Modelling as a driver for the Level-3 curriculum

Peter Osmon
Department of Education and Professional Studies, King's College London

A previous paper identified the potential learning gains from substituting group-project model-making for traditional applied mathematics at Level-3. In this paper I investigate the feasibility of this change by considering a set of recently published project proposals. These range over various application domains and mathematics topics. I suggest subjective criteria for evaluating potential projects from the likely viewpoints of learners and teachers and learners' knowledge of a project's application domain and the appropriate mathematics as objective success criteria. It follows that, except where the application domain is familiar to mathematics students, projects will have to be interdisciplinary - which generally seems impractical. But in the application domain that is familiar to all students - that of twenty-first century everyday life - the mathematics (IT and probability) is increasingly discrete whereas the curriculum still emphasises the mathematics of the continuum- surely evidence of how out of date it has become. The remedy might be for model-making to determine the pure mathematics in the curriculum.

Keywords: Level-3 mathematics curriculum, group-project model-making, application domain, discrete mathematics, pure mathematics.

Introduction

This paper is the latest contribution to my ongoing investigation of the Level-3 mathematics curriculum and its utility. The immediately preceding paper (Osmon 2011) outlined the fundamental role of mathematical modelling in our science and technology based civilisation. It went on to propose that a curriculum where mathematical modelling is substituted for traditional applied mathematics potentially has several advantages for learners. However, I emphasised that merely making use of- applying- existing models greatly dilutes the learning experience. Unfortunately the time likely to be available for modelling at Level-3 probably means that, even as a group activity, it is unrealistic for students to make substantial models. Therefore, providing basic models for them to develop further must be the way to go.

This paper is initially about the qualities that make a good mathematics modelling project. To this end I propose some fairly obvious subjective assessment criteria and apply them to projects proposed by a recent European collaboration. I explain how they satisfy my criteria to varying degrees, identifying various strengths and shortcomings, and select the most interesting one for objective analysis- of feasibility. I go on to show how the current curriculum, with its weaknesses in discrete mathematics and probability, restricts the scope for devising good projects. Finally I ask whether we might get a better mathematics curriculum by reforming it so that it can support a wider range of modelling projects.

Case studies

Real-World Problems for Secondary School Mathematics Students (Maasz 2011) is a recently published European set of Case studies. All contributions emphasise their motivating potential for mathematics learning and their suitability for group work in the classroom. A range of application domains is covered: sport, space flight, environmental issues, lotteries, the information society, growth of populations etc. And various mathematical topics are deployed: geometry, probability and statistics, calculations with large numbers, spreadsheet working with arrays of numbers, etc.

I have reviewed the contributions from twin viewpoints: the kind of model I can imagine wanting to make (of course I am not a seventeen year-old) and the kind of model-making project I can imagine wanting to supervise. To these ends I identified three criteria: (a) Application domain: the subject matter is topical and important, or potentially so, in students' lives, but not nerdy; (b) Group working suitability: scope for argumentation and division of labour; (c) Mathematical difficulty: easy basic mathematics, but open-ended so group members with a range of abilities can gain insights;

