The development of Taiwanese students' understanding of fractions: A problem-based learning approach

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Problem-based learning (PBL) was first implemented in medical education at the McMaster University in Canada in the late 1960s. Now, we are seeing an explosion in the use of PBL in its various adaptations across many levels and subject areas. This paper outlines some preliminary findings from a one-year PBL teaching intervention on students' understanding of fractions in a Taiwanese fifth grade mathematics classroom. The purpose of the study is twofold. Firstly, it seeks to investigate the process of implementing PBL in the context of a Taiwanese elementary school. In doing so, it aims to help others to gain some usable insight by showing them this intervention as it really was. Secondly, it aims to understand what impacts a series of PBL intervention has on the students' understanding of fractions and to add to the knowledge base on the teaching and learning of fractions.

Keywords: problem-based learning (PBL); understanding of fractions; teaching and learning mathematics in Taiwan.

An introduction to problem-based learning (PBL)

PBL has been a popular topic of research and discussion among educators, educational psychologists and researchers. Its importance has been highlighted by many studies and documents. PBL was first implemented in medical education at the McMaster University in Canada in the late 1960s. The model for student-centred, problem-based, small-group learning took shape at McMaster 40 years ago. Now, we are seeing an explosion in the use of PBL in its various adaptations across many levels and subject areas.

PBL stands in the philosophy of social constructivism (Savery and Duffy 1995). That is to say, in a PBL environment students engage in social learning activities that involve hands-on problem-based situations and utilisation of discipline-based cognitive tools, and they work as groups to impose meaning on the knowledge that they construct through the social learning process. Therefore, learning is a process, not a product. The practices compatible with constructivist views through PBL approaches may vary from subject to subject. In general, there are three main characteristics of PBL that contribute to its potential benefits.

The first characteristic of PBL is the problem-driven content structure. The content of a PBL lesson is organised as a problem or a series of problems rather than in a textbook form. These PBL problems are challenging, open-ended and contextualised. The second characteristic of PBL is the inquiry-based and collaborative learning processes in which students work as groups to solve problems and learn from small group collaborative interactions, rather than being taught by the teacher. The third characteristic of PBL is the student-centred situation. In PBL, teachers are not in classrooms to deliver knowledge to students, but are there to

facilitate students' learning. Likewise, students are not in classrooms to wait for their teachers to give them instruction, but are there to construct knowledge and establish a new level of knowledge through the process of working with group members.

Students' difficulties with fractions

Davis, Hunting and Pearn (1993, 63) argue that "the teaching and learning of fractions is not only very hard; it is, in the broader scheme of things, a dismal failure." Such a quote is shocking, but, unfortunately, research has agreed that fractions have continued to be a difficult topic of elementary mathematics over the years. The difficulty that students encounter with fractions is largely attributed to the complex relationships between different representations and basic arithmetic operations. Their symbolic form simultaneously represents many concepts (Kieren 1988). For example, 2/3 is part of a whole, a rational, a ratio, a number in its own right (Mack 1990), a location on a number line, a representation of division, an operator and an operand (Dienes 1969); 2/3 of something is different from the number 2/3.

Studies show that many students hold a limited view of fractions and that is one of the causes of their difficulties in this area. It has been long argued that the concepts that children are encouraged to construct in schools lead to a limited view of fractions, since students are, traditionally, taught algorithms about fractions with little attention to any meaningful grounding in this area. Kamii and Dominick (1997, 59) also claimed that "when we try to teach children to make relationships between numbers by teaching them algorithms, we redirect their attention from trying to make sense of numbers to remember procedures." Schoolchildren often have only a brief exposure to many concepts and procedures of fractions. Li (2006) examines Taiwanese students' conceptual and procedural knowledge of fractions at ages 12 and 13 and concludes that there is a perceived inequality in the procedural and conceptual developments of Taiwanese students regarding fractions. Many of them may be able to use algorithms to do a wide range of operations regarding fractions, but they do not necessarily understand the concept that underlies the operations.

The purpose of the study

Before describing the purpose of this study, it is helpful that the reader be familiar with some educational contexts of Taiwan in which the teaching intervention was located. Mathematics teaching in Taiwan is strongly influenced by the constant preparation for examinations. Birenbaum, Tatsuoka and Xin (2005, 175) describe that the examination culture in many Eastern Asian countries reflects "a nationwide obsession with excelling in exams." Such an examination culture may partially reflect the teaching and learning of mathematics in Taiwan. In most mathematics classrooms in Taiwan, students sit individually and rarely interact and work as groups. Lecture-based instruction dominates classroom activities (Lin and Tsao 1999). Further, the majority of Taiwanese teachers also tend to "prefer traditional settings" (Yang, Chang and Hsu 2008, 527).

