Argumentative activity in different beginning algebra classes and topics

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This study compares students' opportunities to engage in argumentative activity between two classes taught by the same teacher and across two topics in beginning algebra: *forming and investigating algebraic expressions* and *equivalence of algebraic expressions*. The study comprises two case studies, in which each teacher taught two 7th grade classes. All four classes used the same textbook. Analysis of classroom videotapes revealed that the opportunities to engage in argumentative activity related to *forming and investigating algebraic expressions* were similar in each teacher's two classes. By contrast, substantial differences were found between one teacher's classes with regard to the opportunities to engage in argumentative activity related to *equivalence of algebraic expressions*. The discussion highlights the contribution of the topic, the teacher, and the class to shaping argumentative activity.

Keywords: argumentative activity, mathematics, topic, teacher, class, deductive reasoning, inductive reasoning.

Background

In recent years, there has been a growing appreciation of the importance of incorporating argumentation into school mathematics. First, because the principal facets of argumentative activity – justifying claims, generating and justifying conjectures, and evaluating arguments – are all essential components of doing, communicating, and recording mathematics. In addition, accumulating research suggests that participation in argumentative activities – which encourage students to explore, confront, and justify different ideas and hypotheses – promotes mathematical understanding (e.g., Yackel and Hanna 2003).

However, studies have shown that argumentation is not widely used in mathematics classrooms (e.g., Hiebert et al. 2003). Research also shows that students commonly use different kinds of justifications, which often depart from the norms of the field (e.g., Harel and Sowder 2007). Specifically, research shows that deductive reasoning is a source of great difficulties for students, and that students often have difficulties in constructing arguments treating the general case (Harel and Sowder 2007). Instead, students often employ inductive reasoning, which is considered to be the simplest and most pervasive form of everyday problem-solving activities (Nisbett et al. 1983), and is often students' preferred way to form, test, and justify mathematical conjectures (Harel and Sowder 2007).

Studies point to a variety of roles for the teacher in creating opportunities for argumentation (e.g., Yackel 2002). An important role is encouraging students to take an active part in the argumentative activity, e.g., prompting them to generate claims, to provide justifications and to critically evaluate different arguments. Another important role of the teacher involves responding to students' arguments. Thus, for example, the teacher plays a significant role in explicating students' justifications to emphasize the structure of the argument, and in supplying argumentative support that was either omitted or left implicit. Moreover, as a representative of the mathematics community, an important role of the teacher is to present to students what constitutes acceptable mathematical arguments and to model particular ways of constructing and confronting arguments.

However, how the interactions among the teacher, the class, and the mathematical topic shape students' opportunities to engage in argumentative activity is not well understood. The study reported here examines this issue. For this purpose, we use two case studies to compare students' opportunities to engage in argumentative activity between two classes taught by the same teacher, when learning two beginning algebra topics: *forming and investigating algebraic expressions* and *equivalence of algebraic expression*. Each topic requires a different kind of reasoning: Work on *forming and investigating algebraic expressions* by using substitution of numerical values into expressions mainly requires inductive reasoning. In contrast, work on *equivalence of algebraic expressions* requires extensive use of deductive reasoning. The specific research question examined is: How do (1) the contribution of the teacher to the argumentative activity, (2) the contribution of the students to the argumentative activity, and (3) the types of justifications, vary between two classes taught by the same teacher using the same textbook and across two beginning algebra topics – *forming and investigating algebraic expressions*, and *equivalence of algebraic expressions*?

Methodology

Participants, setting, and textbook

Sarah taught two of the classes, S1 and S2, each in a different school. Rebecca taught the other two classes, R1 and R2, each in a different school. Class work in Sarah's and Rebecca's classes consisted almost entirely of work on tasks from the textbook. The textbook used in the four classes was part of the *Everybody Learns Mathematics* program (1995-2002). This study focuses on four central units: Two units deal with *forming and investigating algebraic expressions*, mainly by substituting numerical values into expressions as a means to develop a sense about their behaviour (e.g., task 1 in Figure 1). Work within these units largely requires inductive reasoning. Two additional units focus on *equivalence of algebraic expressions*, dealing with identifying, generating, and justifying the equivalence or non-equivalence of expressions by employing several ideas, such as substituting numerical values into expressions as a means to prove non-equivalence, substituting numerical values into expressions as an inadequate means to prove equivalence, and expanding and simplifying expressions as a means to maintain/prove equivalence (e.g., task 2 in Figure 1). Work within this topic requires extensive use of deductive reasoning, i.e., proving equivalence and non-equivalence of expressions.

