Engineering Students' Understanding Mathematics ESUM

Barbara Jaworski

Loughborough University

ESUM is a developmental research project concerned with innovation designed to improve the teaching of mathematics to first year materials engineering students in a UK university. The main aim of the project is *creating a culture to foster students' more conceptual engagement with mathematics.* Here I mean "more" in the sense of *more* than they have previously had in their earlier studies; *more* than just an instrumental understanding (Skemp 1976) and *more* than previous cohorts of students that I have taught in the past. An important question here is how such understanding can be seen and recognized and this is part of the study. In this short paper, I focus on organizational, theoretical and methodological aspects of the study.

Background and Organisation

Standard provision for Materials Engineering students (49 in the current cohort) in their first year mathematics module is two semesters each involving 2 lectures and 1 tutorial per week for 13/14 weeks; each tutorial is timetabled in a computer laboratory. The first semester curriculum is pre-calculus, including functions and equations (polynomial, rational, trigonometric, exp and log etc.), vectors, complex numbers and matrices. In their second semester they focus on the calculus.

The research described here is taking place in the first semester of their module. It involves implementation and study of an innovation which encompasses a more coherent, integrated use of the following:

- Inquiry-based questions
- GeoGebra representation of functions and equations
- Small group activity in tutorials
- A small group project

Here, the "more" implies "more" than in the last two years during which I have taught this module. During this time I have tried to introduce inquiry-based questions and GeoGebra, in order to promote a more conceptual approach to mathematics. In doing this, I have perceived a rather limited level of success (Jaworski 2010). It is therefore my intention here to integrate the four elements mentioned above and study their implementation. In addition, there is continued use of a Virtual Learning Environment (VLE) LEARN; a series of workbooks known as HELM (Helping Engineers Learn Mathematics) as baseline notes (reference); and exposition/explanation of topics using PowerPoint. Assessment includes Computer Aided Assessment (CAA) tests and a final examination after the second semester. Thus, the innovation involves the following resources:

In lectures (using powerpoint/OHP/GeoGebra)

- use of questions, closed and open (inquiry-based), to involve students
- encouragement of student response/participation
- use of GeoGebra for visualisation of concepts

In tutorials (taking place in a computer lab)

- small group structure
- use of inquiry-based questions and GeoGebra
- project work with inquiry-based tasks

In assessment

- CAA tests (2 rather than, previously, 4) and final examination
- Group project report and poster

Students are also expected to engage in self study, reviewing and working further on material introduced in lectures and tutorials, all available on the LEARN server, and working themselves from the HELM materials.

Theoretical Perspectives

Key concepts relating to ways of working and approaches used are inclusion, engagement, participation, interaction, collaboration, environment, and culture. I seek to *include* all students in activity, *engage* them with mathematics, encourage *participation*, *interaction* and *collaboration* within the educational *environment* of the university and to create a *culture* conducive to conceptual learning of mathematics. This speaks to a sociocultural basis for theorising activity and justifying approaches. It derives from Vygotskian principles that "Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them (Vygotsky 1978, 88), and from Vygotsky's proposal that learning takes place first in the social plane and only later in the individual mental plane.

In accordance with these sociocultural underpinnings I try to create a *community of practice* (Wenger 1998) and extend it to a *community of inquiry* (Jaworski 2006; Wells 1999) in which

- students engage with mathematics (both instrumentally and conceptually) with a focus on meaning and understanding
- "practice" means doing and understanding mathematics
- "inquiry" is intended to engage, raise awareness, draw students into a more conceptual frame.

Rogoff, Matusov and White suggest that "learning involves transformation of participation in collaborative endeavour". (1996, 388). This can be seen as a basic definition of a "learning community". Wenger (1998) talks particularly of a *community of practice* premised on three elements: *mutual engagement*, joint enterprise, shared repertoire. Our community of practice in the mathematics module can be seen to have mutual engagement in that we engage together in mathematical activity; joint enterprise in that we seek the mathematical understanding of the students who are involved; and shared repertoire in our lectures, tutorials, and use of resources. Wenger emphasises the process of *becoming* a member of the community in terms of a growing community identity and a subsequent *belonging* involving *engagement*, *imagination* and *alignment*. We engage with the practice of the community, use imagination in interpreting our own roles within the community and align with the norms and expectations of the community.

