Topologic and topographic features of parameters of functions and meaning transitions within a microworld–microidentity interaction

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Two year 10 English students, one boy and one girl, worked together in a task developed using a GeoGebra software (Dynamic Geometrical System) to promote the development of ideas approaching to the notion of function. During the task, students' voices and the computer screen were recorded to assess how this task promotes the developmental process of what Mason call mathematical being through a perceptual guided activity regarding to parameters notion.

In accordance with Mason, who thinks that teacher can not do the learning for their learners, we assume that learning come about from the activities students develop in a learning episode. Here we will discus the capabilities of the micro-world to direct students' attention to some general topographic and topological features of the geometric enactment of algebraic expressions; and the meaning transitions of the micro-world elements experimented by students regarding to the specific manipulation type (dragging points, sliders, or typing).

The micro-world

Armando Landa Hernández developed the task through GeoGebra, a computer program that is defined as a Dynamic Geometrical System. The task includes a compendium of functions such as $\mathbf{f(x)= a\varphi(x-b)^n+c}$, and it is designed to presents graphical representations of specific randomised functions. This micro-world and others may be freely downloaded via the Parameters of Functions link from the GeoGebra site: http://www.geogebra.org/en/wiki/index.php/Parameters_of_Functions

Two graphics are presented as the main objects. One of the graphics (coloured in green) is taken as the graphic target, the second one (coloured in blue), is a graphic that can be manipulated by students' direct actions in three different ways: 1) dragging points, 2) through slider manipulation, and 3) typing directly sliders' values (see figure 1). The student task was to match the blue graph with the green one. It has also been included into the design of the micro-world the label 'go it', which appears when blue and green graph match. The main activity during the task was the students' direct manipulation of variation in the corresponded graphic representation of every parameter of the algebraic expression. At the end the students were asked to identify and write down the algebraic expression regarded to the graph.

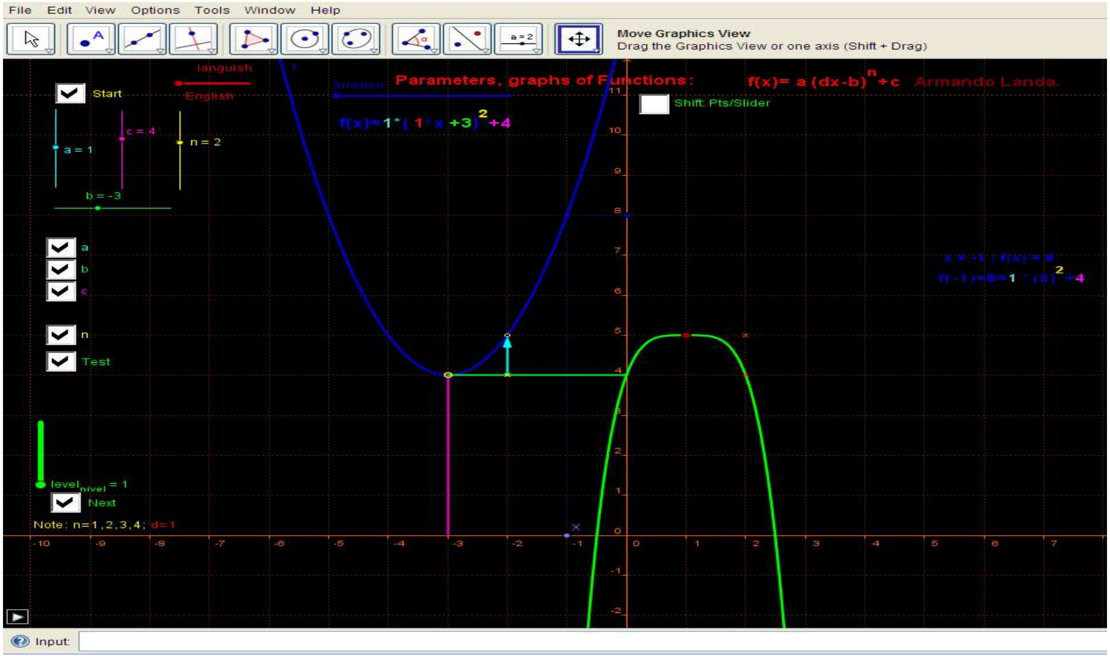

Figure 1. The microworld in 'crane' mode

We use the term "micro-world" as we consider this activity in the way Varela (1999) does, interpreting students' interaction with the task as the couple microworld-microidentity. The assumption that the action is perceptually guided is the central idea supporting the micro-world design.

