Evaluating the impact of a Realistic Mathematics Education project in secondary schools

Paul Dickinson*, Susan Hough*, Jeff Searle** and Patrick Barmby**

**Manchester Metropolitan University, **Durham University*

Over the past 30 years researchers at the Freudenthal Institute in the Netherlands have developed a mathematics curriculum and a theory of pedagogy known as Realistic Mathematics Education (RME). This curriculum uses realisable contexts to help pupils to develop mathematically. In 1991, the University of Wisconsin, in collaboration with the Freudenthal Institute, started to develop a middle school curriculum based on RME called 'Maths in Context'. A related Mathematics in Context (MiC) project was carried out in England in 2004 to 2007 at Manchester Metropolitan University (MMU) with Key Stage 3 pupils. This initial pilot project was evaluated by Anghileri (2006). In 2007, the ideas behind the project were extended to include Key Stage 4 pupils, particularly those studying towards Foundation GCSE Mathematics, and given the project title Making Sense of Mathematics (MSM). MSM has been running as a pilot project in some Manchester schools since 2007. Both these projects were recently evaluated by Durham University, with revaluation of test data from the original MiC project using Rasch analysis, interviews with teachers from both projects, and observations of the RME approach in lessons. This paper presents the findings from the Durham University evaluation, and discusses the impact of RME on both pupils and teachers.

Keywords: realistic mathematics, secondary, understanding, Rasch analysis

Introduction - Background to RME

Realistic Mathematics Education (RME) is an approach to teaching utilised in the Netherlands and developed over a period of thirty years. Based on the work of Freudenthal, and developed by researchers working at the Freudenthal Institute, the approach is significantly different to the approaches used in England in a number of respects. Here we focus on three of these; the use of context, the use of models, and the notion of progressive formalisation. Prior to working with RME, most of our teachers used contexts as a means of providing interesting introductions to topics, and then for testing whether or not pupils could use their knowledge to answer 'applications' questions. Under RME, however, context is seen as both the starting point and as the source for learning mathematics (Treffers 1987). This role of context is seen as crucial in order that pupils continue to make sense of and stay close to their mathematics. Moreover, a particular context is selected not because of its 'real worldness', but because of its richness in giving rise to a variety of solution procedures and reflecting within it the mathematical structures that are being worked on (Gravemeijer 1997). It is being used not for application but for construction (Fosnot and Dolk 2002).

Theoretically, models are given the role of bridging the gap between informal understanding connected to 'reality' on the one hand, and the understanding of more formal systems on the other (van den Heuvel-Panhuizen 2003) Some models are immediately recognisable as such (double number line, ratio table), while others would often be attributed a different label in the UK (repeated subtraction, reallotting). Coming to recognise the importance and the role of these models has been crucial for both teachers and researchers involved in the project. For the models to be useful they must be flexible enough both to emerge initially from the context (often as little more than a picture of the context), and also to support mathematical development. The model should also allow a 'way back' to the original source so that students can continue to 'make sense' of their work as it becomes mathematically more sophisticated. In seeing how models bridge the gap between the formal and informal, the work of Streefland (1991) is crucial. He made the distinction between a 'model of' and a 'model for'. In brief, this means that in the beginning a model is seen as being very close to the contextual problem (the reality), and then later on the model is generalised so that one can reason mathematically. The process of using a model to solve a particular problem is known as 'horizontal mathematisation', while that of using the model to make generalisations, formalisations etc. is known as 'vertical mathematisation'. While both are important, it is, in our experience, relatively rare to see the latter taking place in the UK classroom.Clearly teachers want students to be able to understand formal methods and procedures and RME does not shirk this responsibility. Teachers are always aware of the need for pupils to develop mathematically, and to become more efficient and mathematically more sophisticated over time. What RME does do, however, is offer a very different story of how students and teachers work towards this aim. While formal notions are there, they are seen as being 'on the horizon' (Fosnot and Dolk 2002) or the 'tip of the iceberg' (Webb, Boswinkel, and Dekker 2008). If teachers are not to teach formal procedures, however, they must be given an alternative, and materials based on RME provide this. Within the 'main body' of the iceberg are a range of informal representations and pre-formal strategies which students work on and develop. These are not only seen as desirable but as essential under RME - it is through these that students are able to 'make sense' of formal mathematics, and the time invested in such activities substantially reduces the time we currently use to constantly re-teach and practice formal methods and procedures (Webb, Boswinkel and Dekker 2008).

