Where has all the beauty gone?

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Bertrand Russell famously talked of mathematics as possessing an "austere beauty". It would seem though that the capacity to appreciate the aesthetic aspects of our field is not necessarily the preserve of the mathematical elite. Indeed, a number of educators believe that such considerations have, in conjunction with various cognitive factors, the potential to play a significant role with respect to the student learning of mathematics in the classroom. We consider here the notion of the mathematical aesthetic within this context, drawing on the work of a number of key thinkers in this area. Our preliminary explorations focus on a number of lesson observations, and the intention at this stage is merely to ascertain whether or not aesthetic considerations are playing any part in students' mathematical development in the classroom. We provide a brief discussion of our findings thus far, highlighting potential issues and dichotomies that would appear to arise as a consequence of the current climate of test-score-driven schooling.

Introduction

Aesthetics is generally considered to be a sub-discipline of axiology, which is itself a branch of philosophy concerned with the nature of values and value judgments. More specifically, aesthetics is associated with the nature of beauty in all its many forms. Those who study it are interested in both the creation and the appreciation of objects of beauty. Aesthetics often has connotations of physical beauty (and hence of pleasurable visual sensations), but this is an unnecessarily restricted viewpoint. Indeed, Yuri Borev, a former Professor of Aesthetics at the University of Moscow, gives the following very broad definition of his field of study:

> Any human activity has, besides a purely utilitarian purpose, the grains of what makes it universally important for mankind. It is these grains which lend human activity its aesthetic flavour. (Borev 1985)

However, our primary purpose here is to consider the situation with regard to the mathematical aesthetic in our school classrooms. The above is therefore possibly a little too all-encompassing for our present needs. The somewhat narrower perspective we adopt in this paper in keeping with the following:

> A student's aesthetic capacity is not simply equivalent to her ability to identify formal qualities such as economy, unexpectedness or inevitability in mathematical entities. Rather, her aesthetic capacity relates to her sensibility in combining information and imagination when making purposeful decisions regarding meaning and pleasure. (Sinclair 2004)

It ought to be mentioned at this point that the title of the present paper is intended simply to stimulate thought and discussion; we are certainly not claiming here that the consideration of the mathematical aesthetic has ever played a prominent role in the way that mathematics is taught and learnt in the classroom. In fact, no assumptions whatsoever are being made in this regard, and, as a consequence, we do

not attempt to make any comparisons with the past; all our observations concern the current state of play. An alternative title might have been: Is there evidence in our classrooms for the presence of, or appreciation for, the various aesthetic aspects of mathematics?

In this paper then we make an initial foray into the notion of the mathematical aesthetic within the context of the modern mathematics classroom, bearing in mind all the baggage that accompanies this milieu in terms of the different constraints and pressures that pupils and teachers continually have to work under. We ask a number of questions in this regard, and, in particular, consider the potentially inhibiting influence that current test-score-driven schooling may have on the development of both teachers' and students' aesthetic sensibilities.

The mathematical aesthetic

Let us now consider in a little more detail what is actually meant by the mathematical aesthetic. It is highly likely that any reader of this paper will have experienced some aspect of the beauty that is inherent in mathematics. Besides the obvious visual appeal of depictions of mathematical objects such as the Mandelbrot set, there are several ways in which our aesthetic sensibilities may be touched by mathematics. For example, every one of us will have encountered *beauty in mathematical method*. This is able to manifest itself through the appreciation of an elegant proof of a theorem such as the irrationality of $\sqrt{2}$ or the infinitude of the primes, both of which are extremely succinct and based on very simple notions. Notice how often the word "elegant" appears both in discussions between mathematicians and in mathematical writing.

We may also gain pleasure from the very act of participating in mathematical activities, particularly those of an exploratory nature. In engaging with mathematics this way and adopting a 'hands-on' approach we become intimately acquainted with its beautiful structures and are able gradually to unlock its secrets. This might be termed *beauty via mathematical experience*. In addition, there are many results in mathematics that could be deemed to possess a certain aesthetic quality. One of the most oft-quoted examples in this regard is the equation $e^{i\pi}+1=0$ relating five numbers that play a central role in the field of mathematical endeavour. The Prime Number Theorem, a result giving a degree of order to the apparently-erratic distribution of the prime numbers, is another instance of this. Indeed, we may say that such examples provide us with the opportunity to experience *beauty in mathematical results*. It is of course possible also to enter into any number of debates about the nature of mathematics. Is it, for example, the case that mathematics is invented or simply discovered? Considerations of this type might be referred to as *beauty through philosophical aspects of mathematics*.

