# Investigating the development of prospective mathematics teachers' technological pedagogical content knowledge with regard to student difficulties: the case of radian concept

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This study investigates how two prospective mathematics teachers integrate technology into their lessons to address student difficulties. Prospective teachers took part in a teacher preparation program which aims to develop technological pedagogical content knowledge (TPCK). As part of this program, prospective teachers participated in workshops which aimed to develop TPCK of derivative and function concepts. Following these workshops, prospective teachers conducted their own workshops during which they discussed student difficulties with various mathematical concepts such as limit, continuity, definite integral, probability and radian with their peers. They also discussed how these difficulties could be addressed during a lesson using technological tools. This paper particularly focuses on radian concept and investigates the development of two prospective mathematics teachers throughout the course in integrating technology into their lessons to address student difficulties with radian concept.

**Keywords: Mathematics teacher education; technological pedagogical content knowledge; radian concept; student difficulties**

## Introduction

Recently, teacher education research has focused on the nature of teacher knowledge required for successful technology integration (Mishra and Koehler 2006). In the literature, a theoretical framework called 'Technological Pedagogical Content Knowledge (TPCK)' is proposed to investigate the nature of knowledge to be able to successfully integrate technology into instruction. TPCK framework was originally derived from the idea of 'Pedagogical Content Knowledge (PCK)' which was proposed by Shulman (1986) as a new domain of teacher knowledge. It has been a useful framework for exploring what teachers need to know or to develop for effective teaching of particular content.

Pierson (2001) has added technology category to PCK framework and described TPCK as a combination of three types of knowledge: (a) content knowledge, (b) pedagogical knowledge, that is, the structure, organization, management, and teaching strategies for how particular subject matter is taught, (c) technological knowledge including the basic operational skills of technologies. TPCK is defined as a blend of these three knowledge categories.

Only a few researchers have examined the components of TPCK. Among those, Pierson (2001) and Niess (2005) used four components of PCK suggested by Grossman (1990) to define the components of TPCK. Five components of TPCK are proposed in a research project, part of which constitutes this study. Four of these components were adopted from Grossman (1990). A component regarding multiple representations was added as the fifth component of TPCK:

- Knowledge of using multiple representations of a particular topic with technology
- Knowledge of students' difficulties with a particular topic and addressing them using technology

- Knowledge of instructional strategies and methods for a particular topic using technology
- Knowledge of curricular materials available for teaching a particular topic using technology
- Knowledge of assessment of a particular topic with technology

In the project, TPCK framework with its five components was used to design a course for prospective mathematics teachers. Prospective teachers were participated in the activities concerning each component. This study focuses on a particular component of TPCK, namely *knowledge of students' difficulties with a particular topic and addressing them using technology*. This study aims to bring the content dimension into play focusing on radian concept and investigates how two prospective mathematics teachers integrate technology into their lessons to address student difficulties with radian concept. To do that, considering the relevant research on the learning of radian, three areas of student difficulties were specified.

First, students' difficulties with recognising real numbers as radian measure are reported in the literature. Radian of an angle can be described as the ratio of two lengths: the length of the arc of a central angle of a circle and the radius of the circle. Therefore, a radian angle is expressed as a real number such as sin 30 while angles in degrees are expressed with degree notation such as sin 30º. However, research findings indicate that students find it difficult to make sense of angles if they are expressed without degree notation such as sin30 even though the absence of degree notation was emphasised (Akkoç and Akbaş-Gül 2010). If the expression does not include π, students tend to reject it as an angle in radian.

Second difficulty is concerned with describing radian as a ratio of two lengths: the length of the arc of a central angle of a circle and the radius of the circle. Research indicates that students' concept images of radian are dominated by the formula $\frac{D}{360}=\frac{R}{2\pi}$. They use this formula instead of the definition of radian. In Akkoç and Akbaş's (2010) study, some of the students could relate the radian concept to the notion of an arc. They could only define one radian: the length of an arc on a unit circle which is equal to the length of radius. However, they could not describe radian in a general sense, that is, radian as a ratio of two lengths.

