Fractions in context: The use of ratio tables to develop understanding of fractions in two different school systems

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The project seeks to investigate the implementation of a number of Realistic Mathematics Education lessons on fractions in two different educational systems. Four teacher participants engaged with trialing an agreed sequence of lessons from an RME textbook in their own classrooms - one Primary 6 classroom in Northern Ireland and three fifth classes in Southern Ireland. The teaching of lessons was observed by each researcher in her own school system. Nine of the lessons were video recorded and short video clips were made of children at work during other lessons. Children’s mathematical workings from the lessons were collected and analysed. Similarities and differences in teaching approaches across contexts were examined with a view to identifying some of the supports and constraints experienced by teachers in the implementation of these lessons. In this session we propose to report on the manner in which three RME lesson contexts provided teachers and children with novel ways for thinking about and working with fractions

Keywords: Realistic mathematics education, fractions, ratio, teaching

Background to study

The research reported in this paper is funded by Standing Conference on Teacher Education North and South (SCoTENS). The aim of the study is to compare and evaluate the possible impact of implementing RME curriculum materials in primary classrooms in the North and South of Ireland. The study focuses on two main issues: the impact of the curriculum materials on children’s learning, and the support needs of teachers using these curriculum materials. A sequence of six lessons from *Mathematics in Context* (MiC) – a curriculum designed according to Realistic Mathematics Education (RME) principles for use in American middle schools – was chosen as the focus of the study. The six lessons were based on the topic of fractions. Drawing primarily on the Transformation dimension of the Knowledge Quartet (Rowland, Huckstep and Thwaites 2005) as an analytic framework, each researcher analysed the lessons she observed while adopting what we have called an “RME gaze”.

As researchers we acknowledge that these RME materials are ‘new’ and therefore “boundary objects” to the teachers concerned (Corcoran 2011). More importantly, we acknowledge that teachers are essential stakeholders in the research endeavour (Krainer 2011) and sought their participation as “co-researchers” (Wagner 1997) who met together as a group with us on two occasions to discuss the teaching of the lessons and to view and discuss video-clips from each others’ classrooms. This paper summarises findings on the children’s responses to some of these lessons in which the ratio table is used to organise calculations with fractions, followed by comments on the development of mathematics teaching arising from the implementation of RME contexts.

Key principles of RME

Use of 'realistic' contexts

At a first glance, the material within the MiC textbooks appears to differ significantly from traditional UK or Irish textbooks where real-life contexts may be used initially to interest and engage pupils but are usually followed by series of non-contextual exercises for pupils to practice the ideas and procedures introduced. Later, contextualized problems are given in which pupils are expected to apply the mathematics that has been practiced (Ainley 2011). In contrast, the mathematical content in the MiC materials is grounded in a variety of 'imaginable' contexts. Within RME, contexts are used not simply to engage and motivate learners, nor to illustrate the applicability and relevance of mathematics in the real world, but also as a source for learning mathematics (Van den Heuvel-Panhuizen 2000, 2003). Further, the ratio tables for scaling recipes up and down on which the lesson series under research are based appeared to provide a context in which the *procedures*, *relationships* and *utilities* inherent in the mathematics to be taught are overt to learners (Ainley 2011).

Use of 'models'

According to the RME approach, 'models' are seen as representations of problem situations which serve to bridge the gap between informal, context-related mathematics and more formal mathematics. Concrete materials, pictures, diagrams and even symbols can serve as models. When presented with a realistic context, the intention is that the pupil constructs a 'model of' the problem situation. Initially, this model is context-specific but over time it changes in character, becoming more general, until it serves as a 'model for' mathematical reasoning on a formal level. (Van den Heuvel-Panhuizen 2000, 2003).

The ratio table model

The three lessons reported in this paper are taken from the MiC transition unit *Some of the Parts*. This unit builds on pupils' informal knowledge of ratios and part-whole relationships and introduces operations with fractions. Each of the three lessons is grounded in the context of a recipe, which is intended to support the development of the concept of ratio. Pupils use informal strategies to increase and decrease the amounts of ingredients according to the number of servings required. Fractional amounts, such as ½ cup, 1/3 teaspoon, play an important role in recipes. Therefore, the calculations involve addition, multiplication and division of whole numbers, benchmark fractions and mixed numbers. The ratio table is therefore a tool that is used to organise these calculations. One advantage of the ratio table is that it provides an open structure for pupils to use their own steps when working toward a solution. The numbers in the columns can be placed in any order to suit the calculation.

