An analysis of pre-service mathematics teachers' performance in modelling tasks in terms of spatial visualisation ability

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This study aims to identify pre-service teachers' spatial abilities and to explore the effects of these abilities on performance in mathematical modelling tasks. Following a case study research approach, mixed methods were used for data collection. Participants were 75 pre-service teachers studying an MA degree without dissertation at a state university in Turkey. In order to identify pre-service teachers' spatial abilities, data was collected using a Mental Rotation and Spatial Visualisation Test. In order to investigate the effects of spatial abilities on performance during the solution process and on the visualisation process, pre-service teachers participated in modelling activities. Descriptive statistics were used to analyse the qualitative data. Results indicated that almost half of the pre-service teachers had high level spatial abilities. It was also found that pre-service teachers' mathematical modelling abilities were not sufficiently developed, and that their spatial visualisation abilities were weaker than their mental rotation abilities. Moreover, the result that pre-service teachers who had higher spatial ability also had better performance in modelling tasks than the other pre-service teachers implied a direct relationship.

Keywords: mathematical modelling, spatial ability

Introduction

Skills such as interpreting problem situations encountered in daily life and producing efficient solutions, reasoning and associating can only be acquired and developed through mathematics education (NCTM 1989; Baki 2006). In order to do that, first, the idea that mathematics is an isolated science which is distinct from real life and only used at schools should be abandoned (Aydın 2008). It was this idea that led the way for the modelling approach to mathematics education due to a consideration that in a classroom based on this idea, traditional mathematics teaching would not develop students' skills of applying mathematics to different contexts (Lingefjard 2006).

Mathematical Modelling

Modelling is the process of defining and explaining events or mental organisation of problem situations, using and constructing various schemata and models when encountered with a problem situation (Lesh and Doerr 2003). Mathematical modelling, on the other hand, is a combination of mathematical constructions chosen to represent some real world situations and the relationships among them (Niss 1988) and in the general sense, it is the process of mathematically expressing a real world situation (Kertil 2008). As this process constitutes algorithmic and non-routine problems which are open-ended and which are closely related to a real life context, it is considered as a more suitable problem solving activity for the aims of mathematics education (Blum and Niss 1991; Crouch and Haines 2005 cited in Kertil 2008).

Spatial Abilities

Individuals need visualisation abilities as much as they need the abilities of using already existing or newly constructed mental models, schemata, visual elements, conceptual images and definitions in order to generate solutions to real world events and problems using modelling. Visualisation plays a significant role in the development of thinking and mathematical understanding, in creating associations between relationships mentally or on paper during problem solving and in the transition from abstract thinking to concrete thinking and it is an alternative and powerful resource for learners of mathematics (Dreyfus 1991; Hazzan and Goldenberg 1997; cited in Boz 2005 ; Lavy 2006).

Visualisation, which is a method that allows the invisible to become visible (Zimmermann and Cunningham 1991, 2) can also be defined as the concretisation or illustration of abstract thoughts, or the organisation of abstract thoughts using visual elements (pictures, graphics, etc.) through concretisation so that they can be easily perceived by the sense of sight (Sevimli *et al.* 2008). The most important benefit of visualisation in mathematics is that it transforms a very abstract matter to less abstract or concrete. This is especially important for the students who struggle to understand abstract mathematical concepts.

The process of visualisation as part of the problem solution strategies in the modelling task starts with reading the problem statement. The words that express mathematical terms in the problem or objects that exist in reality stimulate the learner's subconscious information about the concept and thus the learner develops mental images that are accepted as the functional fundamental unit of visualisation (Gutierrez 1996). The learner interprets these images based on prior experiences and spatial abilities. The learner benefits from the content of the problem and constructs a full mental representation of the problem in fragments using these images or a partial mental representation in fragments. The learner transfers the generated mental representation on paper. A mental and physical action which includes mental imagery (Presmeg 1986) is thus materialised. The learner constructs an external representation of the problem which is now concrete on paper. Thereafter, the external representation also affects the interpretation of the mental representation obtained from reading the problem. Learners' spatial abilities, which will be used to interpret images in this process, are also one of the significant and prominent factors (Delice 2004).

Therefore, this study aims to investigate learners' level of visualisation abilities which are observed when they begin the mathematical modelling process and the visual process they go through. Thus the purpose of this research is to find out how pre-service teachers' spatial abilities affect the visualisation process in modelling tasks and as a result how much their abilities affect performance.

Method

This study, which aimed to investigate the effects of pre-service teachers' spatial abilities on their performance in the mathematical modelling process, required an in-depth understanding of problem solving processes and skills. Thus the study had a case study design which is, in its widest sense, defined as an in-depth investigation and analysis of a group or event. Participants were selected on the basis of convenience sampling where individuals or groups are studied as they are. This technique is a type of non-probablity sampling which is widely used in qualitative research (Cohen, Manion and Morrison 2000) and conducts a thorough exploration of the group or event under scrutiny. The participants were 75 pre-service mathematics teachers pursuing a postgraduate degree (MA without dissertation) at the Secondary School Mathematics Education Department of a state university in Turkey.