The third difficulty stems from recognising π as equal to 180. Since students' understanding of radian might be limited to the use of the formula $\frac{D}{360}=\frac{R}{2\pi}$, they might consider π as equal to 180 (Fi 2003). Akkoç's (2008) study indicates that participants marked π as the number 180 on a number line, not as a number around 3.14. Students have two different concept images of π in two different contexts: one in the context of angle and one in the context of number.

The aim of this study is to investigate how two prospective mathematics teachers integrate technology into their lessons to address student difficulties with radian concept which were reported above. For this investigation two research questions are specified: (i) What kinds of student difficulties do prospective teachers foresee during lesson planning and how do they evaluate their lessons with this respect? (ii) How do prospective teachers address these difficulties?

## Methodology

This case study focuses on two prospective mathematics teachers enrolled in a teacher preparation program in a university in Istanbul, Turkey. Forty prospective teachers participated in the program and they will be entitled to a certificate for teaching mathematics in high schools for students aged between 15 and 19. During the program they take courses related to mathematics, pedagogy and content specific pedagogy. The data of this study was collected during two courses: "Teaching Methods in Mathematics" and "Instructional Technology and Material Development". As part of these courses, prospective teachers participated in workshops which aimed to develop TPCK of derivative and function concepts conducted by the author of this paper. The workshops attended to the five components of TPCK which were mentioned above. Following these workshops, prospective teachers conducted their own workshops during which they discussed student difficulties with various mathematical concepts such as limit, continuity, definite integral, probability and radian with their peers. Each concept is discussed by a group of eight prospective teachers. This study focuses on the workshop on radian concept. To guide prospective teachers for their preparation for the workshop, they were provided with a review of literature on students' difficulties with radian. They were given an MSc thesis on the learning of radian in Turkish language (Akbaş 2008). In the workshop, four prospective teachers discussed student difficulties with radian and how these difficulties could be addressed during a lesson using technological tools. Prospective teachers used Cabri Geometry and Geogebra software. Their TPCK were evaluated before and after the workshop. They prepared and taught two lessons on radian: one before the workshop and one after the workshop. They taught these lessons as part of a micro-teaching activity. In other words, they taught these lessons to their peers as if they were in a real classroom environment. They were also interviewed at the end of the lesson. Lesson plans, teaching notes, micro-teaching lesson videos, interview transcripts were analysed to reveal how two prospective teachers integrate technology into a lesson to address student difficulties with radian which were reported above.

## Findings

In this section, findings obtained from the data analysis will be presented in three sub-sections. Each sub-section will report how two prospective teachers (Mutlu and Gamze) take each student difficulty into account before and after the workshop.

*Difficulties with describing radian as a ratio of two lengths*

Before the workshop, during their first micro-teaching lessons, both prospective teachers introduced radian concept by defining one radian only: a central angle facing an arc which has a length of the radius. Neither of them defined radian in a general sense: ratio of the length of the arc of a central angle of a circle and the radius of the circle.

For their second lesson planning, they took this difficulty into account as they mentioned during the interviews. During their second micro-teaching lessons, Gamze and Mutlu not only gave the definition of one radian but also explained radian as the ratio of two lengths.

> How shall we find an angle when the length of the arc and radius are known. How many radiuses does the arc include? Then the angle is L/R (where the L is the length of the arc and R is the length of the radius). Is this really true? Let's run the Geogebra software (see Figure 1) (Gamze).