Project lessons observed in Northern Ireland

Lesson one

In the first lesson, a pizza recipe provides a context for introducing the ratio table model. The recipe makes 4 pizzas and pupils are asked to determine the amounts of each ingredient required for 24 pizzas. A ratio table to show how the amount of one of the ingredients will change as the number of pizzas changes is introduced as an example; two further tables showing alternative strategies are also presented. After studying these tables, pupils are asked to complete the ratio table for all ingredients (Figure 1).

Number of Pizzas	4		
Jars (8 fl oz) of Spaghetti Sauce	1		
Pounds of Ground Beef	1		
Cups of Bread Crumbs	$\frac{1}{3}$		
Teaspoons of Dried Oregano	$\frac{1}{2}$		
Number of Olives	2		
Cups of Shredded Mozzarella Cheese	$\frac{1}{4}$		
Cups of Shredded Cheddar Cheese	$\frac{1}{4}$		
Number of Mushrooms	4		

Figure 1: Ratio table for pizza recipe

About two-thirds (19) of the pupils completed the table in two stages: doubling the first column and then multiplying by 3. Using a similar method, one child multiplied first by 3 and then by 2. A further seven pupils multiplied the first column by 6. One pupil mistakenly chose to multiply repeatedly by two, although she did not apply this rule consistently for all the ingredients. Another pupil appeared to be totally confused with the concept of the ratio table and struggled to complete it. The multiplication of fractions proved to be the most challenging aspect of this work. While the majority of pupils were able to multiply unit fractions by 2, many struggled with the next step which involved multiplying non-unit fractions by 3; the multiplication of thirds proved even more difficult. Pupil responses demonstrated varying levels of confidence in the application of knowledge of fraction equivalents. For example, not all pupils chose to simplify the result when multiplying ¼ by 2 and, when multiplying ½ by 2, one pupil recorded 1 0/2. Following the lesson, the teacher noted that “Ratio alone ok – fraction element confusing.”

Lesson two

The second lesson uses the context of a recipe for chicken and tortilla casserole but this time pupils are required to use a ratio table to find the amounts of ingredients if the number of servings decreases. The recipe caters for 8 servings and pupils are to complete the table for 4 servings and then for 2 servings. Once again, the calculations involving fractions proved problematic. Although some pupils were able to successfully divide halves and quarters by 2, division of thirds and sixths proved much more difficult. The class teacher summarised her reflection of the lesson as follows:

> Class very quick to ‘half and half again’. Completed activity sheet v. quickly…. Overall – pace of lesson seemed v. slow. Plenty of discussion but not enough for the pupils to do. Mixing fractions and ratio difficult.

Lesson three

This lesson uses a more complicated recipe involving whole numbers, fractions and mixed numbers (Figure 2). Pupils used a range of strategies to complete the table. For example, to find the amounts of ingredients required for 10 servings, some pupils multiplied 2 servings by 5 while others combined 8 servings and 2 servings or 6 servings and 4 servings.

Servings	4	2	8	6	10	16
Cups of Flour	$\frac{3}{4}$					
Cups of Margarine	$\frac{1}{4}$					
Tablespoons of Powdered Sugar	3					
Tablespoons of Water	$2\frac{1}{2}$					
Cups of Yogurt	$1\frac{1}{3}$					

Figure 2: Recipe for yoghurt cups

It was interesting to note that pupils tended to work in a preferred order. Some worked horizontally, completing one row at a time. For example, one pupil started with the powdered sugar since it involved whole numbers only; progressing to the ingredients involving fractions only and finally the mixed numbers. Others preferred to work vertically, completing one column at a time. Some pupils worked in a more random order, choosing to work with the 'easier' calculations first.

A large proportion of the class relied on their fraction walls to support the calculations involving less familiar fractions. The more able pupils relied on mental strategies, demonstrating a greater facility with fraction equivalents. However, when adding ½ and 1/8, one pupil recorded 2/12. In relation to the calculations involving mixed numbers, pupils tended to multiply or divide the whole number first and then the fractional part.