As we agree with Stewart (2011) when saying, "that "learning" can only be a modification of the developmental process; this means that what can be "learned is both enabled and constrained by the epigenetic landscape", we consider the complexity of the micro-world a feature to be highlighted. The micro-world includes a compendium of functions distributed in 6 classes and five levels, with two regarding sliders to change between them. All of the elements interact directly between each other. Students' activity in the micro-world can be considered a *visual and kinesthetic experience* (Nuñez 330) about the variation of the parameters on graphs. Every parameter (a, b, c, and n) has visual elements (colours, lines, sliders, numbers, points) that allow the students' identification of interaction between screen elements.

Parameter sliders, points, lines and colors are visual stimulus where students' perceptions and actions are directed. Different positions and colors of sliders allow students to differentiate its effects on blue graph in a way that perception of properties of blue graph came about from dragging sliders, points or typing numbers and its correspondent effect on graph.

By clicking on the check box 'Start' a set of different objects appears on the screen of the computer: a blue graph, sliders in different colours and check boxes among others. By clicking on the check box 'Test' a green graph appears and also a check box labelled 'Next'. In this paper we make reference only to functions $f(x)=a(x-b)^n+c$ which correspond to the '$level_{nive}=1$' one of the 'Test' part of the microworld.

As it has been said, we have assigned different colours to the labels of check boxes in correspondence to the colours of the sliders and objects related to them (segments, vectors, parameters in algebraic expression related to the blue graph).

Written instructions (which disappear when check box 'Test' is clicked on) are: 'Match blue graph with green graph by typing the values for a, b, c, n. into the window Input (ex: a=3 <Enter>) or by dragging sliders a, b, c, n.'

By clicking on the check box ‘Shift: Pts/Slider’ another blue graph appears and sliders disappear (except slider n). The correspondent written instructions are ‘Match blue graph with Test graph by dragging points V, A and slider n’. (Figure. 2)

