature of the subject knowledge of such trainees. Consequently, our current research focuses on the two questions:

- To what extent is teacher trainees' knowledge of fractions dominated by procedural routines?

- What would be the issues in trainee teachers adopting a more conceptual approach to the teaching of fractions

This paper deals specifically with the first of these, the issue of trainees' subject knowledge, and describes research undertaken at Manchester Metropolitan University (MMU) in 2011.

Data Collection

We wanted an instrument that would enable us to measure the trainees' knowledge of fractions and in particular how procedural / conceptual this knowledge was. When trainees are interviewed for the PGCE at MMU, they are usually asked the question, 'work out $\frac{2}{5}+\frac{1}{3}$', followed by: 'How do you know you are right?' Our experiences of this have led us to believe that for many trainees their answers are dominated by a procedural knowledge of fractions. We chose to investigate this further by examining the subject knowledge of 31 trainees who were part-way through a subject knowledge enhancement (SKE) course. Using a test seemed the most appropriate method for collecting our initial data, as it would enable us to quickly gather knowledge about the whole cohort. We were aware of issues of validity and whether the test would actually be able to measure what we intended it to measure (Mertler 2006). This led to us designing a format of test which was in two parts, though the trainees were not initially aware of this.

Initially, the trainees were asked to answer a series of questions on fractions. We emphasised that we were not concerned with correct or incorrect answers but with looking at the methods they had used. After this was completed, we asked the trainees to look at each question again and say how they knew their answers were correct. We suspected that the first responses would be dominated by procedural routines, and hence the second question was introduced to expose other ways of thinking about fractions. We were conscious that sometimes people feel under pressure to use a known procedure because this is what they perceive to be the 'correct method'. So the second question gave each participant an opportunity to expose an alternative strategy.

In designing the questions, we tried to ensure that they covered the range of content knowledge normally expected in school level mathematics. We included 'bare' fractions questions and questions which were set in context, so that in the analysis we might be able to see whether this had any impact on the initial methods used. We also chose questions which had previously been used with pupils (Dickinson and Eade 2005) and questions which had been used as part of continuing professional development (CPD) for experienced teachers (Fosnot and Dolk 2002). This was deliberate as it gave us an opportunity for comparison and also an opportunity to establish the reliability of the questions.

Data Analysis

The test: strategies used to answer the questions

While questions covered all aspects of fractions, for this analysis we focus on the three parts of the first question and the trainees' responses to these. To work out $3\frac{1}{2}\times 14$, the trainees used a variety of approaches. Several converted $3\frac{1}{2}$ into a top-heavy fraction and applied the rules for multiplying fractions, a few used the long multiplication routine to find 14×3.5, some chose to partition it into (3 lots of 14) +

($\frac{1}{2}$ lot of 14) or alternatively (14 lots of 3) + (14 lots of $\frac{1}{2}$), some worked out $3\frac{1}{2} \times 2 = 7$ then $7 \times 7 = 49$.

Fosnot and Dolk (2002) describe two categories of approach to problems of this type: firstly the use of an algorithm or procedure and secondly the use of 'number sense.' Applying 'number sense' involves having an awareness of a number of strategies and making a case-by-case judgement about the best strategy to use. Twenty out of the thirty-one trainees went for an algorithmic approach (either by using a standard procedure to multiply fractions or by using long multiplication). Of those using number sense (some form of partitioning) only three went for the efficiency of doubling $3\frac{1}{2}$ and then multiplying by 7. The results would suggest a strong leaning towards the use of algorithms / procedures. Responses to the second question, $\frac{6}{16} \times \frac{8}{18}$, were also procedural with all 31 trainees employing a version of the strategy 'top x top over bottom x bottom'.

The final part of this first question, $\frac{2}{3} \div \frac{4}{24}$, also revealed a strong preference for the standard procedure with 28 of the 31 trainees applying a version of 'invert and multiply'. In only 3 cases did the trainees appear to be using a 'number sense' approach by choosing to retain the division element of the question.

Use of algorithms versus 'number sense'- some pros and cons

From the responses, trainees showed a clear preference for the use of procedures, as perhaps was to be expected. Procedures are quick and efficient and (provided you remember exactly what to do), can be easy to use and produce accurate answers. There is something quite powerful about knowing, for example, that whenever one sees a division of fractions question, all that needs to be done is to 'invert and multiply'.

Applying 'number sense' on the other hand requires having many strategies at your disposal and deciding on which strategy to use (Fosnot and Dolk 2002). This implies that the user needs to have a deeper understanding of number and of the connections that exist within the world of numbers (for example, an understanding that when multiplying two numbers together, the same result comes from halving the first, doubling the second and then multiplying).

Where learners are able to apply number sense, then their methods have the potential to be even quicker and more efficient than using a procedure. We saw this in the case of $3\frac{1}{2} \times 14$. Seeing this as 7×7 is potentially a lot quicker than converting to fractions and multiplying. Seeing a calculation in an 'easier numbers' form can also minimise the opportunity for calculation errors.