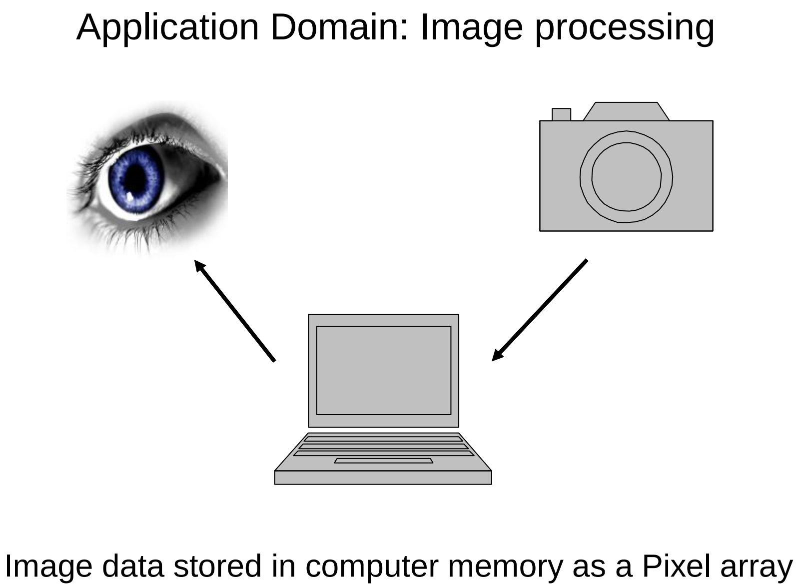

Figure 1 Digital Image processing project: overview of the Application Domain

By these criteria one contribution- Thomas Schiller's "Digital Images: Filters and Edge Detection" in the Digital Image Processing application domain stood out. The students are given a basic model. The project consists of developing the model in ways suggested to them, but with plenty of scope for exploration. Figure 1 shows an image of some kind captured as a digital photograph, transferred to a computer, and then visible on the computer's screen as an array of pixels (picture elements). For simplicity assume a grey-scale image comprising pixels which are 8-bit words i.e. 2^8 = 256 shades of grey where 0 represents black, 255 represents white and shades of grey are represented by the values in between.

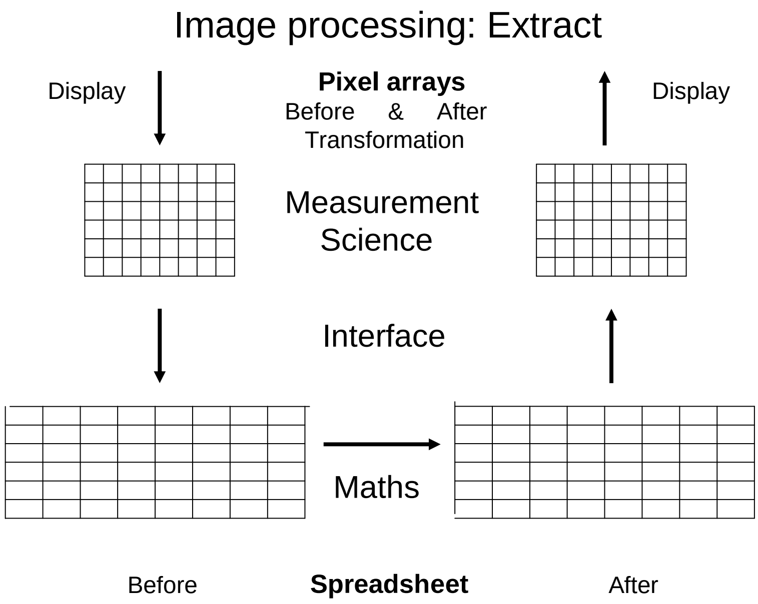

Figure 2 Digital Image processing project: Extract from the Application Domain

Figure 2 (left-hand side) shows the array of pixel values copied to a spreadsheet where the spreadsheet elements are 1:1 with the array of image pixels. The spreadsheet may be transformed by mathematical operations to create a new spreadsheet and Figure 2 (right-hand side) shows the spreadsheet elements copied to the computer's memory for display as a transformed pixel-array (image). (The two transfer routines were provided by a helpful computer scientist.) Figure 3 indicates how some image processing operations may be effected by transforming the spreadsheet data: smoothing and sharpening are fairly basic but edge detection is more challenging.