Differing classroom contexts

Northern Ireland

The pupils whose responses are reported above were in a mixed-ability Year 7 (aged 10-11) class – the final year of primary education in Northern Ireland. Over two-thirds of the pupils had previously finished working towards the high-stakes tests on academic ability which are used to determine whether they will transfer to grammar or secondary schools. The class teacher had over twenty years of teaching experience and had been teaching Year 7 for approximately ten years. When asked about her perspective on the learning environment in her classroom, she explained that at the beginning of the year it is "very assessment and knowledge-centred … and not centred on the learner at all." She blamed this on the fact that the children have so many tests to do, with preparation for the high-stakes tests for academic selection in the first term for Year 7 dominating the curriculum in upper primary. For this reason, she claims that the children:

> have been pushed quite hard probably in Year 6 and moved on without maybe really grasping things ... sometimes by Year 7 you realise they don't have that understanding and then you have to go back over things again.

The teacher from Northern Ireland also conjectured that the pressure of assessment has resulted in a mind-set whereby children want to get everything right and this poses difficulties for tasks which require pupils to investigate and explore different strategies, and to share and discuss their ideas.

Southern Ireland

Three teachers in different classrooms in the south were observed teaching these same lessons. An attempt was made to match the schools according to age of children and socioeconomic status of families. In southern Ireland, primary schooling continues until the children are twelve with the result that the matching classes were all in the penultimate year in primary school. The most obvious disparity in context between the two educational settings is the difference in emphasis on external assessment. During the second of two joint seminars held with teachers and researchers (one in Dublin mid way through the research, one in Belfast towards the end) teachers were invited to locate themselves and describe their classrooms in terms of perspectives on the learning environment (Donovan and Bransford 2005). When the NI teacher identified her classroom environment as shaped through an "assessment-centered lens", the three teachers from the south sympathized with the constraints they imagined such a focus would place on their teaching. In turn, teachers from the south thought their classrooms were more "learner- centered", and "knowledge-centered" (ibid., p. 13). Ironically, since this seminar took place a government circular in the South has mandated that children in second, fourth and sixth classes are to undergo standardized testing in mathematics from May/June 2012, results of which will be made available to parents, school authorities and the inspectorate (DES 2011). While this change in the culture of schooling does not apply to the classes (fifth) observed in this study it may well bring about shifts in emphases and changes in teaching approaches to mathematics, for which these teachers are not yet prepared.

Teacher A

Teacher A was the most experienced teacher of the three and appeared very much at home with the mathematics. He enjoyed teaching the RME lessons and discussed the utility of the ratio-table and its application to a 'sum' that the children had met in their class textbook. His class had adopted an engaging metaphor based on comparing the advantages of a short cut over a roundabout route to the village which became common currency for describing different solution approaches to filling in the ratio tables. Teacher A projected the pupils' activity sheets related to the lessons on to the white board and this idea was subsequently adopted by the other teachers. He extended the task in Pupils' Activity Sheet 7 which required scaling a recipe for Chicken Tortillas down from eight servings to four and then two, by drawing an extra column on the white board and inviting suggestions for how it might be further reduced to make one serving. Such was the classroom climate that one child then suggested that the recipe be further amended to give zero serving, a suggestion Teacher A gleefully invited the class to implement.

Conclusion

Mathematics teaching is a highly complex endeavour. The forgoing paragraphs reporting the responses of pupils in the NI classroom to the mathematical contexts presented by the ratio tables offer evidence of a 'task/results-oriented' approach in the manner in which children engaged with the activities. While the research participants in the south embraced the project with equal enthusiasm, there appeared to be differences in teaching styles which influenced how the material was presented to the children and greater variety in the responses. The research approach fostering "an investigative attitude towards their own practice" (Krainer 2011) could have remained at the level of individual teacher and researcher but because of the 'cross border' nature of the joint seminars interest in comparing practices was heightened with increased potential reflection and for developing teaching. There are saliencies here with RME movement, an integral part of which is the work of *Tussendoelen Annex Leerlijnen* (TAL) project teams. This Dutch concept translates into 'intermediate attainment targets in learning-teaching trajectories' and involves teams of teachers and researchers working together to improve education by "providing insights into the broad out-line of the learning/teaching process and its internal coherence" (Van den Heuvel-Panhuizen, 2008). Above all, such work is time-consuming, on-going, involves many people and is considered worthwhile.

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