In earlier work, I have proposed a community of inquiry, building on the basic ideas of community of practice and incorporating inquiry ways of being and doing. Here inquiry means questioning and seeking solutions, wondering, imagining, inventing, exploring. I relate here particularly to the ideas of Wells (1999) who speaks of *dialogic inquiry*, leading to *meta-knowing* – knowing more about what we do as we engage in doing it. We might begin by using inquiry 'as a tool' and coming to encompass 'inquiry as a way of being' – taking on an *inquiry identity* (Jaworski 2006). Inquiry allows us to question the norms and expectations of the practices with which we align, a process of *critical alignment* (Jaworski 2006). We look critically at our practice, while we engage and align with it; ask questions about what we are doing and why; reveal and question implicit assumptions and expectations; and try out innovative approaches to explore alternative ways of doing and being to achieve our fundamental goals. Practice therefore involves two main elements:

- *Learning*: seen as *participation* in a social setting and consequent *reification* (Wenger 1998)
- *Teaching*: seen as creating the social setting through which the desired learning can develop. It includes developing a resource base through which knowledge and resources inter-relate to create the social setting.

The teaching-learning 'setting' has to work within the broader environment in which *hard* constraints (timetable, organisation, student numbers, lectures, tutorials, physical space, …) limit what is possible. Soft(er) constraints (practices, expectations, beliefs, behaviours, student and staff cultures, …) can possibly be re-framed (through a process of critical alignment) to develop alternative awarenesses in both learners and teachers of what is possible or desirable.

The project therefore brings together a range of elements:

- Resources: questions, tasks, GeoGebra files, small groups, group project;
- Knowledge/experience – forms of pedagogy: design of questions, inquiry-based tasks, group activity;
- Theory: participation, reification, inquiry community;
- Dealing with constraints, hard and soft;
- Community collaboration and growth of awareness.

These elements can be characterised through the theory of *Community Documentational Genesis* introduced by Gueudet and Trouche (e.g., 2009). Here a *document* derives from a set of *resources* together with *a scheme of utilisation*. *Genesis* means *becoming*: becoming a mathematics teacher; becoming a professional user of resources; becoming a knowledgeable professional. In his book *Communities of Practice*, Wenger talks of *learning* as "a process of becoming" (1998, 215). This, he claims, is "an experience of identity" (1998, 215), where identity "serves as a pivot between the social and the individual, so that each can be talked about in terms of the other" (1998, 145). Wenger suggests that "learning as participation … takes place through our engagement in actions and interactions" and "embeds this engagement in culture and history" (1998, 13). *Documentational genesis,* a term which captures the process of the mathematics teacher becoming a professional user of resources and, concomitantly, a knowledgeable professional, navigates the ground between the personhood of the teacher and the teacher's *belonging* (Wenger) to social structures and communities in which resources take meaning.

