Teacher, do you know the answer? Initial attempts at the facilitation of a discourse community.

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My research involves a teaching experiment I undertook in my own primary classroom. The aim of the research was to facilitate a mathematical discourse community where students would explain and justify their mathematical thinking and question the reasoning of others. It was envisaged that students would regularly engage in cognitively demanding tasks and take responsibility for determining what was mathematically correct by discussing different possible solutions. The lesson presented here was the first recording of the experiment and focused on initial attempts at exploring equivalent fractions in the context of sharing pizzas between people. The contributions of students show different levels of mathematical understanding and engagement with the task. The whole class discourse is analysed with reference to the four components of the Math-Talk Learning Community (MTLC) framework (Hufferd-Ackles, Fuson and Sherin 2004). These components are questioning, explaining mathematical thinking, source of mathematical ideas and responsibility for learning. Both teacher and student actions in these key areas are explored. Analysis of teacher questions was carried out using question categories developed by Boaler and Brodie (2004).

Keywords: discourse, community, teaching experiment, questioning, explaining mathematical thinking, source of mathematical ideas, responsibility for learning.

Introduction

The main part of my PhD research was a teaching experiment I conducted in my own classroom in which I hoped to facilitate a discourse community where students would engage in genuine mathematical problem solving activities and discuss their mathematical thinking and the mathematical thinking of others. The experiment took place in a designated disadvantaged boys' school in Dublin with fifth class students. This is the penultimate class of primary school in Ireland and students are generally 10 – 11 years old. The lesson presented here was the first lesson in the teaching experiment and consisted for the most part of whole class discussion around the following problem: Three children shared two pizzas. How much did they get each? This question was shown on the interactive whiteboard with a picture of two pizzas and three children.

Lesson synopsis and initial comments

Students' suggestions for the solution of this problem varied. Edvard and Anthony gave suggestions of two-third or four-sixth per person respectively. Steven and Kevin gave mathematically naïve suggestions involving sharing the pizzas unevenly by giving more to the eldest child or giving a slice "back to the man". I did not evaluate individual student suggestions and continued to press for more solution methods. Steven asked me directly "Do you know?" It is possible that he interpreted the discussion of the multiple possible solutions as a search for one "right" answer and because I refrained from evaluating any of the suggestions to that point, he may have conjectured that I did not know 'the' answer. As a researcher I realised the importance of his question immediately but unfortunately as a teacher I did know how best to deal with his query. It seems obvious in retrospect that it would have been the perfect time to instigate an explicit discussion of my aims for the experiment and my expectations regarding student actions in a discourse community but this did not occur to me at the time and I moved on with the lesson instead. Andrei suggested cutting the pizzas into twenty one pieces and giving each child fourteen slices each. Darragh commented how small those slices would be. Michael then came to the board and showed another solution that involved sharing the pizzas unevenly.

I then explained to students that we would share the pizzas evenly and on the interactive whiteboard I showed a representation of two pizzas cut into thirds being shared between the three children. I explained that the children got one third from each pizza and wrote $\frac{1}{3}+\frac{1}{3}$ on the whiteboard before asking what fraction the children got each. Some students made the classic error of adding the numerators and the denominators to give two-sixth. In some ways my actions set them up for this error and I cannot be sure why I wrote the formal fraction sum on the board as it was never part of my original plans for the lesson. I asked students if they agreed or disagreed with the suggestion of two-sixth. Darragh suggested that "Two sixth is eh, a way of saying it but eh, also two thirds". Jake suggested that it couldn't be two sixth as this would mean each pizza would have been cut into sixths initially. When prompted he repeated his explanation and added more detail. I drew a representation of two pizzas cut into sixths and Jake explained that if the pizzas were cut in this manner the children would receive four-sixth each. Darragh revised his previous contribution and suggested that "Two sixths is equivalent to one third which means that it's the same as one third". He continued on to explain that the fraction in question could be two thirds or four sixths.