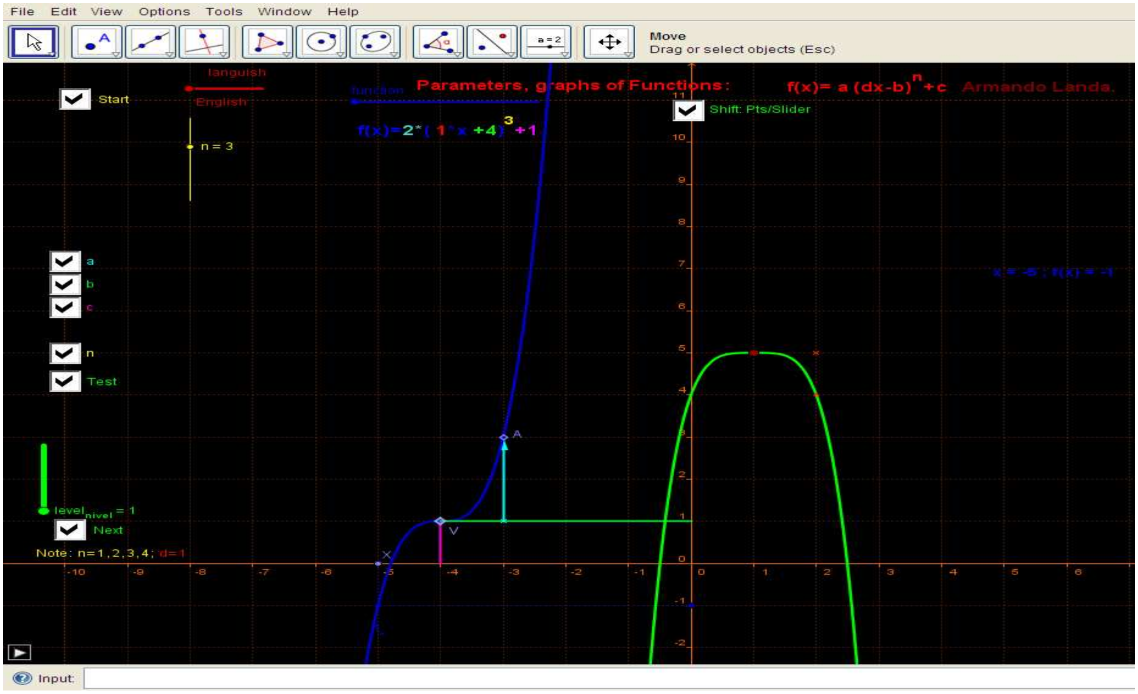

Fig. 2. Micro-world in ‘superman’ mode

The students task

Screen elements, roles and meaning

In order to analyze the learning episode with the two students, we consider the assertion of Spencer-Brown (1969) that mathematics can be derived from the activity of drawing distinctions. As our claim in this paper is that one approach to the notion of function from a geometric perspective may help to make sense of the parameters significance within the algebraic expression and their impact in the graphic enactment, for this paper, we take into account five objects types and their respective identification of topology and topography properties in the task: (1) general and specific algebraic expressions; (2) graphs; (3) modes of manipulation (sliders, points and typing); (4) ‘x’ and ‘y’ axes; and (5) *got it* message

The first activity taken into account in the micro-world is fitting the correspondent parameters identified in theirs graphical representations, in order to match both graphs each other. It is important to realize that the objects distinctions in the micro-world are made from an interdependence perspective that students may or may not draw in the same sense. This means that, as drawing a distinction is an act of absolute freedom, it could be grounded in anything else. Thus, identifying elements in screen is likely linked to perception of properties (Mason 2008) of algebraic expression regarding its geometrical presentation. Having in mind this considerations, lets explores what happened during the beginning of the episode:

Students work with the micro-world around one hour and twenty minutes. Initial instructions given to the students by the teacher were ‘…just play a few minutes with it…’ No names were given to the students about objects on the micro world (sliders, check boxes) or indications how to use those objects or what was the task to be accomplished or the aim to 'play with it'. Moreover the micro-world was given to the students with the software GeoGebra not customized. All tools, commands and properties of the software were left. So one of the remarkable things happened was the fact that some objects were identified and used in the way we expected. Just some seconds had passed after they sat in front of the computer and the girl press the start check box; They began to read instructions, press check box for parameter 'a', familiarity with computer use was evident when he tried to drag the point at the end of the vector –which we name V-. Nevertheless, they could not drag the point, because it was in the sliders mode, this evidence that they did not know what a slider was, because even when the instructions said drag sliders they do tried to drag but the points. After a couple of tries they read again the instructions (53 seconds had passed). She tries to drag point V again, he realize the objects we call sliders and ask her to try one, in trying to do it, she moved the mouse in a way that screen zoom in, then zoom out to the original position and then drag slider; as they noticed the simultaneous movement of the graph he says 'here we go'. Two minutes and forty seconds since they began the task they read again the instructions and press the 'test' check box, a green graph appears. Then they started to manipulate the sliders and took them about 2 minutes to match the graphs by the first time, they realised the 'got it' label appearing when blue graph match the green one. Immediately, they press 'next' check box. During the next 12 minutes they match 14 green graphs.

In the other hand, every element on screen is taken as a possible signifier (Jay 2011) but each one of them related with the others in a different dependence interaction determined for the task. And it is the task, which allow directing attention to different properties when they are relevant for the task. This mean, that distinguishing a slider from a graph in a mathematical sense includes at the same time taken them as different elements but interdependent (E.g. the blue graph is manipulated by means of the sliders). So, after twenty minutes of free activity, students were asked, this time in an explicit verbal way, to match graphs by using the three different modes of manipulate parameter values. This instruction were told using metaphors: for points direct dragging, they were asked to imaging superman carrying the blue graph directly to match the green one; for sliders, teacher asked them to imaging that without superman they had to learn to use cranes; and for typing, they were asked to imaging being programmers and that they need to identify the values and introduce them to change the graph shape to match the graphs. During the next thirty minutes they use, in an alternate way, the 3 modes for varying parameters values, they got 19 matches.

We highlight the fact that in the first moment the cognitive process is guided by the perceptual identification of the reference in the Cartesian plane. Thus, we thing the'X' and 'Y' axes represent the main topological reference, and plays an important roll in the developmental process. As position of graphs is defined by parameters b and c, which taken as the point with position (b, c) (topology), denoting point V (see, figure 2), and parameter 'a' and 'n' determines the shape (topography), we found that perceiving the topological properties of parameters 'b' and 'c' and the topography regarding to 'n' parameter occurred sooner during the manipulation of parameters than perceiving the topographic properties regarding their values and its correspondent effects on the graph of parameters 'a'.

