Relationships between the influences on primary teachers' mathematics knowledge

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This paper reports on one aspect of a small scale research project that aimed to identify areas for improvement in the teaching of mathematics through continuing professional development for primary school teachers in two schools. The findings suggest an emergent conceptual framework of the influences on primary school teachers' mathematical content knowledge. The relationship between these influences reveals a multi-layered belief system that is well-intended and well-informed at the top level but underpinned by less firmly established levels of subject knowledge and consequent pedagogical approaches.

Keywords: primary teachers' mathematics knowledge, primary teachers' pedagogical mathematics knowledge, procedural and conceptual learning.

Introduction

The relationship between the quality of mathematics teaching and the subject-related knowledge of the teacher is widely acknowledged as an important one. It remains the subject of much discussion and various studies continue to illustrate how crucial the dependency of good mathematics teaching is upon the subject knowledge of the teacher if pupils are to gain a comprehensive understanding in their mathematical learning (Williams 2008; Office for Standards in Education Children's Services and Skills 2009). Although such reports continually bring into question the mathematical subject knowledge of in-service primary school teachers, little is known about the knowledge individuals possess or how it is constructed. This small scale research aimed to investigate this knowledge whilst at the same time to identify areas for continuing professional development for the teachers involved.

The study involved interviewing 11 primary school teachers, from two different schools, and focused on three areas: firstly, the content of these primary school teachers' subject and pedagogical knowledge and how it is constructed; secondly, the conceptual and procedural knowledge of the teachers, particularly the balance between the two and how this impacts upon their content knowledge; and, finally, the teachers' self-perceived continuing professional development needs. This paper reports on some of the findings of the first two areas of investigation.

Knowing and Teaching Primary Mathematics

The contexts that were used to explore the content knowledge of the participating teachers were developed from those of Ma (1999) in her investigation of the content knowledge of Chinese and American elementary school teachers. Designed to explore knowledge of mathematics and ideas about teaching and learning mathematics, the contexts identified how the teachers approached the teaching of a topic, how they identified and addressed pupils' misconceptions, how they created and used models, and how they responded to the unexpected. What is reported here focuses on the responses to the first context, identifying the approaches that teachers adopted in the teaching of a topic. Interviewees were asked how they teach a written calculation for subtraction which involves regrouping, such as 845 - 278.

A conceptual framework

The findings allowed a multi-levelled framework to be constructed, which displays the influences on primary teachers' mathematics knowledge and the relationship between aspects of this, see figure 1.

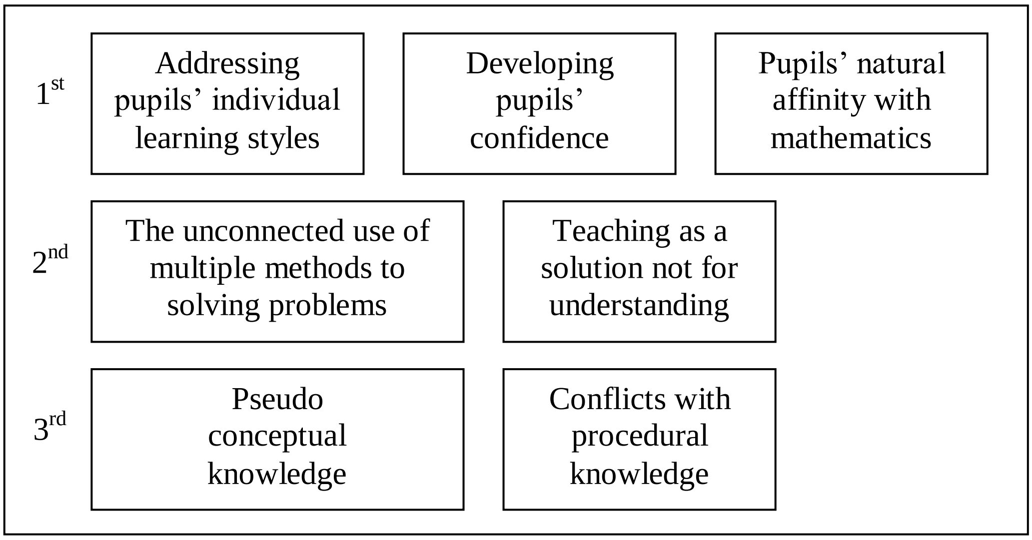

Figure 1: A conceptual framework of the influences of primary teachers' subject knowledge.

