A representational approach to developing primary ITT students' confidence in their mathematics

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Representations of mathematical concepts play an important role in the understanding of learners (Greeno and Hall 1997), and also in the pedagogical processes involved in developing that understanding (Leinhardt et al. 1991; Brophy 1991). In this paper, we report on work with a cohort of pre-service primary teachers, with the aim of developing their understanding of mathematics and their confidence in their subject knowledge and their teaching of mathematics. This was attempted through the introduction and use of representations associated with mathematical concepts covered in primary schools. We present the results of attitude measures and qualitative questionnaire comments in identifying whether and how the use of representations supported pre-service teachers' confidence in teaching mathematics.

Introduction

Shulman (1986) identified representations as being part of teachers' pedagogical knowledge. Specifically in mathematics, Ball et al. (2008) also highlighted representations as being part of the 'specialised content knowledge' of mathematics unique to teaching. In particular, researchers have highlighted the role that representations play in the explanations of mathematical concepts by teachers (Leinhardt et al. 1991; Brophy 1991). Leinhardt et al. (1991) also identified the skill and knowledge required by teachers in considering the suitability of particular representations, as "certain representations will take an instructor further in his or her attempts to explain the to-be-learned material and still remain consistent and useful" (Leinhardt et al. 1991, 108).

Representations also play an important role in the learning of mathematics by students: "An important educational goal is for students to learn to use multiple forms of representation in communicating with one another." (Greeno and Hall 1997, 363) More broadly, multiple representations play an important role in the development of learners' mathematical understanding: "They can be considered as useful tools for constructing understanding and for communicating information and understanding." (Greeno and Hall 1997, 362) In considering the role of representations within understanding, we make the distinction between internal and external manifestations of representations (Pape and Tchoshanov 2001), or 'mental structures' and 'notation systems' respectively as referred to by Kaput (1991). Understanding of a mathematical concept is based on the internal representations of a concept, which are influenced by the external representations of the concept that are presented to learners (Hiebert and Wearne 1992). Researchers (Hiebert and Carpenter 1992; Barmby et al. 2009) have defined mathematical understanding as being a network of internal representations, with more and stronger connections denoting greater understanding.

Representations therefore play an important role in the understanding of learners, and also in the pedagogical processes involved in developing that understanding. For pre-service teachers, who are developing their own understanding and learning how to teach the subject of mathematics, their knowledge of mathematical representations is even more important. However, Turner (2007) highlighted that pre-service teachers' choice and use of representations could be problematic. In this paper, we report on work with a cohort of pre-service primary teachers, with the aim of developing their understanding of mathematics and their confidence in their subject knowledge and their teaching of mathematics. This was attempted through the introduction and use of representations associated with mathematical concepts covered in primary schools. We provide further detail on this input to pre-service teachers in the section below.

Methodology

The sample of pre-service teachers involved in this work was a cohort of 77 students on a 38-week long postgraduate teaching course (PGCE). The programme offered in mathematics is well established and helps the students to explore both pedagogy and content within the primary mathematics curriculum through lectures, seminars and workshops involving leading mathematics teachers from the local authority. However, in 2009/10, the input for students was reorganised with most of the sessions focussing on a 'representational paradigm', based on the research ideas outlined above. In sessions, a variety of representations for a mathematical concept would be introduced to the student teachers. Students would be encouraged to 'explore' what characteristics of a mathematical concept were emphasised by a particular representation; for example, considering the possibility of there being key representations which were more useful for explaining and understanding key ideas, and considering how the representations could be used to make sense of the various procedures (or algorithms) associated with the mathematical concept. In addition, as a medium for exploring ideas on representation, we used a suite of computer programmes that we had devised ourselves and which allowed the representations to be explored in a dynamic and interactive way. The programmes were created as a stimulus and as a scaffold for class discussion.