Data collected for the study were predominantly qualitative and the findings were supported by quantitative data. Multiple methods and research techniques were used for data collection. The research techniques were Spatial Visualisation Abilities Tests and Modelling tasks. Data were analysed using categorisation and descriptive statistics.

In order to identify pre-service teachers spatial abilities, the Spatial Abilities Tests (SST) were administered. The tests were developed by Ekstrom, French, Harman and Derman (1976) and translated into Turkish by Delialioğlu (1996). Spatial ability is a combination of mental transformation and spatial visualisation abilities. Mental transformation ability is identified by using card rotation and cube comparison tests, while spatial visualisation ability is identified by using paper folding and surface development tests. These are multiple-choice tests with a single correct answer. Following the administration of the test, the data was categorised into “correct”, “incorrect” and “no answer” groups and the score of each student was calculated based on the number of correct answers.

After the SST scores were calculated, pre-service teachers’ spatial ability levels (low-average-high) were determined in relation to the closeness of the mean scores to standard deviation. Accordingly, pre-service teachers who had scores between 165 and 229 were identified to have a low, between 231 and 241 to have an average and between 245 and 276 to have a high level of spatial visualisation ability.

Following the identification of pre-service teachers’ spatial visualisation skills, pre-service teachers were given problem solving tasks in line with the model and modelling approach. The tasks required skills such as identifying the variables of real life situations and events using mathematical thinking and stating the relationships between these variables using mathematical expressions. In order to identify pre-service teachers’ performance in modelling tasks, each answer was grouped and evaluated into categories of “correct”, “partially correct”, “incorrect” and “no answer” based on their ability to find the result. For the evaluation of the answers, an answer key was prepared beforehand which, for each problem, identified which answers would be accepted as correct, partially correct, incorrect or as no answer. The descriptive analysis of the data and their percentages are presented in detail according to spatial abilities.

Findings

Quantitative and qualitative data obtained from the study is presented in this section. Spatial ability levels of the participants (low, average, high) and their mathematical modelling skills are identified and the data is discussed.

Data analysis indicated that 44% of pre-service teachers had a high, 22% had an average and the remaining 34% had a low level of spatial ability. Thus, almost half of the pre-service teachers had a high level of spatial ability.

An analysis of pre-service teachers’ answers to modelling tasks demonstrated that 41% gave correct (CA), 22% partially correct (PA) and 29% incorrect answers (IA), while 8% of pre-service teachers could not give any answers (NA).

When pre-service teachers were compared in terms of their performance in modelling tasks according to their spatial abilities (Figure 2), the percentage of incorrect answers given to the modelling tasks by pre-service teachers who had low spatial ability (%41) was higher than that of the other pre-service teachers (%28 and %23). The percentages of correct answers to the modelling tasks by students who had an average and high level of spatial ability were 55% and 52%, while the percentage of correct answers to the modelling tasks by pre-service teachers who had a low level of spatial ability was 41%. 24% of pre-service teachers who had high spatial ability, 17% of those who had an average level and 14% of those who had a low level of spatial ability gave partially correct answers to the modelling tasks. It is worth noting that there were pre-service teachers who could not answer the modelling tasks only among the pre-service teachers who had low spatial ability.