The projects carried out and the evaluation

The Mathematics in Context (MiC) project was carried out in England from 2004 to 2007. This involved using the Mathematics in Context materials originally developed in the USA for International Grades 5-8. They were trialled with Key Stage 3 pupils in 12 Manchester schools and in a limited number of schools in other parts of the UK. The MSM project used materials produced by researchers at Manchester Metropolitan University in collaboration with the Freudenthal Institute in Utrecht. The materials were aimed primarily at Foundation Level GCSE students in years 10 and 11 and have been extensively trialled in schools in the Manchester area.

In 2010, Durham University was asked to re-evaluate both the existing results from the MiC project, and also the more recent work from the MSM project. The methods used for the evaluation are detailed below.

Evaluation methods

The evaluation carried out by Durham University researchers involved a mix of qualitative and quantitative methods.

Looking at the qualitative methods first of all, the transcripts of interviews with teachers who had participated in the original MiC Project 2004-2006 were reviewed to highlight the emerging issues from the project and using the RME approach in the classroom. Teachers had been interviewed at several stages during the project. Further interviews were conducted in the evaluation by Durham University. These interviews were conducted mostly by telephone but included one face-to-face interview and were conducted with teachers who are currently using MiC and/or trialling the MSM materials. These teachers had done their initial teacher training at MMU and had been introduced to the RME approach there and/or had undertaken professional development in using the RME approach at MMU. The interviews used a pro-forma covering a range of aspects of the RME approach. The interviews aimed to discern these teachers' experiences, their views and any issues involved in using the RME approach. In total, thirteen interviews were carried out. These interviews were enhanced through observation of some of these teachers using the RME approach in their classrooms with pupils and also by interviewing some of their pupils.

Quantitatively, as part of the MiC Project in 2004-06, some Year 7 pupils who had been taught through the RME approach were assessed on a range of problems. The same assessment test was also given to a similar number of pupils who had not experienced the RME approach, to act as a control group. As well as solving the problems, the pupils were also asked to explain their strategies for solving them. Correct and incorrect results were coded (1) and (0) respectively, but also explanations were coded in the following way: No explanation (1), Incorrect explanation/diagram (2), Reasonable diagram (3), Correct explanation (4), and Correct explanation and diagram (5). The results were reanalysed in the current evaluation using Rasch modelling, comparing both the number of correct solutions and also the quality of the explanations of the strategies adopted.

Rasch analysis is a one-parameter item response theory (IRT) model, in which the probability of a person being successful on a given item is modelled in terms of a mathematical function involving the difficulty of the item and the ability of the person (Bond and Fox 2007). The Rasch model can be used for dichotomous responses (e.g. right and wrong), or extended to cover more than two responses (Wright and Mok 2004) including missing responses, and also allowing for differing numbers of responses on different items. This so called partial credit analysis estimates not only the person ability and the overall item difficulty, but also provides estimates for the difficulty thresholds between scoring categories. These thresholds should increase in an ordered manner, in line with the ordering of the scoring categories (Bond and Fox 2007). Otherwise adjacent categories should be combined and reanalysed. Therefore, Rasch analysis, using WINSTEPS software was used to confirm the ordering of the categories for the explanation. It was subsequently found that category 3 (Reasonable diagram) and category 4 (Correct explanation) needed to be combined and was done so for further analysis of the results. With the collapsed categories, the estimated reliability of the measure of student ability using all of the questions with explanations was Cronbach $\alpha = 0.79$, above the conventional value for reliable measures of 0.7. The measure of pupil ability using these items, with the explanations accounted for, was therefore considered to be a reliable measure.