The first level identifies the teachers' three main beliefs influencing their mathematics teaching and supporting their pedagogical beliefs about learning. The teachers were fully aware of these aspects and they were very much at the centre of their practice. Underpinning these beliefs, at the second and third levels, are aspects that describe the resultant ways in which those at the first level are manifested within teachers' belief systems. Teachers are apparently unaware of these aspects.

The first level

On the first level are three aspects central to the teachers' pedagogical beliefs. Developing pupils' confidence, addressing their individual learning styles and catering to any natural affinity with number, are all considered by the teachers as important aspects of supporting mathematics learning. Providing learners with opportunities to achieve success in mathematics was a feature common to all the teachers' pedagogical beliefs which seems to stem from their own experiences of learning mathematics. This feature was frequently discussed and was often referred back to when answering other questions.

Another aspect that is prominent in the data, and exists within the first layer, is the importance of teaching that addresses the learning styles of the pupils. The teachers felt that some teaching models suit certain learning styles better than others; for example, they argued that visual learners may prefer the empty number line for the purposes of addition and subtraction calculations, whilst the grid method of multiplication may suit auditory learners as it emphasises the partitioning of the factors to be multiplied, 45 is forty and five.

The third and final aspect contained within the first level is the perception, by these teachers, that some pupils have a natural affinity with numbers. The teachers believe that this affinity enables pupils to understand column methods easier than their peers. Pupils may therefore choose to use column methods in preference to other methods such as the empty number line (ENL) when calculating subtraction or the grid method when calculating multiplication.

These three aspects, at the first level, demonstrate the teachers' commitment to their students. However, the teachers appear to be unaware of other aspects that exist as a result of their efforts to maintain these three first level aspects as part of their practice. They also appear unaware of how these aspects manifest themselves within their own belief system and at the second and third levels of the framework.

The second level

It appears that there is one fundamental aspect of these teachers' pedagogical beliefs that enables them to address all the aspects on the first level. All the teachers acknowledged that pupils need to be presented with many methods of calculation so that, in turn, pupils may express a preference towards a particular method. The teachers felt that the choice pupils make depends upon their confidence, learning style and their level of natural affinity with number. Providing the opportunity to make this choice is central to pedagogical beliefs where aspects of the first level are all considered important in the learning of mathematics. What appears to be driving this is the firm belief that pupils need a method that they are able to use efficiently. Thus, by presenting various methods of calculation, pupils may select the one they can understand and use, so if one method cannot be used successfully another one will be tried, illustrating a belief which favours preference over prescription.

In considering the example, 845 - 278, the teachers suggested the use of the ENL to avoid the learning of algorithmic procedures that fail to develop understanding. However, most teachers recognised that pupils with a natural affinity for number are generally able to readily understand column methods and therefore do not need to use the ENL. They argued that, with increased practice with the ENL, pupils would automatically gain confidence and develop their understanding of number and number relationships in such a way to enable progress to the column methods, understanding the concepts that underpin them. The belief appears to be that, without the ENL experiences, the acquisition of concepts underpinning column methods is too difficult. Criticism of this idea that concepts underpinning the ENL develop directly into those of the column methods is well documented by authors, such as Thompson (2007), who argue that the sequential nature of number associated with the ENL is unconnected to the partitioning strategies used in column methods.