An elitist concept?

Of course, some of the examples given above might not necessarily be suitable for the mathematics classroom. At this point then we may ask ourselves a rather pertinent question: Is the mathematical aesthetic an elitist concept; something that only a select few have the capacity to appreciate? Some professional mathematicians would no doubt answer in the affirmative. In order seemingly to reinforce this point of view, we provide the following relatively well-known quotes from Henri Poincare, Godfrey

Hardy and Bertrand Russell, respectively, each of whom was a leading of mathematician of their time:

> The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful. (Huntley 1970)

> The mathematician's patterns, like the painter's or poet's, must be beautiful. The ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: There is no permanent place in the world for ugly mathematics. (Hardy 1999)

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty. (Russell 1988)

There are, however, a number of distinguished mathematics educators who do not share the belief that only expert mathematicians are truly able to experience the mathematical aesthetic. This more inclusive viewpoint is encapsulated in the following quote from Mary Beth Ruskai:

> We cannot hope that many children will learn mathematics unless we find a way to share our enjoyment and show them its beauty as well as its utility. (Ruskai 1995)

Natalie Sinclair is possibly the most prolific contemporary author on matters associated with aesthetics in mathematics education. In Sinclair (2004) she makes a strong case for the vital role that aesthetic processes play in the development of mathematical knowledge and in the course of mathematical enquiry (in addition to any accompanying cognitive processes that may be taking place). This is irrespective of whether the learning is taking place in a classroom, a lecture theatre or in the rarefied confines of a research mathematician's office.

On the basis of the theoretical work carried out in Dewey (1934), Sinclair identifies three fundamental roles that are played by the aesthetic:

- motivational;
- evaluative;
- generative.

Let us consider each of these in turn. It is important first to emphasise the fact that the motivational aspect of the aesthetic is intrinsic rather extrinsic. That is to say, the reward for pursuing some mathematical activity is the inherent pleasure and personal satisfaction one derives from it rather than any type of material gain one might receive. This role may be considered at least partly responsible for situations in which a person is attracted to a certain mathematical problem or area. The evaluative role is concerned with making judgements about the beauty of the mathematics one is engaged in. For example: Is a particular proof more elegant than another? Is one result deeper than another? Which of the possible lines of enquiry looks the most promising? The answers to these questions will frequently be influenced by aesthetic responses. Finally, the generative role may in some sense be linked with intuitive modes of thought. The aesthetic would appear in this case to operate at a subconscious level, providing tacit guidance for the mathematician. It might be partly responsible for leading to new ideas that would not necessarily have arisen easily from deductive reasoning alone, thereby facilitating generative learning. For more in-depth discussions concerning generative learning see Wittrock (1974a and 1974b).

From the above it would appear that the mathematical aesthetic has the potential to play a significant role in mathematical development at many levels, and that it is not therefore the preserve of the mathematical elite. This might in turn imply that there are tangible benefits to be gained through planning for the presence of aesthetic elements to lessons. Indeed, a number of educators concur with this sentiment. For further research and commentary regarding the aesthetic in the mathematics classroom, see Betts (2005), Gadanidis and Hoogland (2003), Mack (2006); Papert (1978). Furthermore, Sinclair (2009) argues that aesthetic awareness ought to be both a connective and a liberating force in mathematics education.

Aesthetics in the mathematics classroom

By way of an initial exploration in this area, a number of lesson observations took place (each with a different teacher). At this early stage we simply report on what might be seen as 'the current state of play'. We note that in most cases the observed lessons contained several good features, and some would in many respects have been regarded as good to outstanding. It was the case, however, that there was not one single explicit reference to the beauty inherent in the mathematical topics being taught, and in many of the activities provided there seemed to be virtually no scope to facilitate anything even implicitly in this regard.