Alongside the input provided to the pre-service primary teachers, the aim of the study was to examine the impact of the input of this representational approach on the student teachers involved. More specifically, the objectives of the study were to (a) measure the change in pre-service teachers' attitudes towards their subject knowledge in mathematics, and also towards teaching the subject; (b) gain some qualitative insight into why the input incorporating representations might impact on teacher attitudes. In justifying the first objective, past research has highlighted that there is a link between teachers' beliefs/attitudes and instructional practice, although this link can be complex (Thompson 1984). In terms of actually examining teachers' attitudes towards the subject, Ernest (1989) identified the two components of teachers' attitudes towards mathematics and towards teaching mathematics. Relich et al. (1994) similarly identified two dimensions of pre-service teachers' attitudes. The attitudes of the pre-service teachers towards studying mathematics and their attitudes towards teaching mathematics were measured at the start and end of their 38-week course of study using a questionnaire. We developed measures for attitudes towards studying mathematics and towards teaching mathematics. The two attitude measures consisted of 8 and 7 questionnaire items respectively, with responses to the items

elicited on a 5-point Likert Scale of Strongly agree through to Strongly disagree. The Cronbach alpha reliabilities were 0.91 and 0.89 for the attitudes towards studying and teaching mathematics measures respectively, with the unidimensionality of these measures examined through exploratory factor analysis. Using these measures, pre-service teachers in this study were surveyed at the start and end of their course. In addition to surveying this particular cohort of pre-service teachers that received the input on representations, a second cohort of students on the first year of a three-year undergraduate teaching programme was also surveyed to provide a comparison group for the attitude measures. These students were receiving input with regards to both pedagogy and content within the primary mathematics curriculum, but without a specific emphasis on representations. In total, 65 students in the intervention group and 69 students in the comparison group completed the pre- and post-measures of attitude.

In addition to examining the impact of the input on teachers' attitudes, the pre-service teachers receiving the 'representational input' were asked at the end of the course to reflect on their own learning as part of the course. To do so, they were asked to provide written answers to open-ended questions on a questionnaire. These questions asked students for areas of maths where they had deepened their understanding, and also incidents/events which had helped them learn.

Results

The results are discussed in two parts. First we explore the quantitative data from the attitude measures which show the impact of the teaching input on the attitudes of the pre-service teachers. Figure 1 shows the change in the average measures of attitudes for the input group and the comparison group over the course of the year.

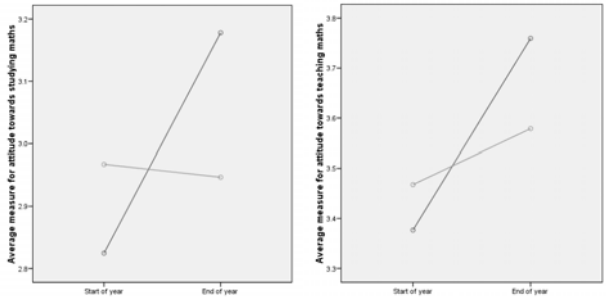

Figure 1: Change in the average attitudes towards studying maths (left) and teaching maths (right).

The average attitudes for the input group showed a greater increase over the year than for the control group for both measures (confidence in studying maths and confidence in teaching maths). Analysing the data using repeated measures ANOVA showed that the change in the attitude measures for the input group as compared to the control group was statistically significant ($p < 0.05$) for both measures. The effect size for the changes for the input group were 0.47 for confidence in studying maths and 0.64 for confidence in teaching maths (compared to -0.03 and 0.19 respectively for the control group).

In addition to the data from the attitude measures, student teachers' responses to the more open-ended questions were examined. The responses to the questions 'State a topic or concept in maths in which you feel you have deepened or modified your understanding' and 'Describe an incident or event which helped you learn' were analysed. The frequencies of responses for different topics identified, and also for different incidents/events mentioned are shown in Figure 2.

A variety of topics were highlighted by students, however two in particular were mentioned by about 30% of the students: multiplication and division, and shape. In terms of incidents and events, the most common category of response was associated with the value of discussion, mentioned by 49 (71%) of the sample. Given below are direct comments taken from individual responses.

> Discussion with university tutors and the class teacher helped to develop both my subject knowledge and knowledge of how to teach it in an engaging way.
>
> Talking to others to develop my understanding. For example speaking to (lecturers) in individual (or small group) sessions and talking to peers to share and compare different strategies for tackling mental maths problems.

Second in importance to the discussion, 43 (62%) of the students referred to the value of visual representations in helping with their learning in mathematics.

> Visual representations helped me understand multiplication and fractions.
>
> Visual representations accompanied by explanations and talking to others helped develop my understanding.

