Level 3 mathematics: a model for the curriculum

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Abstract: A model for a reformed Level 3 mathematics curriculum is derived, assuming its prime purpose is to provide the mathematical foundations for several types of Level 4 courses. Then issues and practicalities along the way are explicitly identified and proposals made for their resolution. Mechanisms for determining detailed content, based on earlier work, are also proposed. The model is structurally similar to current GCE Advanced Level, however routes preparing for the different types of Level 4 course are explicit and the model is intended to be more encouraging for students outside the mainstream and also more economical in its use of scarce teaching resources.

Keywords: Model, Level 3 Mathematics curriculum, Level 4 courses.

Introduction

(Smith 2004) suggests the mathematics 14-19 curriculum is unfit for purpose and proposes its reform should begin with provision of a mathematics double award at Level 2. Preparations for the introduction of Higher Tier and Lower Tier GCSE awards are now underway. Several proposals for reform at Level 3 have been published, generally involving multiple learning pathways aimed at improving take-up of mathematics at Level 3 which is lower in Britain than in comparable countries, (ACME 2010) being the most recent. The author's earlier work, on Post-16 mathematics and the requirements of university courses, suggests a statement of purpose in terms of these requirements and also suggests possible mechanisms for determining and reviewing curriculum content.

The model developed in this paper is a working out of the consequences for the curriculum of this statement of purpose, taking into account the range of university requirements differentiated according to broad subject groups. These consequences are intentionally constrained and focussed by the following considerations: providing opportunities for learners with different abilities and motivations to progress, through Level 3 mathematics, at different speeds and to different extents; providing for part-time learners and for those who have dropped out to readily return to mathematics learning; taking account of the need to make good use of scarce Level 3 teaching resources; while retaining the current A-level structure as far as possible.

Purposes and content

We begin by assuming that the primary purpose for Level 3 mathematics is to lay mathematical foundations for Level 4 courses. This purpose drives our proposals for a reformed curriculum. (Osmon 2009) identified a group of quantitative subjects where Mathematics A-level is a required qualification for entry to all courses across the full range of universities. These are all traditional STEM courses and were labelled Block I.. (Osmon 2010) labelled a further category of quantitative courses- in computer science, economics, business and management- as Block II. Except at the most prestigious universities students are admitted to Block II courses with just GCSE mathematics, but evidence was found for mathematics "top-up" provision in the first undergraduate year, very similar in level and content to the pure mathematics in GCE AS-level (except in the case of computer science where the top-up content was entirely discrete mathematics). Another group of subjects- including geography, biology, psychology, social and health sciences- might be described as semi-quantitative since they may all be presumed to need statistics, but perhaps not any other mathematics beyond GCSE. We give Level 4 courses in these subjects the label Block III. (Actually, the author is aware from direct personal experience, that courses in Blocks I and II also need their students to have knowledge of statistics at Level 3.) There is a further category- the most prestigious universities require students applying for mathematics particularly to take the more challenging Further Mathematics, as well as Mathematics at A-level. This is a valuable option to stretch the most able and mathematically ambitious ten per cent of students.

We can conclude that there is a requirement for four distinct levels of mathematical provision at Level 3 in order to meet the foundational needs of Level 4 courses. Detailed consideration of content is outside the scope of this paper. However, (Osmon 2010) proposes a mechanism, extrapolated from the top-up observation for Block II, for determining appropriate content by investigating the collective needs for pure mathematics across the various Level 4 subject groups.

Curriculum issues: applied mathematics and choice of content

Applied within A-level mathematics

We can distinguish two categories of applied mathematics: generic applied mathematics is needed across all quantitative and semi-quantitative Level 4 courses; subject-specific applied mathematics is needed in courses in only some subjects or groups of subjects at Level 4. Thus, for example, of the various applied mathematics modules offered by Edexcel, the largest of the GCE examination boards, Mechanics (needed only in physics and some engineering courses and perhaps mathematics) is evidently subject-specific applied whereas Statistics (needed at least in principle in all quantitative and semi-quantitative Level 4 courses) alone is generic-applied.