Points cannot do the graphic enactment of 'n'; because of every change in it represent a new shape. That is way there is only two modes for variant the 'n' value. Perceiving 'n' properties depends on identify the shape variation with its correspondent value. But the position of the arrow related to slider 'a' (and its initial and final point) is updated according to the values for 'b', 'c', for the position, and 'n' value, for shape, then its topological properties come about in a latter moment during manipulation. By dragging slider 'a' the shape of blue graph is modified in different way than by dragging slider 'n'. Of the 19 graphs that were matched, five matched by dragging points, 8 by dragging sliders, and 6 by typing directly the parameter values. It is remarkable that in all the trials they start by 'b' and 'c' (defining the position) or by 'n' (defining the shape), but never by defining 'a'.

Another interesting aspect of the way the task direct the attention of students was when in a third stage, they were asked to find the parameters values when the blue graph were hidden; they had to introduce the values of parameters just identifying them in the green graph, without the guide reference of the blue graph. The students by then had got a notion of the coordinates' system used, we think that this is evident when in the first trail, without the blue graph reference, they got match in a single try for each parameter. But the second trail show that the parameter 'a' was not been understood. They started to look for understand what was 'a', he aloud said "but what is 'a'? And then they show the first reference to the blue algebraic expression, the girl point the light blue number in the expression and said: "This is 'a'", the boy answered "yea, I know, but how is that related to the graph? Till this point they had related many objects in the micro-world, even the graph; but the 'a' parameter remained being an incognita. And it is now, when there is some thing that was passed unobserved, when the need took them to reconsider and observe all those properties they had not observed and more. But it took them more than one trail to get it. He show evidence of being referring the 'a' value to the X axe when he said: "I am sure is minus five" pointing the minus five in the axe with the correspondent point for 'a' parameter in the green graph. They were told to use the blue graph reference again but at the end they decide to give up on this graph, and change to the next one. In the third trail the green graph 'a' parameter coincide, as in the first trail, with the distance to the X axe again, they again did not know what 'a' was. This time the first reference they took into consideration it seemed to be were the graph cross the Y axe, because the firs try is 3 and here is where the graph cross Y axe. He showed being disappointed when probing writing 3 and having not got the matching, what happened after show us how he had the idea that 'a' was referred to the axes: She suggested and wrote minus two –the distance to the X axe-, but he said "no, no because all you do with it is move it along that, vertically". Again they were told they might use the blue graph reference and sliders, this time they did, when examined the graphs relation. He seems to return to the axes reference idea because of the relation to the specific position of the graph. The third and four trail contributed to maintain this idea for the relative position of the graph, this time again, coincided with the X axe but now the V point over the X axe in both case. They got matched without more problems, till now they, without the blue graph assistance, had matched 4 graphs, recurred to the blue for a short glance in one, and give up one. The next graph not coincides any more with any of the axes. Now as they could not find 'a' value again they decided to use the blue reference and realize what is 'a', when he done it he said: "Now I see, that's way 'a' goes down from, it's from that line to where it crosses, and how many down is, so 'a' is minus three"

Conclusions

We conclude that manipulating the variation of parameters values attached to immediate perceptual feedback of their consequences in the graphical representation, allowed students to direct their attention to parameters defining the geometrical characteristics of the function of graphs of the type $\mathbf{f(x)= a\varphi(x\text{-}b)^n+c.}$ As data shows the micro-world arise as a powerful mean to promote the perception of topographical and topological properties of the parameters; also showed us how the kind of manipulation direct attention to different properties of the parameters.

By dragging point V to the position of the red dot allowed students to identify the role of parameters in the geometrical representation; with sliders, sliders ‘b’ and ‘c’ separate the movements of blue graph on the screen in two differentiate movements and allowed students to differentiate between parameters. But in both cases, dragging points and sliders, matching the position of the blue graph on the position of the green graph requires only the need of being aware of some visual properties, but it is not required to be aware of its coordinates so no need to pay attention about its numerical values. Typing directs their attention to the numerical properties; and when the blue graph is hidden properties became significant in a different mathematical sense.

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