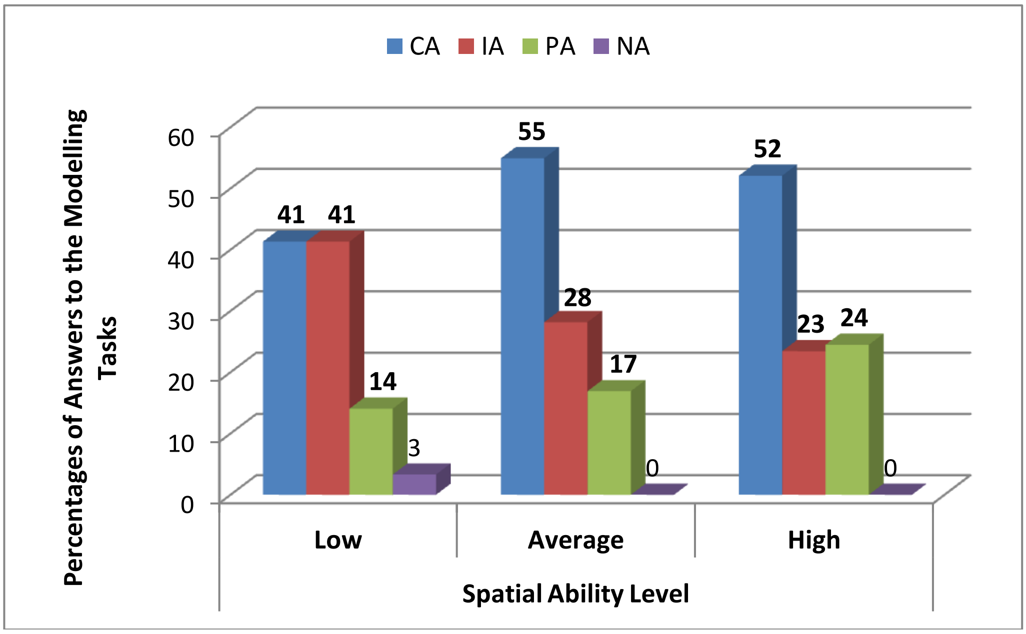

Figure 2: Performance in modelling tasks according to spatial ability

Discussion

Given the data obtained from the modelling tasks, pre-service teachers' performance was, in general, insufficient. Following the reformation of the high school mathematics curriculum, pre-service teachers are expected to be intrinsically and extrinsically motivated to inquire and interpret; to think censoriously and relatively; and to make use of the knowledge and abilities they have at the needed stage of their activities such as problem solving. Thus, the performance in such activities, which assess skills that support the constructivist approach, was expected to be better. Moreover, mathematics pre-service teachers' performance in the modelling tasks cannot be claimed to be good based on the fact that correct answers had the highest percentage. Partially correct answers should also be considered. When pre-service teachers could solve part of the task, but could not decide what to do afterwards, it meant that they neither gave an incorrect nor a correct answer. This increased the importance of the percentage of correct answers, and as this was less than half, it indicated that pre-service teachers modelling performance was not developed sufficiently.

Due to the fact that the modelling tasks were open-ended and had non-routine characteristics, pre-service teachers had to transcend their already existing didactic acceptance, which has a significant dominance on problem solving skills. In other words, for the modelling tasks, when the problem was not understood, in order to choose correct mathematical operations, pre-service teachers had to perform beyond consulting key words or similar problems that were solved before, using each number and data given in the question to find the answer and thinking that each problem given by the teacher was a problem that could be and had to be solved (Reusser and Stebler 1997). This could be the most important reason for why pre-service teachers' level of performance in the modelling task was insufficient (Kertil 2008). Modelling tasks, which are contextually related to real life and which require an integrated use of various skills such as visualisation and algebraic operations in the solution process, can be perceived as an area in which pre-service teachers had difficulties in applying their knowledge. Pre-service teachers' attempts at constructing the abstract (mental) or concrete (drawing on paper) problem were observed in the solution processes. This suggests the potential contribution of their spatial abilities.

Less than half of the pre-service teachers were found to have a high level of spatial ability, whereas more than one third had a low level of spatial ability. The fact that pre-service teachers had three levels of (high, average and low) spatial ability indicated that pre-service teachers could develop their spatial ability during their past experiences and educational life. Moreover, it also emphasised how pre-service teachers projected their spatial ability into the mathematics teaching and learning process and how they could use it in understanding and making sense of problem solving processes and mathematical concepts. Their spatial abilities were observed to have a directly proportional relationship with their performance in the modelling tasks. Pre-service teachers with low spatial ability at modelling activities had the lower success at performance. The reason of this might be their inability to utilise their visual abilities together with abstract/mathematical concepts to approach the solution process of modelling activities from different perspectives. The fact that the pre-service teachers who had a high ability could project it to their performance can be explained by their ability to apply their procedural skills, which would lead them to the solution, by combining the problem that they have constructed mentally and on paper and their mathematical skills. Yet, given that slightly less than half of the pre-service teachers with high ability levels gave correct answers, the need to use visualisation skills in modelling tasks is also foregrounded.

Conclusion and Further Research

Mathematical modelling tasks provide critical evidence on mathematics pre-service teachers' current situation. Pre-service teachers were observed to have difficulties in mathematical modelling and interpreting real life situations using their mathematical knowledge. Therefore, teacher education should include objectives to improve mathematical modelling skills and skills that would enable them to construct mathematical modelling tasks. Modelling concepts-questions-tasks, which could be accepted as applications of adapting mathematics to real life situations and which are also emphasised by the constructivist approach, should be incorporated in the lessons by identifying techniques and categorising pre-service teachers according to their spatial abilities.

Given that pre-service teachers who had a high spatial ability performed well in modelling tasks, activities that will increase the ability to use mental visual-pictorial components could be used in classes. This emphasises the need to focus on activities to support the development of visuality and visual skills. For example, an ability to use algebra/symbols and their integration to visual skills can be studied.

Another important finding is that visualisation ability and performance in modelling tasks are directly proportional. This suggests that more and variety of practice that aims to develop the ability to integrate these two skills in problem solving activities should be incorporated in the teaching learning process.

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