Portuguese pre-service elementary teachers' knowledge of geometric transformations: an exploratory study.

Alexandra Gomes
CIEC/IE – U. Minho

No one questions the fact that teachers' knowledge plays a crucial role in teaching. Research on teachers' knowledge indicates that content knowledge is influential on instruction. Even though there is plenty of research on teachers' knowledge of number and operations, the same doesn't happen with geometry. In Portugal, a new mathematics programme for elementary school introduces geometric transformations from 1st grade. Since this is a rather new topic in the elementary curriculum, it seems important to understand what knowledge (future) teachers have on the topic. In this paper we present findings from an exploratory study, conducted with future elementary teachers designed to evaluate their knowledge on geometric transformations.

Keywords: Teachers' knowledge; Content knowledge; Geometry; Geometric transformations.

Introduction

It seems almost evident that every teacher who has to teach mathematics needs adequate mathematical knowledge. For elementary school teachers this knowledge is critical since they play a crucial role in introducing children to basic but fundamental mathematical ideas and initiating a process of mathematical learning, with every stage highly dependent on the previous. Assuming that elementary mathematics is fundamental mathematics in the sense defended by Ma (1999), then the only sensible path to take seems to be to guarantee solid and efficient mathematical knowledge in future teachers. However, there is evidence that teachers, in particular elementary teachers, do not possess the necessary knowledge to teach mathematics effectively (e.g. Brown, Cooney and Jones 1990; Ponte, Matos and Abrantes 1998). The case of geometry is particularly "worrying". Teachers do not seem to possess the geometric knowledge necessary to teach geometry efficiently.

In Portugal, geometry gained ground and visibility with the mathematics curriculum reform of the 1990's. Currently, new curriculum guidelines in Portugal (DGIDC 2007) give a prominent place to geometry, pointing to the importance of the development of visualization and spatial reasoning, as the main purpose for teaching geometry. Of particular interest is a change in relation to the previous programme that consists of the initiation, in primary school, of the study of different geometrical transformations, first intuitively and then with increasing formalization.

About teachers' knowledge

For more than 30 years, teacher knowledge has been attracting the interest of researchers. Shulman and colleagues developed one of the most influential works concerning teachers' knowledge. In particular, referring to content knowledge (CK) for teachers, Shulman (1986) considers three categories: (a) subject-matter content knowledge (SMK), (b) pedagogical content knowledge (PCK) and (c) curricular knowledge. The notion of PCK is of particular importance since it recognizes that this type of knowledge is exclusive of the teacher and different from the kind of knowledge necessary for a mathematician.

Much work has been done ever since, clarifying and redefining different categories of teachers' knowledge. Ball (1990), in work conducted with primary school teachers, considers that the understanding necessary to teach mathematics encompasses both substantive knowledge of mathematics and knowledge about mathematics. Askew et al. (1997) found that the more effective/competent teachers have a better understanding of the inter-connections among mathematical concepts.

Almost all studies agree that teachers' knowledge is essential for teaching and that the lack of knowledge seems to compromise teaching and therefore learning. Ma, for instance, refers that: "Limited subject matter knowledge restricts a teacher's capacity to promote conceptual learning among students" (1999, 38). Also, an adequate knowledge to ensure effective teaching of mathematics depends not only on a solid mathematical knowledge but also of the nature of that knowledge.

Recent studies have been directed towards a practice-based theory of knowledge for teaching (Davis and Simmt 2006; Turner and Rowland 2008). However, only few have attempted to measure the real influence of the different components of knowledge. The Michigan group (Hill, Rowan and Ball 2005) seems to have been the first one to successfully address this issue and their work gave the first conclusive evidence of the importance of teachers' mathematical knowledge in their teaching.

Baumert et al. (2010) investigated the significance of teachers' content knowledge and pedagogical content knowledge for high-quality instruction and student progress in secondary-level mathematics. One of the findings of their study was that PCK is of key significance for students' mathematical progress. Also that both CK and PCK deserve special attention in teacher training and classroom practice since the deficit in CK may compromise the development of PCK and consequently have negative effects on instruction and student progress.

In summary, it seems adequate to claim that CK plays a crucial role in teaching and, even though it is not sufficient to ensure efficient teaching, it is certainly necessary and therefore worth of further investigation.