This categorization prompts a refined statement of purpose: the role of Level 3 mathematics is to provide mathematical foundations at this level that are needed for *all* Level 4 courses in quantitative and semi-quantitative subjects. This refinement excludes subject-specific-applied mathematics and the rationale is as follows.

There is a Level categorisation issue for applied mathematics, in relation to the pure mathematics on which it relies. Thus, if the applied mathematics relies only on Level 2 pure, should it be categorized as mathematically Level 3? Statistics provides an example. Statistics can be taught in a more or less mathematical way- in the former formulae, for example, are derived or at least presented as results valid only within specified conditions- relying on Level 3 pure mathematics to do so- while in the latter case they are merely introduced and used as rules of thumb for substitution of parameter values- a Level 2 activity. Mechanics provides other examples: deriving the equations of motion requires Level 3 mathematics knowledge, but solving a particle dynamics problem by substitution of values is at Level 2, although of course understanding the problem situation arguably requires *physics* knowledge at Level 3, just as understanding statistics problem situations may require contextual, ie particular subject, knowledge. This interpretation of what constitutes Level 3 applied mathematics would require rethinking of applied syllabuses.

Since there is limited space in the curriculum it is proposed that pure mathematics needed generically at Level 4 should take priority over applied. (At this point it is worth digressing to consider whether the allocation, to the mathematics curriculum, of responsibility for mathematics applications is actually reasonable. The author proposes that, as with the physics example, applications belong in subjects and courses where they occur. Similarly, the mathematical applications in vocational courses should perhaps be taught at the point of use. Perhaps also there is little or no requirement for Level 3 pure mathematics in Level 3 vocational courses, but this needs investigation.)

Choice of content within A-level mathematics

Freedom for individual learners to choose is surely good for their motivation. So is provision of choice a good thing in principle? And do students have the information to make informed choices? The issue of choice may be fogged because two quite different kinds of choice within the curriculum can be envisaged: progression-choice (how much mathematics), and specialization-choice (which branches of mathematics to study).

Progression-choice is not much of an issue if the curriculum structure allows learners to defer their decision about how far to travel until they have done a year of mathematics. Specialisation-choice however is more problematic. Its advocates would no doubt argue that it is desirable in preparation for particular subjects at Level 4. This would only be appropriate, even in principle, if learners were certain, at the time when they had to choose special topics, where they were heading post-16, and this will not be the case for many students. Besides which, some of those who are "certain" will, in the event, want to change direction. And, because of limited teaching resources, students may well not, in practice, be offered a range of specialization choices.

From our viewpoint, that the role of A-level mathematics is to lay the mathematical foundations for Level 4 courses, it is hard to make a case for specialization-choice within the curriculum (unless university admissions tutors specify course entry requirements at the granularity of particular modules- and it must be acknowledged that this does occur occasionally at present). HE courses benefit from their intakes having common foundational mathematics knowledge. The optimal situation both for universities and for post-16 students is to identify generic- across a range of courses- foundational mathematics and teach just this, as proposed here.

Practicalities

Access

Currently only about 10% of the national Level 2 cohort continue to study mathematics at Level 3 and, while this proportion is increasing, its low value is a major driver for curriculum reform. (Smith 2004) identified weaknesses in the Level 2 curriculum as an important factor and the recommended reform of GCSE with a double award in mathematics is ongoing. Mathematics has a reputation as elitist and difficult and this is unhelpful for widening access. (This is reinforced by requiring three modules rather than two for AS and A2 awards and evidently there is a strong case for mathematics to fall into line with other subjects. The author takes this as given in the development of the model.) Everything possible, without lowering standards, needs to be done to encourage both full-time and part-time students to embark on the study of Level 3 mathematics and support them on their journey. This has implications for progression routes and progression stages, and also provision to facilitate the re-entry of returners to Level 3.

Continuity

The disastrous fall in Post-16 mathematics numbers following the Curriculum 2000 reform is a timely reminder of the law of unintended consequences and so the curriculum reform proposed in this paper aims, so far as possible, to maximise structural continuity with current arrangements.

Teaching resources
(Smith 2004) also identified a national shortage of well qualified maths teachers as a constraint on student numbers and evidently reform should aim to make efficient use of this potentially scarce resource.