The teachers demonstrate a lack of awareness of this, drawing heavily upon their teaching experiences to highlight their beliefs that this development of strategies, within the understanding of written calculation, is correct. What appears to be revealed here is a kind of pseudo-learning trajectory within teachers' mathematical content knowledge. They argue for various linear stages of learning through which their pupils will progress, although this may not necessarily be the case.

Multiple methods of calculation

The teachers all allowed their pupils to adopt any system of jumps along the ENL, as it was felt that, in selecting their own methods, pupils were able to build their confidence and explore their own affinity with number. The various jump systems described by teachers in the interviews were accounts of teachers modelling them, which some pupils then adapted. These jump systems have been combined in figure 2.

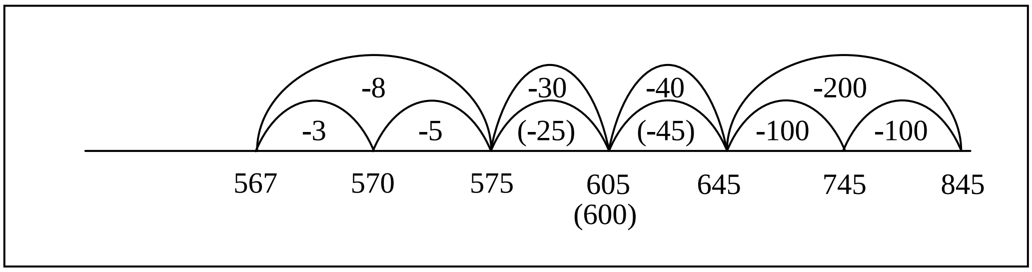

Figure 2: Various jump systems that may be involved in the calculation 845 - 278.

All these different jump systems are based upon the strategy of partitioning of the subtrahend according to the place value of its digits. For example, starting at 845, 200 will first be subtracted, then 70 and then 8. Two observations are made from the analysis of these various jump systems. The first is the level of confusion some teachers experienced when deciding how best to ‘cross the hundreds barrier’ when subtracting 70 and model this to pupils. Most teachers modelled a jump of 40 followed by one of 30 as this would get close to the 600 barrier at 605 without causing too much confusion for pupils. Only one teacher said that he would model ‘hitting the 100’ by partitioning the jump of 70 into one of 45 and one of 25, although he wasn’t sure if some of his less able pupils would be able to calculate the size of this last jump as it involved subtracting 45 from 70, which was the main reason for the other teachers to opting for the jumps of 40 and 30 instead. Only then did he begin to question the suitability of the model for 3-digit subtraction, particularly with pupils who were not ready for it. This realisation that was not apparent in the responses of the other ten teachers.

For one teacher, ‘crossing the hundreds barrier’ problem was solved by making seven jumps of 10, whilst another decided that the jumps were too numerous and unnecessarily complicated the calculation, suggesting she might model this through finding the difference between 278 and 845 by ‘counting on’ or, in this case, ‘counting back’, see figure 3. The process is less complicated than those of figure 2, although the teacher appreciated that, for pupils with little knowledge of number bonds to 100, this may not be an ideal solution.

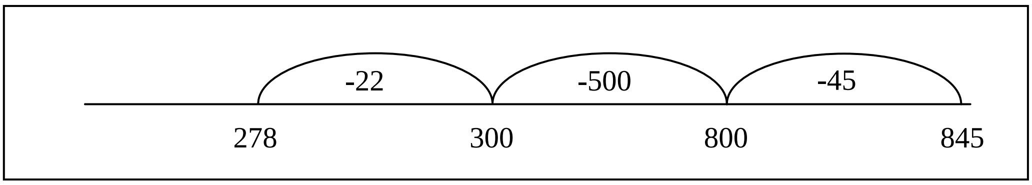

Figure 3: Adaptation of the counting-on strategy in calculating 845 - 278.