Data collection

The main data source was video and audiotapes of the teaching of the four units in each of the four classes.

Data analysis

Detailed data analysis of the lessons included only the whole-class work. The video-taped lessons were transcribed and the argumentative activity in each class during the whole-class work on each topic was then analysed.

1) Consider the algebraic expression $4 - k$:

   Find a positive number and a negative number whose substitution yields a positive result.

   Is there a positive number whose substitution yields a negative result? Demonstrate it.

   Is there a negative number whose substitution yields a negative result? Explain why.

2) Find among the following pairs of expressions a pair in which the expressions are not equivalent:

   $2 \cdot m, m \cdot 2$ $1 \cdot m, m$ $m - 4, 4 - m$ $m + 4, 4 + m$

   For each of the remaining pairs, find a property that shows that the expressions are equivalent.

Figure 1. Examples of textbook tasks (abbreviated from Robinson and Taizi 1997).

The first step of analysis was to examine the teacher's and the students' utterances according to their argumentative function within the whole-class work (e.g., claim, request for claim, justification, request for justification). The second step was to identify the teacher's and students' argumentative moves associated with each claim, indicating them as an argumentative sequence. Two kinds of claims were the focus of the analysis. One was related to generalizations of the behaviour of algebraic expressions in the case of *forming and investigating algebraic expressions* (10 such claims were found in each of the four classes). A second kind of claims was about determining the equivalence of algebraic expressions in the case of *equivalence of algebraic expressions* (13, 11, 33, 30 claims in S1, S2, R1, and R2 respectively). The third step of the analysis involved classifying the types of justifications raised in the argumentative sequences into one of two types: (1) justifications based on a general mathematical rule, and (2) justifications based on a numerical example. We then compared for each topic the two classes taught by each teacher on the contribution of the teacher to the argumentative activity, the contribution of the students to the argumentative activity, and the types of justifications suggested in class.

Argumentative activity in Sarah's classes

Analysis of classroom data revealed that the teacher's contribution to the argumentative activity, the students' contribution to the argumentative activity, and the types of justifications suggested in class were similar in Sarah's two classes, for each of the two mathematics topics.

- *Sarah's contribution*. Sarah prompted her students to establish the claims (generalization for the behaviour of algebraic expressions or determining the equivalence of algebraic expressions). She was the one who usually provided the justifications for the claims, supporting them with proof-related ideas on which they are based.
- *Students' contribution*. The students provided the claims.
- *Types of justifications*. Almost all of the justifications in both classes were based on general mathematical rules.

The following episode from S1 class work on task 2 in Figure 1 illustrates the recurrent argumentative sequence in Sarah's two classes in the two topics. Sarah pointed at the expressions $2 \cdot m$ and $m \cdot 2$:

The contributor	Utterance	The argumentative moves
Sarah	Are they equivalent?	Request for a claim
Student	Equivalent	Claim
Sarah	Right. These expressions are equivalent. In order to prove equivalence we have to use the properties. Here it is the commutative property. We have multiplication so we are permitted to replace the order and it will be the same.	Justification (based on a general mathematical rule) + the proof-related idea on which the justification is based.

Argumentative activity in Rebecca's classes

As in Sarah's classes, analysis of classroom data revealed that the teacher's contribution to the argumentative activity, the students' contribution to the argumentative activity, and the types of justifications suggested in class, were similar in Rebecca's two classes during the whole-class work on *forming and investigating algebraic expressions*.

- *Rebecca's contribution.* Unlike Sarah, in addition to prompting her students to establish claims (generalization for the behaviour of algebraic expressions), Rebecca also requested students to justify the claims and encouraged a dialectical discourse among students, by asking for their opinion about a claim raised in class. Her response to students' arguments was approval.
- *Students' contribution*. Rebecca's students provided the claims, the justifications, and collectively evaluated claims offered in class.
- *Types of justifications*. Almost all the justifications in both classes were based on general mathematical rules.