Geometry

> Geometry (…) is full of interesting problems and surprising theorems. It is open to many different approaches. It has a long history, intimately connected with the development of mathematics. (…). What is more, geometry appeals to our visual, aesthetic and intuitive senses." (Jones 2002)

While it is recognized that the study of geometry is very important since it contributes to the development of visualization, critical thinking, intuition, problem-solving, proof, among others, geometry remains a poor area of research when compared with other fields of research. Nevertheless, findings from research have revealed many deficiencies and errors in teachers' geometric knowledge (Jones, Mooney and Harries 2002; Gomes 2004). Apparently, teachers do not possess the geometric knowledge necessary to teach efficiently.

The study

In Portugal, new programmes for elementary school mathematics (DGIDC, 2007) are just starting to be implemented (school year 2010/2011). In these new programmes, an important role is assigned to geometry and especially to geometric transformations (GT). Since this is a rather new topic in the elementary curriculum, it seems important to understand what knowledge (future) teachers have on the topic. Therefore, our research question is: *What do pre-service elementary teachers (PSET) know about geometrical transformations?* Our aim is to identify and describe the difficulties/mistakes PSET have/make concerning geometric transformations.

Methodology

We developed a questionnaire concerning three GT, translation, reflection and quarter turn rotation, with one question for each GT (25 items in total). PSET were asked to draw the result of a given transformation. In order to access Reflection and Rotation we adapted some items from the Concepts in Secondary mathematics and Science (CSMS) project. For translation, we designed six items to address this transformation.

The participants were 66 female Basic Education Degree[2]finalist students. The questionnaire was answered during a regular class. These PSET already had five semester courses (25 ECTS) in elementary mathematics, one of which focused on geometrical transformations. They hadn't any course on didactics of mathematics yet.

Analysis and results

All answers were classified under four categories: "Correct", "Acceptable" (not accurate enough to be considered correct), "Typical error" (common wrong answers) and "Wrong" (clearly wrong, random answers).

Concerning translation

In the 6 translation items, the median of right answers was only 44%, with the first item being the lowest – only 33% right answers.

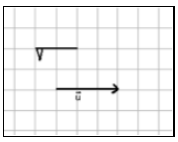
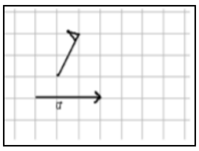
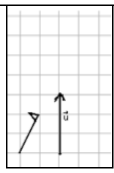
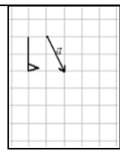
Item	1a 	1b 	1c 	1e 
PSET (% correct)	33	60	50	36

Table 1 – some items on translation and percentage of correct answers given by PSET

As we can see, the items with more correct answers were when the vector was horizontal or vertical and the flag was oblique. On the other hand, the item with more mistakes was the first one, which is with a horizontal vector and also a horizontal flag. This is unexpected in the sense that we tend to consider translations with horizontal (or vertical) vectors as easier and as we can see, that is not always the case. It is also interesting since it shows how a rather small change leads to such different results.

We considered, for translation, the category "typical error" divided into 3 sub-categories: A, B and C. Error A means that students move the object considering an

[2] In Portugal, in order to become an elementary school teacher (children aged 6 to 12), one has to take a 3 years degree in Basic Education and then take a Master Degree.

extreme point, so that the gap between the object and its image corresponds to the vector. This error happens more frequently in items where the vector is horizontal or vertical. In item 1b, also with horizontal vector, this error isn't visible since even with this wrong reasoning, the answer will be right. This is significant because it shows the importance of diversifying the tasks and also how right answers can be deceiving. Error B means that students moved the object to the endpoint of the vector. Error C means that students moved the object keeping it parallel and with the same size and orientation as the original but without considering the vector. We considered this a mistake because it shows that even though students do not cope with translation the majority seems to be aware of some properties of this transformation, namely that it preserves length, orientation and parallelism.

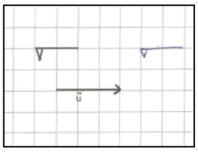
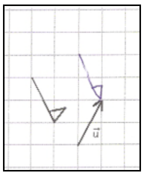
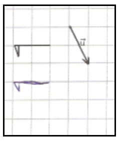
Error A	Error B	Error C
1a 20%	1f 5%	1d 30%
		

Table 2 – examples of errors in Translation and the percentage of each one in the items.