Edvard then asked "Wait can you go up over one-twelfth?" possibly asking if there are fractions with denominators higher than twelve. His confusion may have been triggered by common classroom representations of fractions such as fraction walls that do not show fractions with denominators higher than twelve. Darragh, Andrei, other students and I gave answers to his question. Steven asked about the meaning of the word simplify which Darragh had used and linked it with a similar word from Harry Potter. Darragh explained the term and then noted the multiplicative pattern between two third and four sixth. A student noted that he had done this in a previous lesson too. I repeated his explanation and wrote it the formal equivalence on the board.

Discourse Community Analysis

The lesson was analysed according to the Math talk learning community (MTLC) framework, (Hufferd-Ackles, Fuson and Sherin 2004). The areas of questioning, explaining mathematical thinking, source of mathematical ideas and responsibility for learning were recognised as central by Hufferd-Ackles, Fuson and Sherin during intensive research in classrooms where teachers were attempting to teach in the spirit of reform. The authors identified developmental learning trajectories for both teachers and students across these four key areas. These trajectories are levelled from level 0 to level 3. Level 0 describes a traditional teacher centred classroom with limited mathematical discussion. The following levels describe a gradual devolution of responsibility from teacher to students as the classroom community moves closer to a math-talk learning community or discourse community described above.

Questioning

Teacher Questioning

The teacher questions were analysed using categories developed by Boaler and Brodie (2004). I categorised all questions where I asked for the name of a fraction as a type 2, inserting terminology questions. Questions about what the denominator and numerator represent were categorised at type 3 questions, exploring mathematical meanings and relationships. The results are shown on the table below.

Question Type	Number
1. Gathering information, leading students through a method	15
2. Inserting terminology	6
3. Exploring mathematical meanings and/or relationships	11
4. Probing, getting students to explain their thinking	18
5. Generating discussion	24
6. Linking and applying 7.Extending thinking 8. Orientating and focussing 9. Establishing context	0
Total	74

Table 1: Analysis of teacher questions according to Boaler and Brodie's question categories (2004).

Boaler and Brodie's (2004) type 4 questions that probe student thinking and type 5 questions aiming to generate discussion can be associated with reform orientated lessons. Both types of question are relatively common in this lesson accounting for a combined total of 57% altogether. There were a small proportion of type 2, inserting terminology questions, and also a number of type 3 questions that explored mathematical relationships and representations (8% and 15% respectively). One might expect a higher proportion of questions focussed on mathematical relationships. However it is likely that methodology of not counting repeated questions influenced this total. Individual questions were discussed in a lot of depth in this lesson often with multiple student contributors to a single teacher question. For example, after successfully partitioning two pizzas into thirds to share between three people, I asked "how would we write that though? I'm saying that he's got one third and another third so …" at turn 201. The discussion that followed continued until turn 278, this section itself being a substantial segment of the 488 turn transcript. This section also contained type 4 and 5 questions where students were asked to contribute to the discussion and were encouraged to articulate their thinking.

Student Questioning

Student questions were coded as questions seeking clarification about the mathematics being discussed or questions seeking organisational clarification. In general the category of student questions was easily identifiable. The nature of the content of the students' questions about mathematical issues is also interesting and for this reason some examples of their questions are included in the table below.

Question type	Questions seeking clarification about the mathematics being discussed	Questions seeking organisational clarification
Examples	Steven: So all I've to do is do a one and then a five? Darragh: What does she get Steven? Steven: Do you know? Steven: What's equivalent? Edvard: Teacher, it should be higher than one sixth because you can't go higher than one twelves…. Wait can you go up over one twelfth? Steven: What does simplify mean?	Michael: Can I clear this? Student: Why is it green?
Total	14	2

Table 2: Analysis of student questions by type with examples.

As can be seen from the examples of student questions, students participated at different levels. Some students asked basic questions such as Steven's "So all I've to do is do a one and then a five?" when asking how to write a fifth. Other students questioned the solutions posed by their peers, for example Darragh's questioning of Steven's solution. Edvard posed a question about general properties of fractions when he asked about limits to the size of fraction denominators.