Results

Looking firstly at the results from the reanalysis of the quantitative data, Table 1 shows the percentages of the project and the control group getting particular items correct. As can be seen, students in the project group were more likely to get each item correct except for Q4c (note that these were the labels used in the test – no question 3 was included). We can also look at the quality of explanations as categorized above (Table 2). In each case, the project students were more likely to provide higher quality explanations.

Table 1: Proportions of correct answers

	Group	
Question	Control	Project
Q1a	4.2%	17.0%
Q1b	2.0%	20.8%
Q2a	8.2%	32.7%
Q2b	34.7%	55.1%
Q4a	39.5%	40.8%
Q4b	16.3%	32.7%
Q4c	2.3%	0%
Q4d	39.5%	63.3%
Q4e	37.2%	44.9%

Table 2: Quality of explanations

		Quality of explanation			
Question	Group	1	2	3 or 4	5
Q1a	Control	6.3%	79.2%	10.5%	4.2%
	Project	6.4%	57.4%	25.5%	10.6%
Q1b	Control	20.4%	69.4%	10.2%	-
	Project	8.3%	52.1%	39.6%	-
Q2a	Control	38.8%	49.0%	12.2%	-
	Project	18.4%	44.9%	20.4%	16.3%
Q2b	Control	6.1%	63.3%	24.5%	6.1%
	Project	6.1%	38.8%	24.5%	30.6%
Q4a	Control	30.2%	53.5%	16.3%	-
	Project	34.7%	38.8%	26.5%	-
Q4b	Control	55.8%	37.2%	7.0%	-
	Project	46.9%	30.6%	20.4%	2.0%
Q4c	Control	55.8%	44.2%	-	-
	Project	44.9%	44.9%	10.2%	-
Q4d	Control	27.9%	44.2%	23.2%	4.7%
	Project	12.2%	26.5%	42.9%	18.4%
Q4e	Control	41.9%	32.6%	23.3%	2.3%
	Project	34.7%	20.4%	24.4%	20.4%

Calculating the overall student abilities using the Rasch Analysis, the average measures (in logits) for the project and control groups are given in Table 3.

Table 3: Results of Rasch Analysis

Group	Mean	Std. Deviation
Control	-1.37	.66
Project	-.69	.65

Using independent samples t-test, the difference in means between the two groups was found to be significant at the 1% level ($p < 0.001$), with the project group students having a higher average ability value. In terms of effect size, the difference between the two groups corresponded to an effect size of 1.05 or a difference of over one standard deviation in favour of the project students. Cohen (1969) categorises effect sizes of 0.3, 0.5 and 0.7 standard deviations as 'small', 'medium' and 'large' respectively. The difference between the two groups could therefore be considered to be very large.

Looking at the qualitative results, the interviews with teachers in the current evaluation showed they are enthusiastic about the RME approach, and believe in its philosophy. Some teachers noted that they find that the RME approach is a natural way for children to learn mathematics. They emphasised that it is essential that teachers understand the philosophy and are trained in the use of the materials, highlighting that "you can't just pick up the books and use them; it will not be effective". These teachers believe the RME approach develops a deeper understanding of mathematics in their pupils than more traditional methods.

Teachers reported that the contexts in the problems and related activities interest the pupils and so engage them in a lesson. Their pupils experience a range of activities, including practical work and discussion. Discussion at various levels in which pupils share their ideas, in pairs, in a group or as a whole class is an essential part of the RME approach and can occur several times during a lesson. The various contexts enable pupils to make links in mathematics through recognising the use of the same models in different contexts. Teacher noted that using RME encourages an intuitive approach, in which pupils can visualise problems and try things out for themselves and think about different approaches to a problem, rather than having a teacher demonstrate an algorithmic technique, which pupils then practice, probably with little understanding. Teachers compared the procedural nature of the traditional approach and its dependence on memory with the RME contexts and the models that evolve from them, noting these provide building blocks that pupils can fall back on, but also recognise they can use them in a new problem solving context. Teachers noted that the RME approach also gives pupils the confidence both to share their solutions with others and also accept that their solution may be incorrect.