Concerning reflection

Given that reflection is one of the most used isometries it may come as no surprise that more than 50% of the answers were right for almost all items. Within reflection, PSET coped more easily in items involving a single point rather than a line or flag. Also, the performance was slightly better when the reflection line is either horizontal or vertical. The grid doesn't seem to have any sort of consequence.

Table 3 presents some items, the percentage of right answers given by PSET and the results from CSMS test on the same items (with 15 years old children).

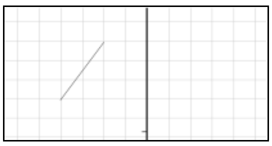
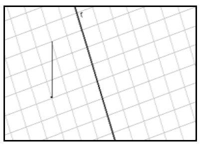
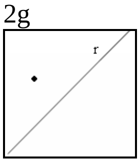
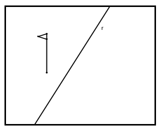
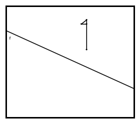
Item	2b 	2c 	2g 	2h 	2i 
PSET	74	71	**91**	36	21
CSMS	83	82	**65**	33	41

Table 3 – items on Reflection and percentage of correct answers given by PSET and CSMS

Comparing the results from PSET with those of CSMS we notice that they are very similar.

We identified two typical errors. Error A means that the image is drawn parallel to the object but not horizontally nor vertically aligned. This error only appears in 3 items, all related with reflecting an object on a slanted axis. Error B means that that the image is drawn parallel to the object and either horizontally or vertically aligned. This error is more frequent and particularly relevant in items 2h and 2i.

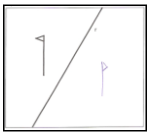
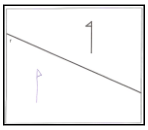
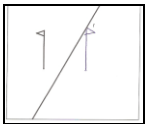
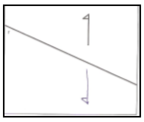
Error A		Error B	
2h 8%	2i 11%	2h 24%	2i 32%
			

Table 4 - examples of errors in Reflection and percentage of this type of errors in the items.

Concerning rotations

This GT appears to be the one where students have more difficulties. Only when the centre of rotation was on the object was the percentage of correct answers above 50%. A grid doesn't seem to be helpful. Table 5 presents some items, the percentage of right answers given by PSET and the results from CSMS test on the same items (with 15 years old children).

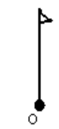
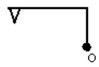
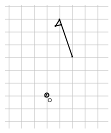
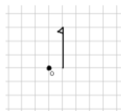
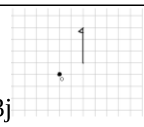
Item	3a 	3b 	3e 	3h 	3i 	3j 
PSET	85	67	61	**9**	12	15
CSMS	91	77	71	**24**	25	21

Table 5 - items on Rotation and percentage of correct answers given by PSET and CSMS

These results are quite similar to the ones on CSMS test, although in some cases a bit lower, especially in 3h.

We considered only one type of typical error, which refers to answers where PSET transformed correctly the slope of the object (vertical into horizontal and vice-versa) but incorrectly positioned endpoint of the object. Table 6 shows examples of these errors together with the percentage of this type of answers both from PSET and CSMS (PSET results/CSMS results).

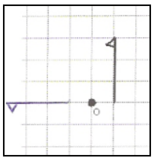
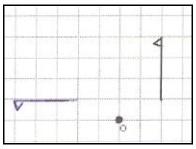
3i 14% / 8%	3j 6% / 7%
	

Table 6 - examples of errors in Rotation

This error seems to indicate that some students are aware that if the object is vertical/horizontal its image, by quarter turn rotation, will be horizontal/vertical.

Concluding remarks

This study shows that these PSET don't seem to be prepared to teach geometric transformations. Although they already had contact with the subject, they still revealed several difficulties in all studied GT. The results concerning reflection and rotation were similar to the ones achieved by 15 years old students in the CSMS test. As for translation, we found some difficulties that make us wonder about its apparent simplicity.

More research is needed on the geometrical knowledge of pre-service elementary teachers. In particular, it would be interesting to find out the impact of pedagogic training in their knowledge of this topic.

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