Instead, the dominant discourse, in terms of motivation at least, concerned forthcoming examinations, despite the fact that in some cases these were many months away. Phrases such as "The examiner will be looking for …" or "We are doing this because you will get asked about it in your GCSE" were frequently to be heard. In fact, we recorded 28 comments of this nature (remembering, from above, that there were none at all associated with aesthetic considerations). This would appear to indicate that the current climate of test-score-driven schooling is very much driving the way that the curriculum is being delivered. It is a question of deciding whether or not this is an entirely good thing.

In addition to the fact that there were no real opportunities for students to experience or explore mathematics in a way that would allow their aesthetic sensibilities to develop and even flourish, there appeared to be a lack of vocabulary associated with the aesthetic. Although plenty of good learning certainly did take place in many of the lessons that were observed, the atmosphere and learning environment often seemed somewhat flat in the sense that there was a complete absence of any "Aha!" moments amongst the students.

Given that aesthetic considerations do indeed have a part to play in the teaching and learning mathematics at school, then some might be a little concerned that, from our limited observations at least, test-score-driven schooling is dominating classroom discourse to such an extent that motives for teaching and learning are purely utilitarian as opposed to aesthetic. There are thus some stark questions to be asked in this regard: Is there the will amongst stakeholders to remedy this situation, or have we now gone beyond the point of no return? Is it now the case that only the mathematical elite will have the opportunity to appreciate the aesthetic aspects of mathematics in the classroom? Are we, in schools, simply disregarding aesthetic values in mathematics? If so, is this process self perpetuating? Will this affect the mathematical creativity and engagement of future generations to the extent that most students become 'mathematical parrots', capable only of regurgitating the work of others as opposed to being creative themselves?

Concluding remarks

In this paper we are merely highlighting some findings and expressing possible concerns over what might be perceived as just one of many potential issues in contemporary mathematics education. At this preliminary stage we are certainly not claiming to offer any solutions, although several suggestions will be mooted here; there is indeed plenty of scope for future research in this area. As mathematicians and teachers, we occupy a privileged position in the sense that each of us is likely to be endowed with a heightened awareness of just what it means for a piece of mathematics to possess 'beauty'. This raises yet another question: Are students benefitting from this in their learning? Surely, in our roles as teachers and educators, it is our duty to pass on the sheer delight our subject gives us at so many levels.

In order to nurture students' aesthetic sensibilities with respect to mathematics, the learning environment must be conducive both to facilitating the requisite intellectual involvement and to encouraging the independent and creative exploration of mathematics. It does seem that the current climate of test-score-driven schooling is in fact inhibiting teachers' natural tendencies toward the aesthetic, thereby making the establishment of such learning environments very rare events indeed. Since this phenomenon would appear to be here to stay, some thought may be needed as to how to counteract some of its potentially negative effects.

First, this issue might be made explicit on PGCE programmes. Some educators believe that teachers should plan for aesthetic activity to take place in any mathematics lesson. There is of course the potential here for a dichotomy between the educators' ideals and student teachers' realities, and some open discussion in this regard would be helpful in preparing trainees for the dilemmas and different pressures they will encounter when attempting to cater for aesthetic dimensions in their lessons. Indeed, specific guidance may be needed in this area.

Second, are the curriculum and the accompanying resources (textbooks, for example) also conspiring against the aesthetic? To deflect the focus from this constant test-score-driven approach to teaching, it would seem important for teachers to select tasks that give students the opportunity to have aesthetics experiences. There are in fact a number of examples of good practice in this area. In Griffiths (2010) is an activity that was designed to allow students to experience the mathematical aesthetic in several different ways, and at several different levels. The starting point is something as simple as the logo of the Fibonacci Association. In Unal (2008 and 2009) may be found examples relating diagrams to sums of infinite series in surprising and beautiful ways. An example of a beautifully simple visual model describing the way the universe is expanding in given in Griffiths (2009). In Lesmoir-Gordon (2010) can be seen some wonderful exposition and accompanying graphics associated with fractals. For further specific examples aimed at younger students, see Gadanidis and Hoogland (2003) and Sinclair (2002 and 2006).

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