Teachers noted that it may take several lessons for pupils to internalise the models they work with, but once they do they can understand how these models can be applied in a variety of contexts. This was contrasted with the more traditional approach and the need to move onto the next topic. Some schools where the RME approach is being adopted had rewritten their scheme of work for Key Stage 3, to reflect an integrated problem solving approach to mathematics rather than one with specific topics and teaching time allocated to them.

Teachers find that pupils are generally receptive to the RME approach and are more positive about mathematics compared to those who are taught by traditional methods. This was reinforced in the observed lessons where pupils were seen to enjoy working together to solve the problems and sharing their strategies and solutions with each other.

The transcripts of interviews with teachers who participated in the original MiC project showed teachers then had much the same views about RME as the current interviewees, some of whom were the same teachers.

Discussion

One outcome of this study is the exemplification of the use of Rasch Analysis to enhance the information that can be gained from pupils' assessments in mathematics. Not only was the analysis used to quantify the quality of the explanations provided by the pupils in the test, but was also used to confirm the validity of the codings used to categorise the explanations in terms of them being separate, distinct categories. Callingham and Bond highlighted the lack of use of methods such as Rasch analysis in mathematics education research, and also the possibilities it provides in bridging the "qualitative-quantitative divide" (2006, 2). This study has further illustrated the potential of Rasch analysis in mathematics education research.

The qualitative data provided by the interviews with teachers serve to explain why RME had such an impact on the test results of the project pupils in comparison to the control pupils. The contexts that are used can raise pupils' interest levels and enable them to bring their own ideas and intuitions into the classroom. These are then shared, discussed, and developed. This progressive development towards more formal procedures means that they ‘stay connected’ in the minds of the pupils; too often in more traditional approaches, pupils ideas are replaced by the formal, with the result that such procedures are remembered and then, all too often, forgotten. This, together with the amount of discussion in lessons, seemed to raise the confidence levels of pupils. This has the effect of making pupils more willing to ‘have a go’ at problems and also improves their ability to articulate mathematically. With this increased articulation, we also see the development of pupils’ explanations

In addition, the recurrence of familiar mathematical models in different contexts (for example, the number line and the ratio table) provides a structure for pupils and allows them to see connections between different curriculum areas. There is also an emphasis on visualization and activity which contrasts with more traditional approaches which teachers report as all too often being auditory and numerical. Teachers report seeing pupils using drawings and models not just in the classroom, but also in tests and examinations, and even in questions where no drawing was specifically required. This use of drawings and models, in addition to the above willingness to provide explanations, seemed to support the project pupils in the explanations asked for in the assessment given to them.

Acknowledgements

The authors would like to thank Mathematics in Education and Industry (MEI) for funding this evaluation.

References

Bond, T. G., and C. M. Fox. 2007. *Applying the Rasch model (2nd Edition)*. Mahwah, NJ: Lawrence Erlbaum Associates.

Callingham, R., and T. G. Bond. 2006. Research in mathematics education and Rasch measurement. *Mathematics Education Research Journal* 18(2): 1-10.

Cohen, J. 1969. *Statistical power analysis for the behavioral sciences*. New York: Academic Press.

Fosnot, C.T. and M. Dolk. 2002. *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann.

Gravemeijer, K. P. E. 1997. Mediating between concrete and abstract. In *Learning and teaching mathematics: An international perspective*, eds. T. Nunes and P. Bryant, 315-45 . East Sussex: Psychology Press

Streefland, L. 1991. *Fractions in realistic mathematics education. A paradigm of developmental research*. Kluwer academic publisher, Dordrecht.

Treffers, A. 1987. *Three dimensions. A model of goal and theory description In mathematics instruction – The Wiskobas project*. Reidel

Van den Heuvel-Panhuizen, M. 2003. The didactical use of models in Realistic Mathematics Education: an example from a Longitudinal trajectory on percentage. *Educational Studies in Mathematics* 54(1): 9-35.

Webb, D., N. Boswinkel, and T. Dekker. 2008. Beneath the tip of the iceberg: Using representations to support student understanding. *Mathematics teaching in the middle school* 14(2): 110–13.