May mathematical thinking type be a reason to decide what representations to use in definite integral problems?

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In this study, we focused on whether mathematical thinking type affects what representations to use in definite integral problems. The participants were three of thirty seven first year undergraduate mathematics students who were selected through a purposeful sampling technique. Data collection techniques were tests and interviews. Tests were used for determining which students going to be selected for interviews and main data were collected by interviews. Results show that students' mathematical thinking types have some effect on their representation preferences. On the other hand it seems that students' problem solving behaviours are more affected by teaching processes than thinking types.

Keywords: Problem solving behaviour, mathematical thinking type, multiple representations

Introduction

One of the most important goals of mathematics education research may be said as aiming to improve students' problem solving performance. However it has always been difficult to reach this goal due to insufficient knowledge of processes and strategies used by different type of students to solve various problems (Schoenfeld 1992). There are some types of variables that have an influence on problem solving behaviours and performance. According to Days, Wheatley and Kulm (1979) the two most important ones are subject variables and task variables. Subject variables can be mathematical experience, cognitive level or thinking type, whereas task variables can be language and structure of the problem, context of problem and placement of the problem. In this study we focused on the students' mathematical thinking types as a subject variable and language of the problem as a task variable with respect to the teaching and learning of definite integral problems in university level.

Literature on mathematical thinking differences includes various studies such as its role in the problem solving process and success at problem solving (Lowrie and Kay 2001). Krutetskii (1976) categorised mathematical processes into three: in addition to the analytic and geometric preferences, he drew attention to the existence of harmonic processes which use both preferences together. Analytic learners can easily work with abstract diagrams and tend to use verbal-logical components more than the visual-pictorial components in the problem-solving process (Presmeg 1985). Visual learners tend to present the problems using components they can understand visually. Harmonic learners, on the other hand, are capable of using both the analytic and geometric approaches together in a well-balanced way. Some studies reported that analytic learners were more successful at problem solving than visual learners (Lean and Clements 1981). However, this could be related to the difficulty level of the test and students' prior experiences (Lowrie and Kay 2001). As may be retrieved from the relevant research, more research is necessary to explore students' problem solving tendencies in terms of their experience when tackled with mathematics problems, particularly, in terms of multiple representations.

The knowledge and use of multiple representations have been dealt with many researchers in various contexts. On the other hands, various studies exist in the literature investigating the effect of multiple representations on the comprehension process (Goldin and Kaput 1996), the development process of teaching environments focused on multiple representations (Kendal and Stacey 2003), the consistency between the preferred and used representations and the investigation of representation knowledge-beliefs (Sevimli and Delice 2011). Research in literature confirms that differences in multiple representations are an important factor in the learning environment. Moreover, when identifying mathematical process types, the approach and the components used in the solution are examined. Previous studies tried to investigate students' mathematical process types through the representations they used. Galindo-Morales (1994) studied traditional and CAS supported teaching environments in relation to mathematical differences and found that, like analytic learners, visual learners also used algebraic methods. Moreover, Sevimli and Delice (2011) reported that the students tend to prefer algebraic representations, and that input representations in a problem statement could affect preferences, especially for visual learners. These findings have inspired us to do research on students' problem solving behaviours who have different thinking types. By this aim, we tried to find an answer for that problem statement "Does Mathematical Thinking Type Decide What Representations to use in Definite Integral Problems?" This study is significant in that it was focused on the relationship between calculus students' differences in mathematical processes and their problem solving behaviours in terms of multiple representations.

Method

Research Design and Participants

This study investigated a certain case within its boundaries with an interpretivist paradigm and thus had a qualitative research approach. As the case of the potential impact of learners' mathematical process differences on the process of using multiple representations was explored within the concept of the definite integral, the research was a type of case study. The participants were three of thirty eight second-year undergraduate mathematics education students who selected using purposeful sampling technique according to their mathematical process type (Patton 1990).

Data Collection Tools

Data collection techniques were tests and interviews. Two different tests were used for two different purposes. The Mathematical Process Instrument (MPI) was developed by Presmeg (1985) based on Krutetskii's (1976) thinking structures theoretical framework. The participants were expected to identify their way of thinking during the mathematical problem-solving process. Each problem in the MPI could be solved using either visual or non-visual methods. The participants were asked to choose the method that was similar to their own solutions. The instrument was used to categorise the participants as visual, analytic and harmonic according to their mathematical process types in the mathematical problem-solving process.

