A response to the JMC Working Group Report "Digital technologies and mathematics education"

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A multi-layered model of STEM subjects extending upwards from mathematics to applications, with separate classical and digital branches (classical STEM is underpinned by continuum mathematics, digital STEM by discrete mathematics), is used to interpret the JMC Report. The Report recognises that today's schoolchildren are wholeheartedly embracing the digital applications with which they are surrounded. But perhaps more significantly they are the first generation to grow up in a society being shaped by digital STEM, whereas their parents and their teachers have lived their lives in a classical STEM world. There is always a generation gap but for this generation a widening gulf separates teachers and pupils. The Report recognises a danger that school mathematics will seem like a dead language and suggests three remedial steps: bring digital technologies (such as Dynamic Geometry and Computer Algebra Systems) into mathematics classrooms, emphasise student-led modelling and problem solving, and include a programming language in the curriculum. In other respects the Report is silent about curriculum content. The traditional mathematics curriculum emphasises continuum mathematics but future generations will need more emphasis on discrete mathematics if they are to understand their world, model applications in it, and become application innovators.

Keywords: STEM subjects, continuum mathematics, discrete mathematics, application modelling, curriculum

Introduction

This paper uses the Report from a working group of the Joint Mathematical Council (JMC) of the United Kingdom (JMC 2011) as a framework for the latest in a series of papers, most recently (Osmon 2012) arguing for mathematics curriculum reform. The Report is apparently developed from two premises:

1. Today's schoolchildren are actively engaged with the IT-application-rich environment in which they find themselves- and this is something beyond the experience of any previous generation, and
2. The UK needs a new generation of computer scientists and IT-application innovators and they will need mathematics knowledge for these roles.

These are my words but are, I believe, a fair summary.

I suggest that both premises are likely to be widely accepted, but their implications may discomfort mathematics teachers. The Report suggests that mathematics is in danger of looking like a "dead language" to these schoolchildren. Of course there is always a generation gap between teachers and pupils, but for the first time, as I explain below, a cultural change is occurring that makes bridging hard. With regard to the second premise, during my career I have been an IT product innovator and can confirm from experience the importance of mathematics knowledge.

Starting from these premises, the Report's message is that three kinds of reform of mathematics education are needed:

(A) Integration of digital mathematical tools, for example Dynamic Geometry and Computer Algebra Systems, into teaching/learning and assessment practice.

(B) Emphasis on modelling, specifically student-led modelling, and problem solving.

(C) Learning programming. There has been a recent revival of interest in inclusion of programming in the curriculum, coinciding with condemnation of ICT as insufficiently challenging.

These three reforms seem progressively more ambitious. (A) will greatly affect examination as well as mathematics classroom practice. European Schools (2010a), referenced in the Report, gives some possible implementation details. My two BSRLM papers on Tablets in mathematics learning, for example Osmon (2011), predict every child will soon have a Tablet, and this will render school PC labs obsolete, as well as greatly reducing infrastructure costs. Classrooms will need wireless networking, schools will need a server and broadband connection and that is about it. We should be optimistic that teachers will be able to cope with these changes but (B) and (C) particularly will present significant challenges.

Curriculum content

Besides mention of "an algorithmic programming language"- no curriculum content changes seem to be envisaged in the Report and I find this curious. Elsewhere the air is thick with content proposals for example:

European Schools (European Schools 2010b), referenced in the Report

Tony Gardiner in The De Morgan Journal (Gardiner 2012)

Imminent National Curriculum changes

But generally these proposals "tweak" the "traditional" curriculum, with its emphasis on mathematics of the continuum. There is no mention, for example, of set theory, except in the European Schools curriculum.

The mathematics I needed in my career, as a physicist and then as an electronic engineer, was indeed *continuum mathematics* (functions of continuous variables, differentiation and integration, differential equations), but then later, as a computer scientist and IT product innovator, it was *discrete mathematics* (set theory, Boolean algebra, graph theory, state machines. *So I have come to believe that today's IT-application-rich world, inhabited by our children, is underpinned by discrete mathematics, in contrast with the continuum mathematics of the traditional curriculum that underpinned the knowledge-world of earlier generations.*

Thus, I argue that our children are not learning very much of the discrete mathematics that supports their IT-application-rich world, because it is largely absent from the traditional curriculum. But this is the mathematics they will likely need if they are to become IT-application innovators.

STEM knowledge hierarchies

I found it remarkable that, while there was an overlap in the mathematics I used as physicist and electronic engineer and another overlap in the mathematics for computer science and IT product innovation, there was little in common mathematically between these two pairs of subjects. Rather there was a gulf between them. With hindsight it seems I have experienced two parallel STEM hierarchies:

c-STEM (continuous/classical)	d-STEM (discrete/digital)
Engineering applications	IT applications
Engineering technology	Information Technology
Physics	Computer Science
Continuum mathematics	Discrete mathematics

In each hierarchy the bottom layer is the relevant pure mathematics, the layer above is the characteristic science, above that lie the hierarchy's characteristic technologies, and the top layer contains the applications of those technologies.