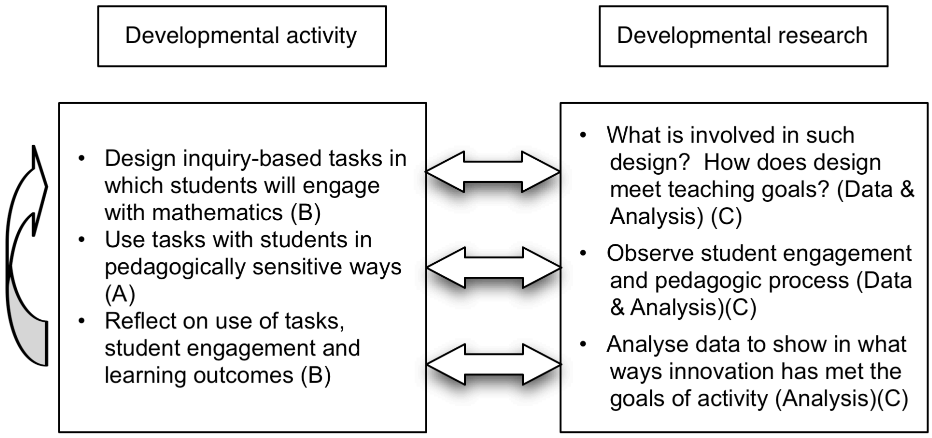
In a book devoted to the roles of resources in mathematics curriculum material and teacher development (Gueudet, Pepin and Trouche in press), Visnovska, Cobb and Dean express the idea of documentational genesis as follows:

> According to Gueudet and Trouche, teachers' documentation work includes looking for resources (e.g., instructional materials, tools, but also time for planning, colleagues with whom to discuss instructional issues, and workshops dedicated to specific themes) and making sense and use of them (e.g., planning instructional tasks and sequences, aligning instruction with the objectives and standards to which the teachers are held accountable). The products of this work at a given point in time are characterized as documents (e.g.,

records of the big mathematical ideas that are the overall goals of an instructional unit; a sequence of tasks along with a justification of their selection). These documents can in turn become resources in teachers' subsequent documentation work. The process of *documentational genesis* therefore foregrounds interactions of teachers and resources, and highlights how both are transformed in the course of these interactions.

Developmental research methodology

It seems therefore, that theory of community documentational genesis both fits well with activity in ESUM and offers a ways to make sense of the interplay between creating learning opportunities for students and the concomitant development of knowledge and understanding of the teacher in doing so. Research and development go hand in hand both to chart progress and stimulate knowledge in practice. Below we see, on the left, a focus on inquiry-based tasks, their use with students, and the teacher's reflection on their use – a cyclic process in which feedback from reflection leads to modification of the tasks to suit students learning. Research analyses the process in its different stages.



In this analysis of ESUM in relation to inquiry-based tasks, I recognize the three layers of inquiry designated in my previous research into teaching at school level (Jaworski, 2006):

A: Inquiry in learning mathematics:

Students engaging with inquiry-based tasks in mathematics to encourage conceptual engagement, learning and understanding

B. Inquiry in teaching mathematics:

Teachers using inquiry in the design and implementation of tasks, problems and mathematical activity with students, possibly in a project with other teachers

C. Inquiry in developing the teaching of mathematics:

Teachers, researching the processes of using inquiry in mathematics and in the teaching and learning of mathematics – possibly together with outsider researchers.

Thus within the project we see two developmental (learning) levels:

- *Student development:* Through participation in lectures and tutorials and group project work; inquiry in mathematics through questions and tasks, working together, being drawn into the community of doing and understanding mathematics …
- *Teaching/teacher development:* Through engagement with the didactic process: designing, using and developing resources; observing and responding to students; discussing with colleagues; analysing data …

The STEM funding pays for an outsider researcher to collect and analyse data alongside developments in teaching. The teacher is an insider researcher, being involved in practitioner research through designing teaching, reflecting on teaching activity and contributing to analysis of data. Data is collected from observations (and audio recording) of lectures and tutorials, documents produced, student surveys and interviews. A first level of analysis, involving analysis of student questionnaires and drawing on teacher reflections leads to modifications to the module as the module progresses. At the time of writing, the module is still in progress. A phase of qualitative analysis will follow completion of the module. This analysis will relate to theoretical perspectives to analysis of data and issues arising.