Earlier, Leu (1996) examined the teaching and learning of fractions in Taiwan and concluded that working with fractions often challenged students and teachers alike. For Taiwanese students, they may have learned a lot about fractions both procedurally and conceptually; however, their ability to use this knowledge may remain under-developed. In view of the potential benefits of PBL to students' development of a higher-order understanding of mathematical ideas, I therefore

proposed to undertake a one-year PBL teaching intervention on students' understanding of fractions.

While a great number of studies have offered a wealth of empirical evidence to the potential benefits of PBL, it remains like a "black box" (Hak and Maguire 2000) without sufficient information and details of implementation varied among different programmes. Many challenges and problems may arise to obstruct the development of cooperation and interaction in real-life PBL classroom situations, since cooperation does not necessarily occur when students are put together and asked to work in groups. Recently, a new focus of research attention has been paid to understanding the essential mechanisms underlying the situation being investigated and, therefore, framing research as a case of a more encompassing phenomenon with the aim of providing more educational value to the understanding of the relevance in real-life classroom settings. Therefore, the purpose of the study is twofold. Firstly, it seeks to investigate the process of implementing PBL in the context of Taiwan. In doing so, it aims to help others to gain some usable insight by showing them this intervention as it really was. Secondly, it attempts to understand what impacts a series of PBL intervention has on the students' understanding of fractions and to add to the knowledge base on the teaching and learning of fractions.

Participants

The participants in this study were one fifth grade teacher (Miss Lee) and her thirty-five students (19 boys and 16 girls). Both the teacher and students encountered PBL for the first time. Miss Lee considered her teaching as a primarily lecture-based and textbook-driven style. The results from the student interviews also provided some confirmations of it. They all described that their regular mathematics lessons were that Miss Lee talked and they listened. They hardly asked questions, expressed their thoughts or shared ideas with other students during lessons.

Four teaching units during the intervention

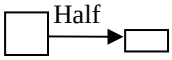
The main structure of this intervention was designed by me, in discussion with Miss Lee, and carried out by her. In the classroom I involved myself in the role of 'observer-as-participant', in which my identity as a researcher was clear to the students and I was there to observe what was going on in the classroom without taking part in the teaching activities.

According to the fraction-related studies and the Taiwanese mathematics curriculum, four teaching units were designed: (1) Basic Constructs of Fractions; (2) Equivalent Fractions; (3) Multiplication of Fractions; (4) Fractions, Decimals and Percentages.

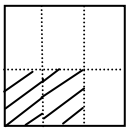
In general, the PBL process in this study followed three steps. I use the following problem, which was drawn from the teaching unit of fraction multiplication, to further illustrate these three steps. The objective of this problem was to help students conceptualise the rule of multiplying fractions (multiplying tops and multiplying bottoms).

> Problem: Jenny's mum made a square-shaped cake for her birthday. At the party, half of the cake was eaten and then the rest put in the fridge. The next day, Jenny's brother ate 2/3 of the remaining part of the cake.
>
> Please first work in a pair to discuss how you would fold the coloured paper provided to represent how much cake Jenny's brother had eaten and then share the pair work in groups.

The first step was to launch a problem. Miss Lee introduced the problem/activity to students. The second step was to solve the problem through group/paired discussion. Students worked as groups/pairs to explore and solve the problems and Miss Lee circulated among groups at all times. The following quotes were drawn from group/paired discussion over the aforementioned problem.

> 'This is easy. Jenny's brother ate $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$, but how do we fold the paper to present it?'
>
> 'First fold the paper in half, like this: 
>
> , but what we do next? Erm, a bit challenging.'
>
> 'Miss Lee, are we allowed to use a ruler?'

The third step was to conduct a whole class discussion over the problem being investigated. Students articulated their solutions to the whole class and gave/received feedback to further reason the problem. During the whole class discussion over the problem above, Miss Lee invited students to come forward to show how they folded. Several students raised their hands. Miss Lee picked three students to come forward. Miss Lee used one of their folding approaches, as shown below, to further guide students to discuss why $\frac{1}{2}\times\frac{2}{3}=\frac{1\times 2}{2\times 3}=\frac{2}{6}$.



