Calculating: What can Year 5 children do now?

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In 2006, 2008 and 2010 we collected and analysed answers from a Year 5 QCA test paper to explore the range of calculation strategies used by a sample of approximately 1000 Year 5 children. Once again in 2012 we have repeated this research using the same group of 22 schools. This paper explores the findings from the 2012 data, including case studies. It examines the range of strategies used by the children. We conclude by considering if and how the use of particular calculation strategies has impacted on the overall results and we ask if this shows greater clarity about which strategies lead children to success.

Keywords: calculations, strategies, primary mathematics

Introduction

This Year 5 research emerged (Borthwick and Harcourt-Heath 2007) when the National Numeracy Strategy (NNS) (DfEE 1999) was the main framework that teachers used to support them in planning and delivering mathematics in the English National Curriculum. Prior to the introduction of the NNS the mathematics curriculum had focused more on the applications of mathematics and less on written calculation strategies. However, the NNS placed more emphasis on arithmetic skills and children were exposed to perhaps alternative methods for calculating than they had been shown before (e.g. number lines and the grid method). The UK now has a renewed Primary Framework for Mathematics (DfES 2006), which still includes this emphasis on written calculation strategies. One of the main aims of both the original and revised mathematics curriculum was to provide children with the "ability to calculate accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies" (DfEE 1999, 4). We have also retained this aim as our benchmark when analysing the Year 5 data.

However, while our longitudinal study continues to follow the progress of children's success with written calculation strategies, other research shows that this proficiency with calculations is not yet secure for many pupils. Howat (2006) reported that children (aged 8 years old) were still failing in arithmetic because they were unable to sufficiently understand that a ten in a two-digit number could be ten ones or one ten. While there is a plethora of research that examines children's progress and understanding in specific calculation strategies (e.g. Anghileri 2001; Anghileri, Beishuizen and van Putten 2002), our study is unique in that it involves large scale data spanning across the last six years which looks at strategies for all four operations. However, this paper concentrates only on the 2012 outcomes rather than the previous data (Borthwick and Harcourt-Heath 2007; 2010).

Methodology and context

Data was collected from test papers completed by Year 5 children from 22 schools throughout Norfolk. A range of primary and junior schools were selected. Responses to four questions from each of the papers were analysed for their calculation strategies. One question each for addition, subtraction, multiplication and division was used.

Calculation	Question
Addition	546 + 423
Subtraction	317 – 180
Multiplication	56 x 24
Division	222 ÷ 3

Table 1. Questions from QCA Year 5 test paper

The four questions we selected were chosen as they had no context and required children to perform a calculation, as opposed to less abstract problems that involve children in some interpretation before a calculation can be carried out. The categories used for analysis were determined by the National Numeracy Strategy (DfEE 1999) and other research (e.g. Beishuizen 1999).

Findings and discussion

Each of the following sections looks at proportions of children using the range of strategies for the four questions and includes examples of children's work.

Addition

94% correct / 6% incorrect

546 + 423	Number Correct	Number Incorrect	Percentage Correct	Percentage Incorrect
Not attempted				
Standard algorithm	430	10	98%	2%
Number Line	32	7	95%	5%
Partitioning	179	9	95%	5%
Expanded vertical	168	6	97%	3%
Answer only	114	22	84%	16%
Other	14	8	64%	36%
Totals	937	62	94%	6%

Table 2: Results from 999 children for addition question.

This question was by its very nature the least useful because it did not require bridging through ten or one hundred. As a result, a number of different strategies were identified.

Subtraction

69% correct / 31% incorrect

317 – 180	Number Correct	Number Incorrect	Percentage Correct	Percentage Incorrect
Not attempted		11		
Standard Algorithm – decomposition	106	71	60%	40%
Standard Algorithm – equal addition	0	2	0%	100%
Number Line	484	70	87%	13%
Negative Number	13	5	72%	28%
Counting Up	20	65	24%	76%
Counting Back	16	1	94%	6%
Answer only	28	9	76%	24%
Other	24	74	24%	76%
Totals	691	308	69%	31%

Table 3: Results from 999 children for subtraction question.

Almost all children attempted to answer this question, with the number line emerging as the most often selected and successful strategy (see Figure 2 below for an example). However, those children who employed the counting up strategy but did not record the number line were not as successful as those who drew it to aid their thinking. Almost one fifth of children selected the standard algorithm but this was much less successfully employed.

As illustrated in Figure 1 below, some children still demonstrate a lack of understanding about subtraction by using partitioning inappropriately and incorrectly.

Figure 1

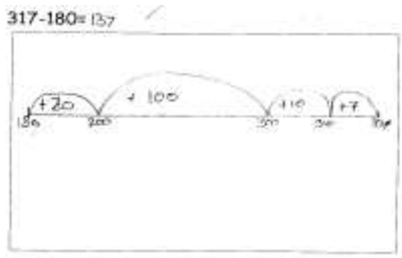

Figure 2

Multiplication

42% correct / 58% incorrect

56 x 24	Number Correct	Number Incorrect	Percentage Correct	Percentage Incorrect

Not attempted		109		
Standard Algorithm	5	19	21%	79%
Grid Method	397	283	58%	42%
Expanded Vertical	7	7	50%	50%
Two partial products only	0	70		100%
Answer Only	0	14	0%	100%
Other	11	77	13%	87%
Totals	420	579	42%	58%

Table 4: Results from 999 children for multiplication question.