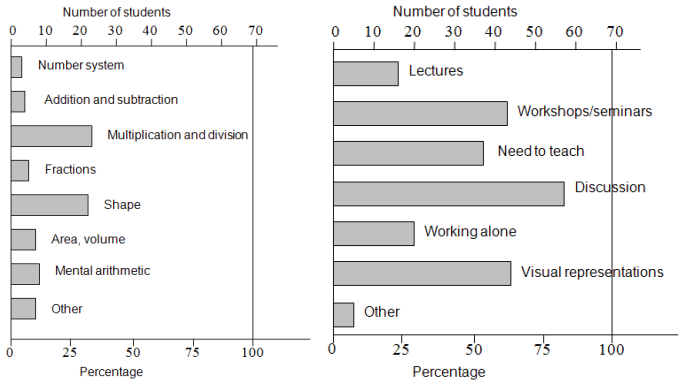

Figure 2: Topics identified (left) and incidents/events mentioned by student teachers (right).

Discussion

In this study, the 'representational approach' that we used with the particular group of pre-service primary teachers had a possibly significant impact on their attitudes towards studying and teaching mathematics. We identified the important roles that discussion and visual representations played in developing these attitudes to mathematics. In order to explain why these elements might be important for pre-service teachers, we first examine the issue of developing pre-service teachers' understanding of mathematics.

Ball (1990, 458) emphasised the importance of understanding the subject for teachers: "Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways". We can conceptualise 'understanding' in terms of connections made between (internalised) representations of mathematical concepts through reasoning processes (Hiebert and Carpenter 1992; Barmby et al. 2009). Therefore, developing the range of representations (in the case of this study, visual representations) that pre-service teachers have available to them will develop their understanding of a mathematical concept. However, increasing the range of representations for teachers is not enough in itself – teachers also need to develop the connections that they have between representations, for example the connection between visual representations and symbolic representations or algorithms.

It is with regards to this development of 'connections' between representations that discussion can play a role. Drawing on Hoyles' (1985) work on discussion and learning mathematics, she highlighted three aspects to discussion: articulating ideas brings about reflection on those ideas; discussion involves framing ideas in a way that will be accepted by others; listening to others modifies your own thoughts. Interpreting these ideas in terms of our view of understanding, in discussing our mathematical ideas, we modify the representations that we have and the connections that we have made, both through our own reflection and as a result of articulating our understanding, and also through comparisons with other people's understanding. Therefore, we see the importance of representations for understanding, and the process of discussion in developing that understanding, as going hand-in-hand.

For pre-service teachers however, representations of mathematical concepts have an additional importance. As highlighted in the introduction, representations are important for the explanation of mathematical concepts in the classroom as well (Leinhardt et al. 1991; Brophy 1991). In terms of our view of understanding, we are developing pupil understanding through the introduction of representations from which they can reason to symbolic or procedural representations. Therefore, representations have the dual role as tools for developing teachers' own understandings, and also tools for explanation in developing pupils' understanding. It is for these reasons based on the research, and the qualitative comments made by the pre-service teachers, that we see why visual representations and a discussion-based approach might develop pre-service teachers' attitudes towards studying and teaching mathematics.

We have argued that a 'representational approach' with pre-service teachers may be affective for developing their attitudes towards the subject of mathematics. However, we need to also introduce a note of caution in our discussion. Firstly, we draw further on Ball's (1990, 458) ideas: "(Teachers) need to understand the subject flexibly enough so that they can interpret and appraise students' ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions". This quote states that teachers need to build on pupils' existing understanding, and the implication of this is that teachers also need to be aware of suitable representations to introduce to pupils at their level of understanding. As stated by Cobb et al. (1992, 2), "meanings given to these representations are the product of students' interpretive activity." Therefore, a further development in pre-service teachers' use of representations would be this awareness of representations for different pupils (e.g. different ages) (Barmby et al. 2009). Furthermore, as instructors on university courses, we in turn need to be aware of different levels of understanding for different student teachers and in different areas of mathematics. It is noticeable in this study that the impact on pre-service teachers' knowledge was perceived to be less in fractions (a common area of difficulty for pre-service teachers, Ball 1990; Tirosh 2000). Therefore, further work is required in using representations to develop pre-service teachers' knowledge and confidence in particular areas of mathematics.

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