The second data collection tool was a Representation Preferences Test (RPT) which was developed by the researchers. RPT was designed in order to determine participants' tendencies to use different representations for the definite integral and was used in earlier studies (Sevimli and Delice 2011). The test consisted of nine definite integration items each of which represented a different objective of the course. The test was found to have face and content validity after analysis by five experts in mathematics education in our university.

After administering the tests, semi-structured interviews were conducted with three respondents for a deeper understanding of the mathematical processes and skills involved in representation preferences. These three participants for the interviews were selected using the purposeful sampling technique. Main selection criteria were that each participant had a different mathematical process type and that they were typical examples of the mathematical process type they had. Interviews were conducted based on the RPT answers.

Data analysis

Participants' mathematical process types were determined by a standard deviation value added to and subtracted from the average. The participants were thus grouped into three categories of visual, harmonic and analytic according to their mathematical process types. Each participant’s representation preferences for each question were analysed separately within categories of numeric, graphic, algebraic or mixed. The data was analysed by descriptive statistics. Main data collected by interviews and interview data was tagged for analysis using an open coding method. Responses to the same interview questions of the students who had different mathematical styles were coded under a number of themes.

Findings

MPI scores were used in order to determine the participants' mathematical process types. The standard deviation value was added and subtracted to the average and the participants whose scores were 20 or more were coded as “Visual”, whose scores were 9 or less were coded as “Analytic and whose scores were between 10 and 19 were coded as “Harmonic” participants. An interview was conducted with a participant who could represent the each mathematical process types.

Case of Analytic Participant

The participant was coded as AP (Analytic Participant) and the researcher who conducted the interview as I (Interviewer). The characteristics of AP were that his MPI score was 6 and his general preference tendency was algebraic representation with 52%. Important extracts from the interview is presented below.

> *I:* Do you consider alternative solution methods (apart from algebraic) when solving a calculus problem?
>
> AP: I like problems that can be solved using equations more. We also use this representation more at school; actually our lessons are almost dependent on this representation. I think in any case a mathematician can obtain more correct answers more easily as long as the algebraic representation is used. So, I do not consider any alternative way for solving calculus problems.
>
> *I:* When solving a calculus problem, is the representation used in the problem statement important for you?
>
> *AP:* In fact, it is important when trying to interpret the data. When solving the problem, what the problem asks is important.

AP thought that different representations were used to simplify the data and believed that algebraic representations were sufficient for problem solving. While implying that the representations used in the problem-solving process changed in relation to the topic, this participant stated that algebraic representations were enough for calculus problems.

Case of Harmonic Participant

One of the harmonic participants was interviewed in order to investigate the process thoroughly; this participant was named as the Harmonic Participant and was coded as HP. MPI score of HP was 16, general preference tendencies were algebraic and graphical representations with 42% and 34% respectively.

> *I:* Do you consider alternative solution methods when solving a calculus problem?
>
> *HP:* For some mathematical problems there could be more than a single solution method. I usually start the process with my most preferred one. If I am not successful or sure, I try another one. However, in higher education mathematics, it is not something I do frequently because usually I can only know a single solution method.
>
> *I:* When solving a calculus problem, is the representation used in the problem statement important for you?
>
> *HP:* I tend to think more on the problem types which are presented in different representations. For problems which are stated algebraically, using systematic calculation techniques is enough for the solution. However, especially if tables are used, I think the question becomes more difficult although the data is easy to understand.

HP stated that she had to think more about the problems that were expressed in different representations and that the input representation could affect the representation used in the solution. Explaining that she was used to algebraic representation in calculus problems, HP mentioned that in advanced mathematics problems she generally knew a single solution and followed this solution method.

Case of Visual Participant

The visual participant interviewed who represented the visual type was named as Visual Participant and coded as VP. The MPI score of VP was 24 and her general preference tendencies were algebraic and graphical with 30% and 34% respectively.

> *I*: Do you consider alternative solution methods when solving a calculus problem?
>
> *VP*: Yes, I do, but I give up easily this new approach if I could not tackle with it. The structure of the problem, my previous experiences and habits influence the solution way I would use.
>
> *I*: When solving a calculus problem, is the representation used in the problem statement important for you?
>
> *VP*: Of course it is, the visual qualities of the problem are important for me and when it is visual, it is easier for me to understand the problem. If not, I try to visualise the statement during the solution process. But it is much more difficult for me to do this visualisation in mathematics problems at the higher education level.