Image processing:
Spreadsheet transformations

Filters:

A. Smoothing-

Scale spreadsheet elements by averaging over nearest neighbours:

$$1/9. \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

B. Sharpening-

Scale spreadsheet elements by negative weighting of nearest neighbours:

$$\begin{vmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{vmatrix}$$

Edge detection (Static to identify objects, dynamic for motion detection):

Construct a spreadsheet of differences from x and y neighbours

Figure 3 Digital Image Processing project: Spreadsheet transformations

This Image processing project clearly satisfies all three criteria, as follows:

(a) Application domain: is indeed topical and important in students' lives.; typically they will have cameras built into their mobile phones and transfer photographs to computers routinely, and perhaps manipulate them with software such as Photo-shop.

(b) Group working: there is clearly scope for argumentation and division of labour.

(c) Mathematical difficulty: as Figure 4 indicates the mathematics of basic image processing is easy, with scope for exploring more challenging applications.

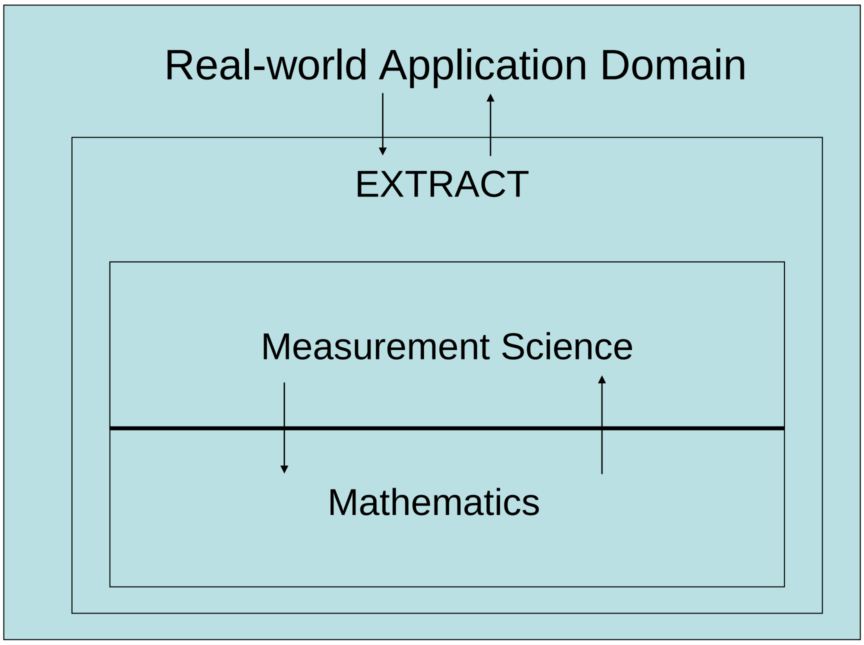

Figure 4 Generic model: showing model structure within the Extract from the Application Domain

A generic model of mathematics applications

Figure 4 describes this generic model. I derived it for my own use after working with models in various branches of science and technology and introduce it into this paper to help clarify what models are, by separating their constituent parts, so that the kinds of knowledge needed for making any particular model is apparent. While the generic model is evidently related to the process of model-making described by mathematics educators, for example (Niss 2007), it serves a different purpose: it is a description of models not a prescription for making them: a WHAT-IS not a HOW-TO.

Figure 4 shows a set of nested boxes. The outermost represents the real-world application domain of the model, for example growth of populations. Inside it is the particular EXTRACT from the application domain that is to be modelled - leaving behind what is irrelevant to this particular model. The actual model is represented by the innermost double-box. The box is double because it contains the two principal parts of the model - the (pure) mathematics and what I call the measurement science. All application domains have characteristic measurement science (for example processes and units of measurement). The line separating measurement science and mathematics is the interface translating between them - it is also part of the model. Thus, a mathematical model has three parts: some pure mathematics, some measurement science, and an interface between them.

Correspondingly three kinds of knowledge are needed to make a model. Thus, in the case of the Image processing project, the Application Domain is Image processing of a captured digital image (Figure 1) and the Extract (Figure2) is a model in three parts: (a) the mathematics: formulas in the spreadsheets (Figure 3), (b) the measurement science: pixels, and (c) the two-way interface (a pair of given software routines) converting pixel arrays into spreadsheet arrays, and vice-versa. In order to develop this model learners need to know the relevant mathematics and be familiar with idea of pixels - we can tick both these boxes - and the interface is provided for them.