Explaining Mathematical Thinking (EMT)

It seems clear that the nature of teacher questions will influence the manner in which students explain their mathematical thinking. As shown in the first table there were a sizeable proportion of teacher questions aimed at probing students thinking, 24% in total. There were also a number of teacher prompts, not in question form, to explain and justify mathematical thinking. For example when I was about to call on a student to the board to present a solution, I stated "I don't just want the person to come up here and cut it up. I'd really like to hear why you're doing it, why you're cutting it in that way". There is some evidence that students responded to these prompts. In the example just mentioned, I called on Alex to present his solution. He did this successfully and explained his reasoning clearly. When he was finished other students evaluated not only his mathematical thinking but also his way of explaining his mathematical thinking with Michael commenting "That's a good way" and Luke adding "He explained it in a good way."

Another issue that became apparent when examining the nature of students' explanations of mathematical thinking was their inappropriate use of non-mathematical or real-life ideas. For example, the contributions of Steven and Kevin discussed in the lesson synopsis above involving unequal sharing methods. The other issue that must addressed as part of the explaining mathematical thinking component is the nature of teacher explanations or teacher telling. In this lesson although I often restated or re-voiced student contributions, I did not partake in direct telling of answers. In fact it would seem that this approach combined with the fact that multiple possible solutions were considered was novel for students and may have prompted Steven's question about whether I knew the answer or not.

Source of Mathematical Ideas (SMI) and Responsibility for Learning (RFL)

The teaching approach in this lesson was to solicit multiple possible solutions from students. In this way students' ideas were central to the lesson. On considering the nature of the teacher questions aimed at eliciting student solutions I realise that I could have done more to encourage students to build on the ideas of previous contributors to the class discussion. In particular I posed the question "what do you think?" over ten times. It may have been more effective to ask "what do you think of his solution?" or a variation of this. So although I was effective at positioning students as the source of mathematical ideas but I was possibly less effective at encouraging responsibility for learning (RFL) as described in the MTLC framework. High levels of student RFL in the MTLC framework imply that students will attend to and build on the mathematical thinking of their peers. There is evidence of RFL in the contributions of some students in particular. For example, when Steven presented his erroneous solution, students including Darragh, Andrei and Jonathan either questioned him or commented on his solution. For much of this lesson, it was students who explained the mathematics with certain higher achieving students being particularly vocal. Darragh introduced the terms 'simplify' and 'equivalent'. His many contributions showed high levels of RFL and allowed for whole-class discussion and consideration of relevant mathematical terminology and concepts.

MTLC level

It seems clear that this was not a traditional lesson revolving around invitation-response-evaluation (IRE) iterations (Meehan 1979). It therefore cannot be described by level 0 of the MTLC level descriptors. One of the defining features of level 3 of the MTLC framework is unprompted student-student mathematical discussion. Although this is present at times, it cannot be considered robust or regular enough to describe this community as level 3. The strongest argument for describing this lesson as an example of a community operating at level 2 of MTLC framework is the large role that students' multiple solution strategies played in the course of the lesson. This is a feature of the level 2 descriptors for EMT, SMI and RFL in particular. However it would be misleading to state that this lesson included all of the level 2 descriptors. In particular, the teacher actions around the facilitation of student to student dialogue were not met. In this way although there were elements of a level 2 MTLC discourse community, the requirements have not been fully met in this lesson.

Conclusion

This lesson was the first in a teaching experiment that ran over the course of a school year. Analysing this lesson within the constraints of this paper requires the "focussing of a lens" (Lerman 2001, 90) in which certain issues are highlighted while others remain part of the background detail. For example, issues around the achievement levels of students and the nature of their participation are emerging themes in the larger study but the focus here was on how the interplay of teacher and student actions shaped the classroom discussion on this occasion. The mathematical task had multiple possible solutions. The open nature of the task, combined with the teaching approach of pursuing the mathematical thinking of students without directly evaluating it, seemed to create a space for genuine mathematical discussion rather than the 'number talk' described by Richards (1991).

The dual role of teacher-researcher is an important part of the larger project and the experience of recording and analysing my own lesson according to the MTLC framework has been complex but revealing. The role of the MTLC framework in both understanding what is happening in the classroom and seeking to shape the nature of classroom discussions is also complex. The nature of classroom discussion described by each level of the framework and how these different means of communicating mathematical ideas may be appropriate in different circumstances and for different students will also be a topic for future research.

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