VP stated that the input representation was very important for her. VP paid attention to the visual perception of the problem and expressed that this visualisation facilitated understanding. VP experienced a paradox in her answers to RPT. VP mentioned that she wanted to solve the problems using different representations, but her prior knowledge and habits could sometimes prevent this.

Problem Solving Behaviour

The participants were asked some RPT problems to observe their problem solving behaviour. The answers which are illustrated below show that AP interpreted roots of equation as boundaries. *HP* drew a proper graphic but she could not decide right integral formula for the area between the graphs. *VP* drew a wrong graphic and she stated that, she uses graph as a confirmative. In brief, none of the participants who were interviewed could reach a correct solution for this problem. Moreover, participants' thinking types have not shown a clear influence on problem solving strategies. Finally it seems that, all participants except the algebraic ones tended to use representations as a confirmative tool.

Question: Calculate the area bounded by $y=x-3$ and $y^2= x-1$?

AP	***HP***	***VP***

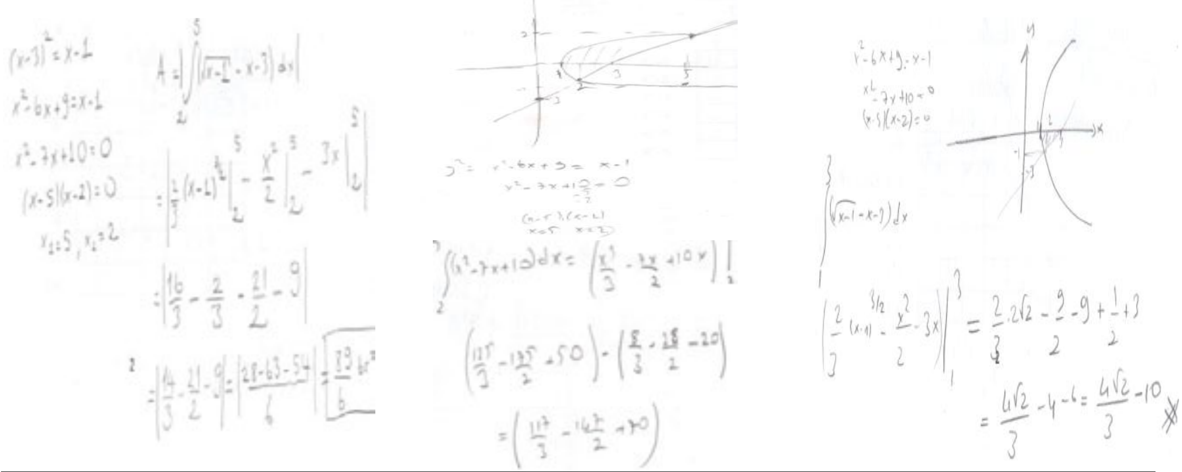

Figure 1: The answer of participants for problem of area in integral

Discussion

The participants exhibited different representation preference tendencies in definite integral problems. Namely, what representations students think to use before the problem solving might be different throughout the actual problem solving. Sevimli and Delice (2011) reported that students, each with their own preference, tended to use a single representation when they were solving the problem. This could be due to a lack of knowledge and skills in using different representations. The participants' general preference for algebraic representation in their answers to RPT could be due to a dominance of algebraic representations in the lessons within the current system and to their algebra dominated experiences also in the content of the education programmes prior to their university education (Kendal and Stacey 2003).

In the interviews, AP and HP stated that integral problems required a similar approach (integral operations), different representations were used to facilitate the comprehension of the data and it was easier to solve problems with algebraic representations. Also, previous studies shown that mathematicians used diagrams: noticing properties, generating conjectures and understanding a mathematical claim (Samkoff, Lai and Weber 2012).The concept of multiple representations is not used to facilitate the presentation of the data in a problem, but to create interdisciplinary relationships and to create an awareness of the different definitions of the concept (Goldin and Kaput 1996). Thus, students who lack a sufficient level of representation knowledge and awareness, may not exhibit a positive performance in choosing the appropriate representation in the problem-solving process.

The findings revealed that students' mathematical thinking types might affect their representation preferences. On the other hand it seems that students' problem solving behaviours are influenced by teaching processes rather than thinking types. The analytic participant felt confident at solving problems by using algebraic representations, the harmonic participant pointed out that she tended to use representation which she had experienced before. The visual participant preferred the use of more visual representation but she complained about her insufficient knowledge of representations. This study may highlight a perspective for future studies on the development of educational designs.

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