The following episode from R2 class work on task 1 in Figure 1 illustrates the recurrent argumentative sequence in both of Rebecca's classes on this topic. After substituting positive and negative numbers into the expression $4 - k$, Rebecca asked the class to generalize the outcomes produced by the substitutions, and a student suggested a generalization. Rebecca asked for the other students' opinion about it, which led to students' collective evaluation:

The contributor	Utterance	The argumentative moves
Rebecca	Which numbers will give positive results?	Request for a claim
Student 1	Any number smaller than four	Claim
Rebecca	Did you hear what she said? Is she right?	Challenge for evaluation
Student 2	I don't think she is right	Objection
Student 3	Because if she substitutes half…	Justification (for the opposition)
Student 4	If she substitutes half it will be okay	Opposition
Student 5	Minus one?	Justification (for the first opposition)
Rebecca	Substitute minus one here [points to the algebraic expression]	Challenge for examination

Later on, the students accepted the initial generalization and justified it.

In contrast to the previous results, in the case of *equivalence of algebraic expressions*, while similarity was found in both classes in the teacher's contribution to the argumentative activity during the whole-class work, considerable differences were found between the classes with regard to the contribution of the students to the argumentative activity and the types of justifications suggested in class.

- *Rebecca's contribution.* As in the previous case, in both her classes, Rebecca encouraged her students to establish claims (determining the equivalence of algebraic expressions), to justify claims, and to evaluate claims. Her response to students' justifications was approving the correct ones or encouraging a different justification in cases of the incorrect ones, with no explicit distinction between adequate and inadequate justifications.
- *Students' contribution*. In both classes students provided the claims and the justifications. However, whereas in R1 students' arguments were frequently challenged and evaluated by their peers, in R2, despite of Rebecca's encouragement, no critical evaluation among students developed.
- *Types of justifications*. In R1, all justifications relied on general rules – simplifying and expanding algebraic expressions by using properties of real numbers. In contrast, in R2, students repeatedly suggested substituting numerical values into expressions to prove equivalence (a specific case of supportive examples for universal statements as mathematically invalid).

The following episode from R2 class work on task 2 in Figure 1 illustrates the recurrent argumentative sequence in R2. Rebecca pointed at the pair of expressions $m + 4$ and $4 + m$ written on the board:

The contributor	**Utterance**	**The argumentative moves**
Rebecca	Are they equivalent?	Request for a claim
Student	Equivalent	Claim
Rebecca	How can I prove it?	Request for justification
Student	Because if you substitute 2 you get 6 in both	Justification
Rebecca	Okay. But maybe it is by coincidence?	Request for justification
Student	Substitute 3	justification
Rebecca	[Substituting 3 in both expressions and obtaining 7 in both]. Do we have to substitute more numbers in order to prove that they are equivalent? What do you think?	Request for justification
Student	Substitute 4	justification

Discussion

One main finding was the identification of a typical approach to argumentation of each teacher, as manifested in both her classes and during the teaching of both topics. Sarah's argumentation approach exposed students to mathematical arguments and explicit ideas of proving, but it did not give the students a significant role in their generation and evaluation. Rebecca's approach to argumentation largely shifted to students the responsibility for justifying and evaluating claims, but she seldom discussed the arguments raised in class or offered an explicit distinction between adequate and inadequate ones. While restricted to the cases of this study, this finding of a “constant” teaching approach to argumentation that hardly changes even when the situations change provides new information for research dealing with teaching mathematics in general and in encouraging class argumentation in particular, and requires further examination.

Still, another main finding was the identification of differences in the opportunities for argumentative activity in the case of one of the teachers in the teaching of one of the topics. The argumentative activity during work on *forming and investigating algebraic expressions* was similar in Sarah’s two classes as well as in Rebecca’s two classes. However, whereas the argumentative activity during work on *equivalence of algebraic expressions* was similar in Sarah’s classes, there were substantial differences during work on this topic between Rebecca’s classes. These differences were expressed in different types of justifications provided by students in each class and in the extent to which dialectical discourse developed in each class. These differences can be related to the intersection of mathematical situations that involve deductive reasoning, known to be difficult for students (Harel and Sowder 2007), and Rebecca’s approach to argumentation, which included students but hardly acted on their contributions. In contrast, work associated with *forming and investigating algebraic expressions* basically involves inductive reasoning, known to be students’ usual preferred way to form and test mathematical conjectures (e.g., Harel and Sowder 2007). Consequently, it is possible that the use of inductive reasoning suited students’ preferences. In Sarah’s case, however, her “non-inclusive” approach apparently prevented the class and the mathematical topic from playing a dominant role; thus they did not serve as a source of differences in neither of the topics. These findings emphasize the need for further research into the role of the mathematical topic – in addition to the teacher – and in particular inductive- and deductive-related topics, in shaping the argumentative activity in class. Likewise, they highlight the need to incorporate attention to another factor: the classroom.

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