So, analysis of this project using the generic model confirms that it is appropriate. Evidently, we can do a similar analysis for any candidate model-making project to check its feasibility: the mathematics knowledge requirement can be decided by checking against the curriculum; the application requirement is less straightforward - where the application domain is everyday, this knowledge can be assumed, but if it lies within another curriculum subject, then it seems the project has to be cross-curricular.

Other Case Studies projects

By my subjective criteria two other Information Society projects - dealing with error detection and correction in the context of information communication - are very attractive: reading Bar-codes and QR-codes, and playing CDs/DVDs. Both projects demonstrate, and allow the students to investigate, the important property possessed by discrete (digital) data that, unlike continuous (analogue) data, it can be communicated reliably even in adverse conditions - by means of error detection and correction. The mathematics of check-sums is easy, Hamming distances and Solomon-Reed codes are more challenging. These two projects are about information communication, while the Image processing project is about information transformation. I looked in vain for a project concerned with information storage (the third dimension of Information Society Technology), ideally in the vitally important field of databases. But of course the students have no knowledge of even basic set theory and hence relational databases are a closed book.

The longest contribution includes proposals for a variety of projects in the area of probability, showing how context leads to depth of understanding, and uses these to argue for reform of probability teaching- deploring teaching of probability by mechanical rules, and proposing probability should have greater emphasis in the curriculum, including developing working knowledge of the various common probability distributions. Other probability based contributions- on lotteries and surveys- demonstrate the social importance of this area of mathematics although the particular problems proposed did not score highly on my subjective scale.

Surprisingly there was only one, rather briefly outlined, project concerned with growth of populations, surely a subject with the potential for a high score, and also, like the Image processing suited to doing the mathematics in a spreadsheet, but also using its graphics capability for the output interface.

Subjectively the projects with more traditional mathematics content- generally geometry or particle dynamics- are less attractive than the Information Society, Probability, and Population-growth modelling examples.

Conclusions

Model-making projects should be attractive to both learners and teachers, but the learners will also need requisite mathematics and application domain knowledge. Mathematics students cannot be relied on to have domain knowledge beyond their everyday lives and cross-curricular collaboration is not generally practical. It follows that most projects will have to be situated in the everyday application domain. But the pure mathematics in the traditional curriculum emphasises the continuum to the exclusion of discrete mathematics which was appropriate for supporting most nineteenth and twentieth century applications. However, increasingly twenty-first century applications rest on discrete mathematics, typified by the Information Society examples described above, and many of them involve probabilities. Thus today's students are effectively barred from a wide range of the applications on which our society depends - most obviously relational databases - by the paucity of their discrete mathematics knowledge.

Nothing demonstrates more clearly how school mathematics is stumbling into the twenty-first century than its continuing emphasis on the continuum while using spreadsheets in the classroom in place of slide rules. The weakness is not just at Level 3, and remedial updating would necessarily affect Levels 1 and 2: some set theory would have to be taught at Level 2 with the ground prepared at Level 1 and the nettle of probability would have to be grasped at Level 2. Perhaps the way to fix this is to let the needs of model-making determine the pure mathematics curriculum.

References

Maasz, J., and J. O'Donoghue, eds. 2011. *Modelling and Applications in Mathematics Real-World Problems for Secondary School mathematics Students.* Rotterdam: Sense Publishers.

Niss, M., W. Blum, and P Galbraith. 2007. Introduction. In *Modelling and Applications in Mathematics Education: the 14th ICMI study.* Springer Verlag.

Osmon, P. 2011. Applied mathematics = Modeling > Problem solving? *Proceedings of the British Society for Research into Learning Mathematics* 31 (3): 125-130.