Over two thirds of the children chose to use the grid method for completing the multiplication calculation. We were surprised to note that this category had both the highest number of correct (397) *and* the highest number of incorrect (283) responses.

While Figure 3 below shows an appropriate grid structure, the presentation of the multiples of tens numbers (e.g. 50 and 120) might cause us to question issues of place value. It could be suggested that children had been taught to think when multiplying 20 by 50 that you simply multiply 2 by 5 and add two zeros. The particular example shown also demonstrates the impact of incorrect partial product calculations on the overall answer. The second example, Figure 4, shows a typical representation of the 'two partial products' category that more than 7% of the children used.

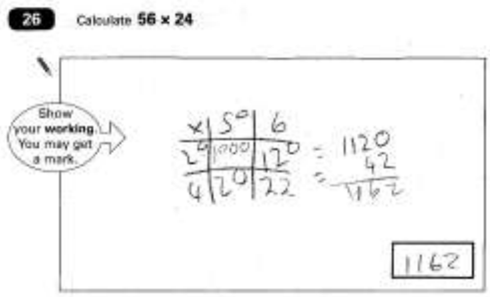

Figure 3

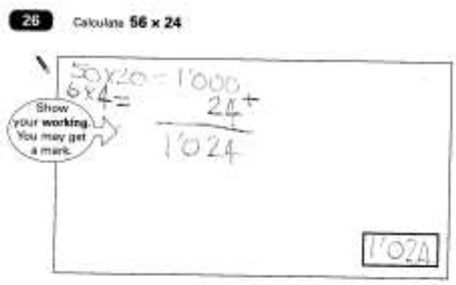

Figure 4

Division

38% correct / 62% incorrect

222 ÷ 3	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		151		
Standard Algorithm	48	38	56%	44%

Chunking Down	58	19	75%	25%
Chunking Up	83	47	64%	36%
Number Line	152	142	52%	48%
Answer Only	19	96	17%	83%
Other	23	123	16%	84%
Totals	383	616	38%	62%

Table 5: Results from 999 children for division question.

This calculation was the least well answered with 15% not even attempting it. Although the number line was selected most often, a significant proportion of children did not gain a correct answer through its use. Examination of the children's responses revealed that the underlying strategies are not secure, for example, children repeat the subtraction of 3 but they seem not to be moving to the next stage where they are subtracting multiple 'chunks' of 3. This leads to inefficiency and simple errors in calculation.

Case study

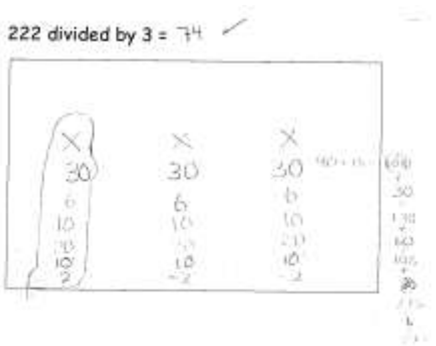

Figure 5

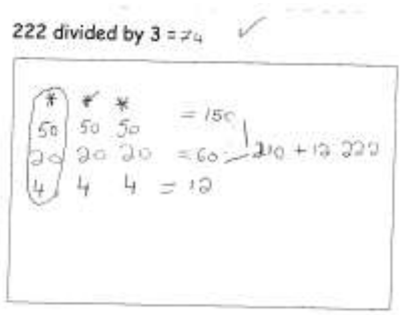

Figure 6

The two examples shown in Figures 5 and 6 are taken from children in the same class. 95% of the 21 children in this group answered the division question correctly and they all used the same 'sharing' strategy and similar layout. Whilst this could demonstrate that the children had been taught to answer this algorithmically, closer analysis showed that children had chosen different sized 'chunks'. This would suggest that while children have been taught this particular method, they have also been given the associated mental skills and understanding to make it their own, even to the point where the first child has used a negative chunk to readjust.

It is interesting to note that in the same school the parallel class employed a range of division strategies with a lower 65% of children answering correctly.

Conclusion

The overarching aim of this paper was to report on the findings from the 2012 data and examine the range of strategies used by the children. From this analysis it is clear that the number line still seems to be under-utilised, despite the wealth of research that reinforces the strength of this particular method (e.g. Beishuizen 1999; Anghileri, Beishuizen and van Putten 2002). While more informal strategies are being used, according to this research, there is some evidence to suggest that these are being sometimes taught algorithmically. This would suggest that teachers' subject knowledge is still weak, despite the fact that it is widely recognised that the mathematical subject knowledge of teachers is an important factor in the teaching and learning of mathematics (Williams 2008). Indeed it was Williams (2008) who recommended that every school should have access to a maths specialist teacher (MaST). This has not yet been fulfilled. Division remains the weakest calculation in terms of success in the 2012 data and does seem to be as Watson and Mason (2012) describe, for many children, 'the odd one out'. While Watson and Mason talk of children developing 'coping strategies' to 'get away with it' our research would show that for many children, they simply do not even tackle this calculation. This research tells us that there are some Year 5 children who are still not able to complete age related calculation questions for all four rules. This continues to have implications for schools with regard to the policies they adopt for calculations but also the importance they place on other aspects of learning mathematics, such as representation (e.g. Barmby et al. 2011).

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