Progression in the use of jumps on the ENL was another factor in teachers’ pedagogical misunderstandings. According to van den Heuvel-Panhuizen (2001), the use of fewer and larger jumps on the ENL represents shifts in pupils’ mathematical understanding. The jump systems described by the teachers fail to recognise this. They suggest, for example, subtracting 70 in one jump using knowledge of number bonds but subtracting 8 through making four jumps of 2. It appears that, in offering a multitude of jump systems to pupils, these teachers are unaware of the progression within pupils' mathematical understanding illustrated by the different uses of the ENL model (Treffers 2008). Some teachers also commented that the level of understanding of lower attaining pupils may mean that they are not able to progress beyond the use of the ENL, which they considered perfectly acceptable provided the pupils calculated correctly.

So, in addressing aspects of the first level, aspects of the second are formed, comprising the unconnected use of multiple methods which are, in turn, driven by the need to solve the problem of ensuring pupils have a method for calculating.

The third level

As already highlighted, these teachers held the belief that one essential area of prior knowledge for pupils to successfully calculate 845-278 is a competent use of the ENL. They claimed that this competent use leads to improvements in pupils' number sense which, in turn, allow the conceptual understanding of column methods to be readily achieved. They believed the procedures involved in the algorithms are remembered by pupils consequently. Two further areas of prior knowledge were also identified by the teachers, namely, place value and partitioning.

All the teachers discussed how they considered their pupils' appreciation of the value of each digit to be of vital importance. Perceiving a tens digit as complete in itself was important, as this, they felt, would avoid the procedural trap of "borrowing one from the next column." However, despite their conscious attempts to avoid procedural learning, these teachers weren't always successful in doing so.

One teacher, whilst demonstrating her modelling and discussing the problems of subtracting 8 from 5 in the calculation 845 – 278, attempted to avoid the "borrowing" procedure. Deliberate changes in her explanation to avoid "borrowing one" resulted in "taking a ten from the next column to leave 30 and moving this ten to the next column, where it could be added to the 5." The 40 and the 5 were still treated as two separate numbers instead of two components of the expanded number which could be regrouped into 30 and 15. This made "borrowing" an inevitable outcome, significant in this case as this particular teacher was one of only two who outlined the importance of pupils being able to regroup numbers differently.

For most teachers, much discussion focussed on the dangers of teaching the procedures without understanding. What they failed to realise was their modelling still focused on "borrowing" rather than the "regrouping" they believed their teaching supported. Language used to support this perceived shift in focus, and avoid the procedural connotations of "borrowing", included "steal", "take off ten and adjust", "move a ten over" and "use a ten from the tens and bring it back".

Prominent at the third level, then, is teachers' content knowledge which is a pseudo-conceptual knowledge that leads them to believe that they are teaching conceptually and avoiding procedural teaching. It is evident from the interviews that most of the teachers are aware of the dangers of relying on procedural teaching, to such an extent that they view procedural teaching as a practice to avoid. Yet, there appears to be conflict with this belief and their experiences. For example, when discussing the use of open-ended tasks in the classroom, most teachers believed that pupils first needed to be taught skills, often procedurally, in order to be able to apply these in the open-ended task. This is something with which they appeared somewhat uncomfortable, as they felt they should be teaching conceptually.

Concluding remarks

The mathematical content knowledge of in-service primary school teachers has been described, here, within a multi-levelled theoretical framework where relationships between each level are not yet clear. Subsequent analysis suggests that relationships between the aspects within each level and between the levels are more likely to form a fluid and inter-related web than static successive levels dependent upon those that exist above and beneath another.

Firstly, these teachers are unable to relate successfully the first-level aspects to either their second-level pedagogical approaches or their third level mathematics subject knowledge. At the second level, the use of a multiplicity of methods by these teachers to support students solving calculations is misconstrued as a means to mathematical efficiency. A lack of conceptual understanding becomes evident at this level. At the third level, the teachers are conscious that procedural learning can lead to conceptual learning, as illustrated by Hiebert and Lefevre (1986), but they are not sure how to achieve this and they lack sufficient conceptual confidence themselves to utilise pedagogical approaches which support this.

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