In the case of c-STEM, continuum mathematics (most obviously differential equations) supports classical physics, which has several branches. For example Maxwell's equations of the electromagnetic field is a set of four differential equations, and on these rests the whole range of electrical and electronic engineering: heavy current machines and their applications at one end, through electronic devices and circuits, to communication of radio waves and their applications at the other.

In the case of d–STEM the layers are not so clearly differentiated and I suggest this is because of the rapid pace of development: whereas c–STEM developed, and to an extent crystallised, over centuries, it has been barely two generations from construction of the first stored program computers to today's digital IT-rich society. But, it is clear that, just like physics, computer science has several branches. One such, for example, is databases. The pure mathematics that these rest on is set theory and they are constructed and interrogated using relational programming languages, like SQL (Structured Query Language), which are a branch of computer science. IT applications built on them include Internet shopping and data mining. But already there are many branches of computer science and its dependent technologies and applications are proliferating.

Terminology: STEM and STEMs

STEM is a well-known acronym for generic Science, Technology, Engineering and Mathematics. I envisage its analysis into distinct four-layer vertically integrated bodies of knowledge: individual STEMs. (A bottom-up acronym: MaSTAs for Mathematics, Science. Technology and Applications would be more informative but less resonant.) Practitioners at the different levels within a STEM and possessing the STEM's characteristic knowledge to varying degrees, form a community of initiates. STEMs vary in size and degree of development and correspondingly have more or less important roles in our civilization. Until the end of the twentieth century c-STEM was pre-eminent. It now seems clear d-STEM is rapidly becoming the new pre-eminent STEM and bringing far-reaching industrial and socio-economic change.

In my oral presentation I used the term "i-STEM" (information-STEM), rather than "d-STEM" (discrete/digital-STEM) as here, and for consistency I should have called the older one "p-STEM" (physical-matter-STEM), rather than c-STEM. But this is a paper about mathematics, not science and technology, and so it is more appropriate to distinguish these two STEMs by their mathematical roots than by the "stuff" they work with. Besides, there are some continuum-based information applications, most obviously music, and using the "i" label would tend to blur the boundaries and so demonstrate less clearly the STEM pre-eminence change that is occurring.

Modeling

The Report's second message is increase the emphasis on modelling. Model-making is a hot topic in mathematics education across all developed countries (Maasz 2011). This was also a message in Osmon (2012). We make models to gain understanding of something complicated. I would say it is our general way of making sense of the world. A model simplifies, by focusing on (extracting) only certain aspects of some Real-world domain.

A mathematical model may be constructed in any STEM layer. Thus Maxwell's equations are a scientific model. But most models are made in the applications layer, in some particular domain of application. It is often helpful to regard a mathematical model as having the structure shown in the Figure. The Extract is the property or process to be modelled and the essence of modelling is considering the Extract in isolation from the rest of the world. It is then (somehow) analysed by the modeller into two portions, the domain D and the relevant mathematics M, so that relevant domain properties, typically in the form of textual descriptions and measurement units, are separated from the pure mathematical properties.

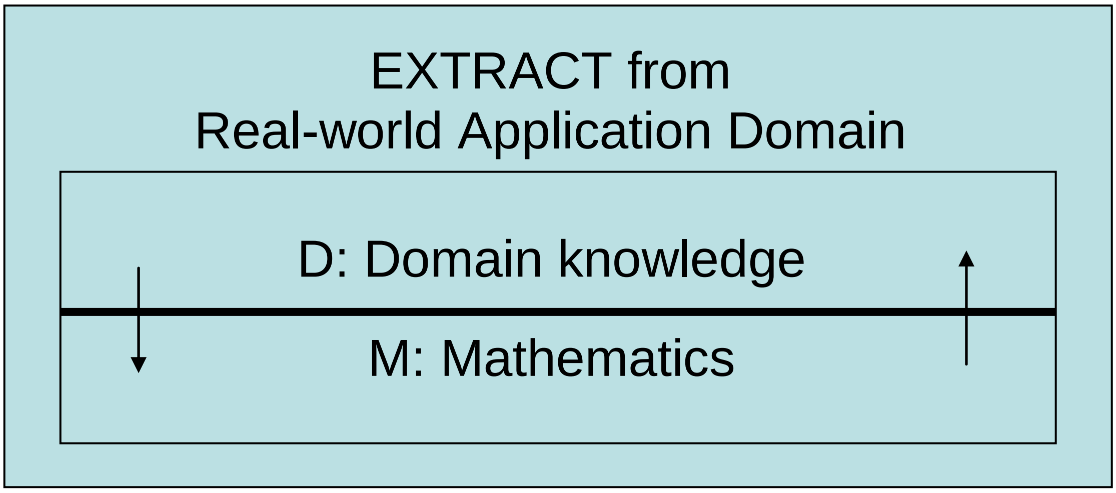

Figure. Two-part structure of mathematical models.