Proposed Level 3 Highway Model

A Highway metaphor for Level 3 progression in mathematics is proposed rather than the network of qualification pathways, characteristic of other models, some of which (for example FSMQs) are actually cul de sacs. The Model assumes the outcome of GCSE reform will be a mathematics double award. The Highway comprises three learning "lanes" as shown in the Figure. Progression is from left to right. Entry requires a Level 2 qualification: GCSE Lower Tier (LT), LT upgraded to HT via a Bridging Module (BM), or GCSE Higher Tier (HT). LT provides access only to the Highway's "slow lane" (bottom row in the Figure). HT and BM give access to all three lanes. (The metaphor is inaccurate because learners may travel on two of the lanes simultaneously!)

Five awards are envisaged at GCE Advanced Level: three at AS and two at A2, as follows, with each award recognising achievement in two units:

Statistics Applications: GCE AS-level (2 units)
Mathematics: AS (2 units of Pure Mathematics)
A2 (1 unit of Pure Mathematics and 1 unit of Mathematical Statistics)
Further Mathematics:
AS and A2 (each 2 units, Pure Mathematics assumed).

Continuing with the highway metaphor, the middle lane is expected to carry the main learner traffic- the Mathematics A-level course. It is envisaged that, as at present, the most able and ambitious mathematicians will also take Further Mathematics (the fast lane). The slow lane contains the mathematically least demanding course: Statistics Applications. It is envisaged that some students in the middle lane may take Mathematics only as far as AS, but will then take the Statistics Applications AS rather than Mathematics at A2. However, if they continue with A2 Mathematics this includes a Mathematical Statistics module covering the Level 3 mathematical content of statistics. By these means the foundational mathematics and statistics needs of HE courses in Blocks I, II and III are all covered.

A successful learner in the full-time education main stream will be able to complete any of the AS and A2 courses in one year as at present. But the Model can also meet the needs of those travelling more slowly, in part-time education, or whose progress has been interrupted for some reason, since exit from and re-entry to any lane is assumed to be possible between modules and after an AS. Such facility is important if the number qualifying in maths at Level 3 is to be maximised. (The model offers two options to students with LT, who complete the Statistics Applications AS and who want to do more Level 3 maths. These are shown as dashed lines in the Figure: either take BM or, if they are now confident enough with mathematics, take the Mathematics AS.)

The author has experience of modular schemes in higher education that meet the needs of a variety of learner types: full-time, part-time, and returners. Attention to details of the Highway's implementation not shown in the Figure could help part-timers and also stragglers to catch up and complete their course. These details include division of the teaching year into two halves and, so far as resources permit, teaching each unit in both halves and with three assessment points for each unit- at the end of each teaching term and at the end of the summer break.

The Model is potentially more economical of teaching and examining resources than present arrangements: ten GCE Advanced Level units instead of the eighteen currently in the Edexcel catalogue, for example. The proposals, above, for multiple teaching and assessment of units, to benefit learners outside the main stream, are potentially more expensive, but these are qualified by the remark “so far as resources permit”. So, a more accurate claim for the Model is that it facilitates multiple teaching and assessment provision on a cost-effective basis.