Possible reasons for the widespread use of procedures

This is the way they were taught at school

Perhaps the most influential factor is the way the trainees were taught at school (Bramald, Hardman and Leat 1995) and whether the focus was on learning procedures or on developing mathematical concepts. Much has been written about the tension between these two approaches, for example Brown (1999, 3) refers to the "swings of the pendulum" between approaches which stress the "accurate use of calculating procedures" and those which favour developing "number sense". Others (see Thompson 1994, 1997; Sugarman 1997; Beishuizen 1999) discuss the issues relating to the teaching of algorithms for number operations compared with the teaching of mental strategies, informal written methods and a development of number sense. Despite the heavy investment into research of this nature and indeed the emphasis placed by the National Numeracy Strategy (NNS 1999) on the use of models such as the empty number line, the overriding emphasis in UK textbooks is still to show a procedure and then produce questions which practice that procedure (Haggarty and Pepin 2001). Consequently, it is likely that, in whichever era the trainees were taught their school mathematics, the goal will have been to have knowledge of the standard written procedures. It is also possible that this may have been the only approach they were taught.

The trainee's awareness of non-procedural approaches to working with fractions is limited.

One question to consider here is: 'Were the trainees making a positive choice to use a procedural approach over a number sense approach or did they not possess the facility to try the questions any other way?' Swan (2006, 16) refers to the fact that despite spending many years practising techniques, it is possible to gain very little "substantial understanding of the underlying concepts". We suspect that many of the trainees would have little conceptual (or 'relational' (Skemp 1976)) understanding of why their procedures worked. The second element of the test, whereby trainees had to say how they knew they were right was designed to expose some of the issues relating to the type of understanding they possessed.

A discussion of the responses to the questions: How do you know you are right?

Having analysed the responses we were able to distinguish four main categories. Categories 1 and 2 link closely to the responses one might expect from someone who has predominantly an 'instrumental understanding' (Skemp 1976) of maths. Categories 3 and 4 relate to having a 'conceptual understanding' of maths.

Category 1 – Uses the algorithm to justify the algorithm

This was a common occurrence across all the questions. One trainee stated 'I know I am right because I trust the method'; another said "I've always done it this way". Some repeated exactly the same calculations; some reversed the sum and then applied another procedure. Several (see Figure 1) simply described the procedure they had used as a justification for why they must be right. These trainees could be said to be displaying 'instrumental understanding' as described by Skemp (1976) in that they can apply the rule but their only justification for this rule is the rule itself.

Category 2 – Acknowledges that they don't know why they are right

Many trainees were not able to offer any explanation as to how they knew they were right. Unlike those in Category 1, these trainees seemed to recognise the limitations of their algorithm as a means of giving credence to their answer. Several referred to the fact that their method was 'just a rule'; something they had learned at school and never really questioned.

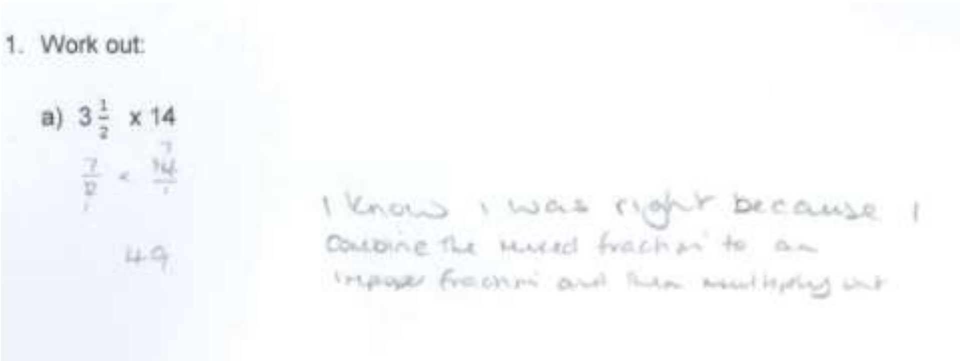

Figure 1: Trainee teachers use of algorithm to justify the use of an algorithm

A couple of the trainees said to us later that it was only since coming on the SKE course that they had started to question the way in which they understood maths. This is another indicator that for many the experience of learning maths at school may be almost exclusively procedural.

Category 3 – Uses an alternative 'sense making' strategy

Having first used a procedure to answer the question, these trainees answered the question again using a 'number sense' approach. Like those in category 2, the algorithm provided them with little sense of whether they were right or not, but these trainees looked for, and were able to find, an alternative way of looking at the problem. Given the brevity and relative simplicity of many of these approaches, it seems strange that more trainees did not adopt these strategies initially. It would appear that sometimes people feel compelled to use the 'standard method', even when more complicated, as this is what is believed to be 'proper maths'.

Category 4 – Draws a picture

This strategy was rarely used despite the fact that classic early notions of fractions are developed around pictures (Lamon 1999).Even when the question was set in the context of "How many $^1/_4$ inches can be fitted into $^4/_3$ of an inch?", only five out of 31 drew pictures. Two of these drew a circular diagram, showing no affinity with the linear representation inferred by the context. When asked to find another way of proving that $\frac{1}{2}+\frac{1}{3}$ is not equal to $\frac{3}{5}$, only 11% drew pictures and yet this is relatively easy to see if you do draw a picture. It was clear from analysing these tests that:

1. Most trainees demonstrated a procedural (rather that conceptual) knowledge of fractions
2. Most trainees appeared satisfied to use the authority of a procedure to justify a procedure, although others did recognise the need to find other ways to justify their procedures (even if they did not yet know what these were).

It is important to recognise that many of these trainees may not see a need to teach fractions in any way other than how they learned them at school. Countering this represents a huge challenge, as it involves changing people's beliefs. According to Swan (2006), central beliefs are often established young, firmly held onto, and are incredibly difficult to change, particularly once one reaches adulthood. Working with the trainees on these issues represents the second part of this research study.

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