Brief insights from ongoing practice

It is too early as yet to report on findings. However, a central focus of innovative practice so far has been on questioning – on the kinds of questions that can be offered (and are offered) to seek to promote student engagement with and understanding of mathematics. A number of types of questions have been used – either as pre-designed tasks planned for use in a lecture or tutorial, or as the teacher's spontaneous questions to involve students and discern what students are making of a lecture. The latter are sometimes of a rather more direct or closed nature (focusing on immediate concepts) while those pre-designed take a more open or investigative form. For example, the following account is taken from the teacher's weekly reflection on the week's teaching, referring to one particular lecture:

> In the first example on Tuesday, I asked students to draw a triangle of given dimensions before going on to consider use of sine or cosine rules. In fact two triangles were possible for the given dimensions. This turned out to be a very good question, since different students wanted to approach it in different ways and we achieved a discussion across the lecture with students in different parts of the room arguing their approach. This seems worth analyzing to reveal the characteristics of a question which achieved this involvement (especially on a Tuesday when students seem more sluggish).

I have included the last remark, since it points to one aspect of the wider environment that has to be taken into account in analysis. We should analyse not only the questions but the wide range of factors that influence their impact.

Task 1 below shows three pre-designed questions. The first was offered in a lecture for students to tackle during the lecture; students were invited to talk with their neighbours to suggest possible lines. The other two questions were offered in the succeeding tutorial in which students were asked to work in groups of four, using GeoGebra to explore together. Analysis of observational data will look at the nature of the spontaneous questions and students' responses to them and at the kinds of activity generated by the more exploratory questions. Interviews are also planned to talk with students about their responses to such questions and perceptions of their associated learning/understanding.

The use of GeoGebra has elicited differing responses as observed in tutorials or revealed through responses to a questionnaire. Some students can be seen to explore and discuss as the design of questions envisaged. In other cases students input functions and produce lots of curves without evidence of real consideration of the mathematics behind their drawing.

Task 1

1a) Consider the function $f(x) = x^2 + 2x$ (x is real)
Give an equation of a line that intersects the graph of this function

- Twice
- Once
- Never (Adapted from Pilzer et al. 2003, 7)

1b) If we have the function $f(x) = ax^2 + bx + c$.
What can you say about lines which intersect this function twice?

1c) Write down equations for three straight lines and draw them in GeoGebra
Find a (quadratic) function such that the graph of the function cuts one of your lines *twice*, one of them *only once*, and the third *not at all* and show the result in GeoGebra.
Repeat for three *different* lines (what does it mean to be different?)

Returning briefly to theory; teacher and students here form a community within a complex setting in which teaching and learning are constituted. Questions and GeoGebra are just two of the many resources linking teacher and students, teaching and learning, and contributing to satisfying teacher and students' goals. The roles of the teacher in relation to such resources will be explored further using Documentation Genesis as an analytical tool.

References

Gueudet, G. and L. Trouche. In press. Communities, documents and professional geneses: interrelated stories. In *Mathematics Curriculum Material and Teacher Development: from text to 'lived' resources, ed*. G. Gueudet, B. Pepin and L. Trouche. N.Y.: Springer

Jaworski, B. 2010. Challenge and support in Undergraduate Mathematics for Engineers in a GeoGebra medium. *MSOR Connections* 10: 1.

———2006. Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education* 9: 187-211.

Pilzer, S., M. Robinson, D. Lomen, D. Flath, D. Hughes Hallet, B. Lahme, J. Morris, W. McCallum, J. Thrash. 2003. *ConcepTests to Accompany Calculus*, Third Edition. Hoboken NJ: John Wiley & Son.

Rogoff, B., E. Matusov, and C. White, 1996. Models of teaching and learning: Participation in a community of learners. In *The handbook of education and human development,* ed. D. R. Olson and N. Torrance, 388-414. Oxford, UK: Blackwell.

Vygotsky, L. 1978. *Mind in Society*. Cambridge: Harvard University Press.

Wells, G. 1999. *Dialogic inquiry: Toward a sociocultural practice and theory of education*. Cambridge, UK: Cambridge University Press.

Wenger, E. 1998. *Communities of practice: Learning, meaning and identity*. Cambridge, UK: Cambridge University Press.