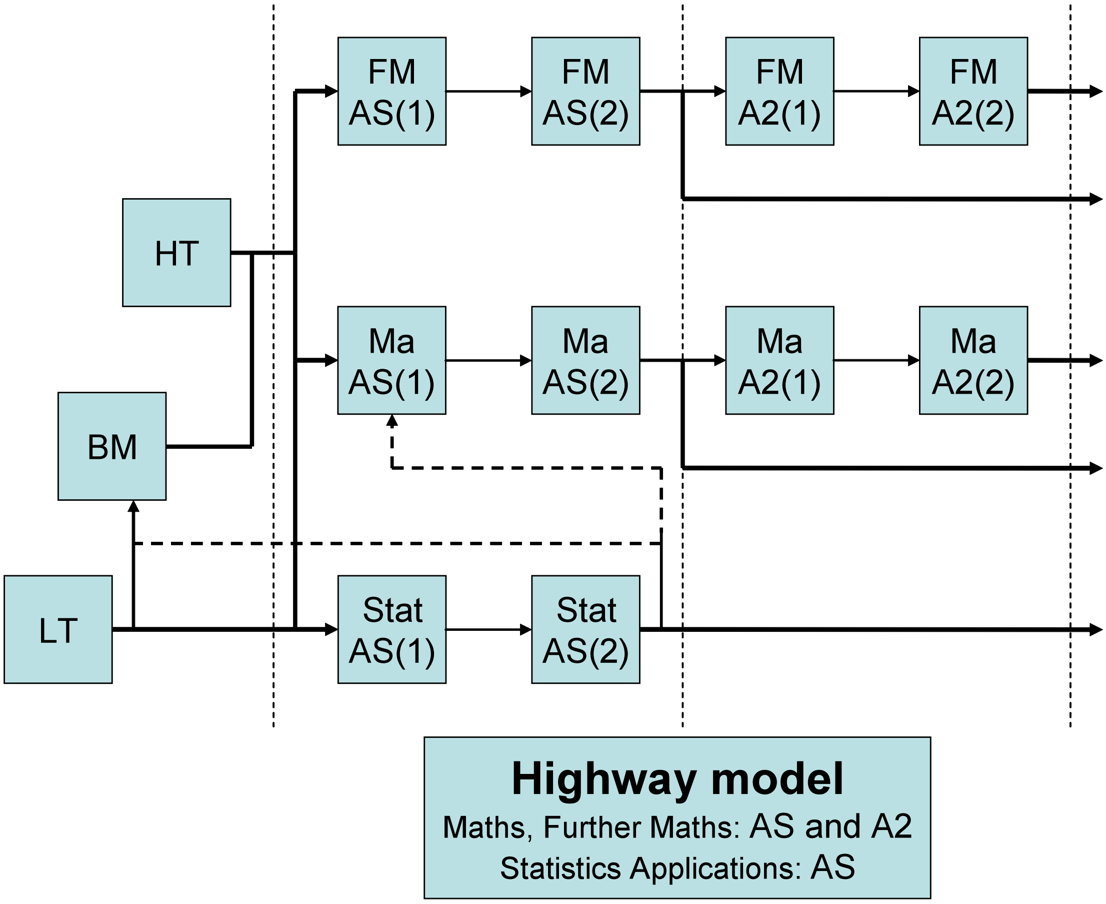
Summary and conclusions

A model for Level 3 mathematics provision has been developed starting from the premise that the primary purpose of Level 3 mathematics is to lay the mathematical foundations needed by academic and vocational courses. The requirements of various subjects at Level 4 have been identified and grouped accordingly into blocks with common requirements. The issue of applied mathematics provision within Level 3 mathematics has been explored, with the conclusion that statistics should have a place in the mathematics curriculum because there is a generic requirement whereas perhaps subject-specific applications of mathematics properly belong in the particular subjects where they arise. The related issue of choice relating to specialist content, as distinct from choice relating to speed and extent of progression, has been explored separately with the conclusion that it is unhelpful. And practicalities of access, continuity with established Level 3 structures, and use of scarce teaching resources have all been taken into account.

The model is more concerned with the structure of mathematics provision than content. Content is assumed to be determined by the collective needs of the groups of Level 4 courses. Teaching and assessment issues, beyond the desirability of continuing the current practice of unit assessment at A-level and having multiple assessment points in the year, to help maximise mathematics take-up and success have not been considered.

The model for Level 3 mathematics provision that has been developed from these premises and considerations arranges progression as a three lane highway. It offers a range of GCE Advanced Level courses to meet the Level 3 needs of learners across the range abilities while having a high degree of compatibility with present mainstream arrangements, and also offering flexible opportunities for part-time students, including those returning to maths learning as well as main-stream stragglers, all of whom need encouragement if the number qualifying in mathematics at Level 3 is to be maximised.

For these proposed curriculum reforms to be effective, Universities must play their part, by requiring that the students they admit have the appropriate minimal level of foundational mathematics: A-level Mathematics for Block I courses, AS-level for Block II, Statistics for Block III.



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