Model structures are similar in c-STEM and d-STEM and model-making is a similar activity. For example, consider a c-STEM model that calculates the trajectory of a ball thrown in the air. Within its domain D are velocity and projection angle in appropriate units. These parameters are passed, as pure numbers, to M (equations for horizontal and vertical motion etc) to be worked with, and the resulting equation to a parabola is passed back into D for units of length to be appended. Now consider digital image processing as a d-STEM modelling example. The domain D is a digital image displayed on a screen. This comprises an array of pixels (picture elements) of varying brightness, which may be transformed into a corresponding array of numbers, which may in turn be manipulated mathematically so that when transformed back into pixel arrays the image has been altered in some systematic way. This example is described in more detail in Maasz (2011) and Osmon (2012).

Model-making potentially provides fine learning situations, superior to disembodied exercises or word problems, because models demonstrate the value of mathematics for gaining understanding of a real problem situation and not merely "solving" it.

Student-led modeling

The Report advocates student-led modelling. Model-making projects with small student-led teams is emerging as good education practice, for example Maasz (2011). The Figure partitions a model into domain D and mathematics M. Evidently for a model-making task to be meaningful, the modellers need relevant application domain knowledge, as well as the relevant pure mathematics knowledge, and there are precisely three domains where we may expect students to be knowledgeable. Here are the three domains together with suggestions for two model-making projects in each knowledge domain together with the requisite mathematics.

a. Everyday environment (increasingly this means IT products and services):
Vending machines: automata: Moore machines;
Codes (Bar-codes, QR-codes, reliable recovery of streamed information from CDs/DVDs): coding theory- digital data can be communicated reliably despite unknown errors- not so for continuous data.

b. Personal interests (increasingly, as remarked in the Report, these have an IT flavour):
Music Database: relational algebra, compare with a tree-structure solution from graph theory;
Image Processing: array processing example in (Maasz 2011) and (Osmon 2012).

c. Curricula of other subjects (most obviously science):
Mendel's pea plants: Monte Carlo simulation, hypothesis testing;

Population growth: large population behaviours can be treated as continuous distributions, but small ones need probability: Monte Carlo again. The predominance of discrete mathematics in these example applications supports the case for giving it increased emphasis in a reformed curriculum.

Programming

The Report's third message is that learning programming should become part of a mathematics education. I have long been puzzled that we don't teach some form of algorithmic language in mathematics, as a vehicle for learners to describe their mathematical reasoning. This could be a pseudocode as used in some introductory programming courses, for example (Albey, nd). However real programming like model-making is a creative activity and this makes both qualitatively different from the rest of school mathematics. And in my opinion teaching programming within mathematics is the more ambitious of these two goals.

Conclusions

The current mathematics curriculum has an important role imparting knowledge of the foundation layer in c-STEM: for long the pre-eminent STEM. Now d-STEM is rapidly gaining eminence, becoming culturally and economically important for the rising generation and the case for reform of the mathematics curriculum so that it also imparts the discrete mathematics needed by d-STEM seems strong. If mathematics is to be a live language for future generations, and the UK is to grow a generation of IT product innovators, as the authors of the Report hope, I believe I have shown that schoolchildren will need a foundation of discrete mathematics knowledge: otherwise, within the terms of the Report, the range of student-led modeling opportunities will be greatly restricted. Of course the curriculum already has some discrete as well as continuous content, traceable back to the beginnings of mathematics in counting and measurement respectively, and I am certainly not proposing elimination of continuum mathematics from the curriculum but rather a rebalancing of content to meet the needs of future generations of d-STEM novitiates. I envisage that, besides the discrete topics mentioned above: Set theory, Boolean algebra, Graph theory, and State machines, there will be others, including a deeper understanding of probability than is presently expected and, presumably, programming in some form.

My work on the curriculum has been focused on Level-3. But the world is changing, rapidly, and all learners, not least those whose mathematics education ends at Level-2, will need an appropriate knowledge of discrete mathematics and I would expect this requirement would begin to be met in the Level-1 curriculum. When I say these things, one response is “What content should be dropped from the traditional curriculum?” How to rebalance the curriculum is a matter of priorities and practicalities and not something that can be decided all at once. The purpose of this paper is to say that the issue should not be ducked, a divide is opening between teachers and their pupils and rebalancing needs to happen, else mathematics will indeed come to seem like a dead language and we will not grow the future generations of IT- applications innovators the UK needs.

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