Data collection methods

In terms of investigating the implementing process of the intervention, data were collected from school and classroom observations, teacher and student interviews, audio- and video-recordings of all the lessons, field notes, researcher diaries and the post-implementation open-ended student questionnaire. In examining the students' understanding of fractions, I collected data from classroom observations, audio- and video-recordings, field notes, group discussion notes, the post-unit (PU) tests and the post-implementation (PI) test. Group discussion notes were that each group was required to note down about what they had discussed during group discussions. The PU tests comprised a series of questions, each of which contained both open and closed questions that related to the concepts of the teaching unit; they were given to students at the end of each unit. The PI test, incorporating identical questions in the PU tests, was administered to all students three weeks after the intervention had been completed. The comparison between the results from the PU tests and the PI test provide an insight into the retentive effect of the intervention on students' understanding of fraction sense.

Preliminary findings

Some preliminary findings from this one-year PBL intervention are presented here. In terms of the implementing process, there were three main phases: the beginning (Lessons 1 to 5), the middle (Lessons 6 to 12) and the later parts (Lessons 13 to 19) of the intervention. During the beginning part of the intervention, it was evident during classroom observations that after a problem was launched, although some students might remain indifferent or quiet, most students got together for the discussion and looked like they were enjoying the discussion. The classroom

atmosphere appeared harmonious and relaxed. Miss Lee also told me that she was satisfied with the involvement of students' participation in the activities during this beginning phase. Nevertheless, many difficulties and challenges were gradually arising and needed to be addressed during the middle part of the intervention; for example, "students talk off topic", "students don't get along", "students' disruptive behaviour" and "teacher being angry".

It was not surprising to know that many things affected the process of the implementation when creating a more learner-driven classroom. However, for Miss Lee who was accustomed to being a traditional teacher, she appeared to be struggling with the aforementioned challenges. She thus tended to use a more custodial pupil ideology and a generally more authoritarian approach to demand immediate compliance from students for maintaining control over the class and running lessons smoothly As Clark (2006) notes, due to the long-held pedagogical beliefs, even the best-intentioned teachers can easily slip into traditional teaching approaches. It was also observed that, during the middle part of the intervention, Miss Lee struggled to balance the teachers' teaching desire and the PBL principles. Often she ran counter to the principles of PBL, of which she seemed hardly aware. For example, a whole class discussion became teacher summary time.

As Reeve et al. (2004) suggest, the possibility of creating learner-driven classrooms is in the hands of the teacher; however, dealing with challenges in a PBL environment is not a single competence which suddenly emerges in the teacher. By the later part of the intervention, it was observed that Miss Lee appeared to gradually gain in classroom experience; suggesting that previous experiences helped her develop some strategies and take more appropriate teacher action. For example, in terms of her classroom management, there was a change from straightforward coercion to a better communication style.

Earlier, when Miss Lee saw dysfunctional group work, she would usually immediately deliver punitive measures to the group (e.g. asking them to stand up until they were told to sit down) as a consequence of their failure. During the later part of the intervention, she would try to explain to students why they were being asked to do the tasks in groups and emphasise the importance of discussion to their learning. The attitude used by Miss Lee to deal with the situation above showed a hint that she was changing from merely 'doing things' to the students towards 'working with' them. Further, instead of acting as only one authoritarian figure in the classroom, she began to share some of her authority with the students to take charge of their group off-task behaviours. The frequency that she gave direct solutions to students' problems also decreased over time. She would redirect the questions addressed to her back to the group. This suggests that she appeared to act as less of a mathematical-content expert in class.

In terms of the impacts of the intervention on the students' understanding of fractions, the results of the students' performances between the PU tests and the PI test show that the students performed better in the PI test than the PU tests. It is important to note here that there could be many factors that would have contributed to the fact that the students generally did better in the PI test than earlier in the PU tests. Nevertheless, as far as the scores were concerned, the increased scores might imply that this PBL intervention may have a positive impact on the students' understanding of fractions. Further results will be offered in the near future. Incidentally, this test result seemed to be a source of encouragement for Miss Lee. She mentioned,

> I am glad to know this result. I didn't expect that they would be able to do better in the PI test. I thought they would forget most of the concepts introduced during

the lessons and perform much lower in the PI. It seems to work, doesn't it? I may involve this kind of teaching more in my teaching, although it is challenging.

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