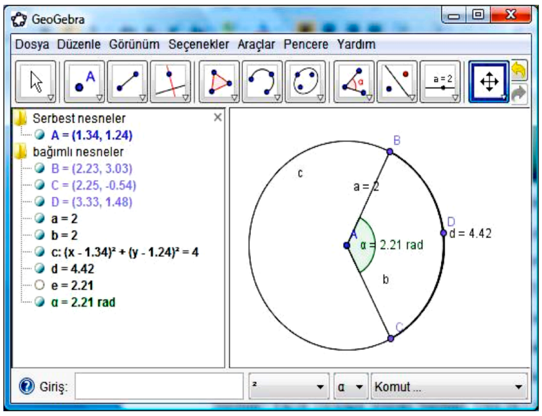


Figure 1. Gamze's activity with Geogebra during her second lesson

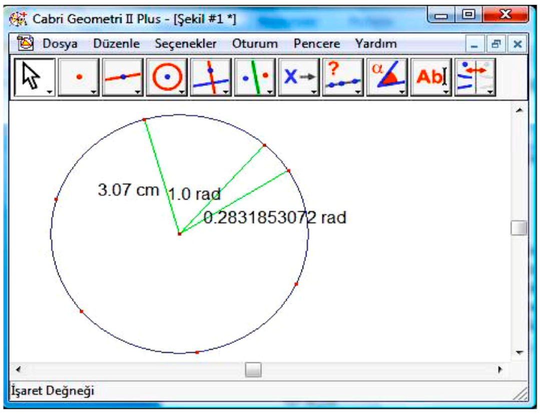


Figure 2. Mutlu's activity with Cabri Geometry during her second lesson

Mutlu first defined one radian and used Cabri Geometry to show that the round angle is approximately 6.28 radians (6+0.28=6.28) which is 2π (see Figure 2).

During the interviews, both prospective teachers evaluated software they used in their second lessons. They both emphasised that defining one radian only might prevent students from understanding the radian concept in a general sense, that is radian as the ratio of the length of the arc of a central angle of a circle and the radius of the circle. They both thought that the software they used could enhance students' understanding of radian in that sense.

*Difficulties with recognising real numbers as radian measure*

Before the workshop, neither of prospective teachers considered this difficulty in their planning. However, some aspects of their lessons related radian to real numbers. Both prospective teachers explained that there is approximately 6.28 radians around a circle. Gamze used a piece of string to measure the radius of a circle which she made from card board and then measured the circumference to find how many pieces of string are there on the circumference. However, she did not make any emphasis on real numbers. For example, as she was explaining the sine function, she did not mention that the elements of the domain of sine function (which is the set of real numbers) are in radian measurement not in degrees. Mutlu used Cabri Geometry to show that there are six 57.3's and one 16.2 degrees in a round angle (see Figure 3) and she evaluated 16.2/57.3 nearly as 0.28. She then concluded that there are 6.28 radians in a round angle. Although she expressed radian as a real number, she reached that conclusion using the degree measurement. Furthermore, for all the other examples in her first micro-teaching lesson, she used angles in radian which includes π only. This might lead students to reject angles in radian if they do not include π.

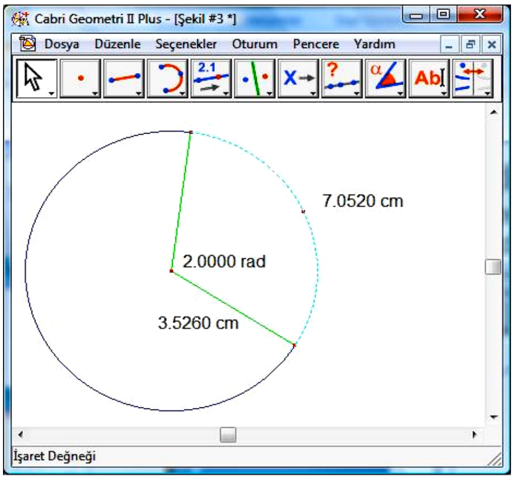


Figure 3. Mutlu's activity with Cabri Geometry during her first lesson

After the workshop, both prospective teachers took students' difficulties with seeing radian as a real number into account as they mentioned during their reflections for their second lesson planning. Likewise, the analysis of their second micro-teaching lesson videos indicates that they integrate technology into their lessons to address this difficulty. Mutlu asked her peers to construct 2 radians using Cabri Geometry as shown in Figure 4. During the interview, she said:

> I deliberately asked them to find 2 radians. Radian could be any number, not just π. It could be any real number" (Mutlu).

Gamze similarly emphasised that "π is not a symbol for radian angle. It's a special kind of real number used to avoid using big numbers". In summary, both prospective teachers addressed this difficulty using technology in their second lesson.

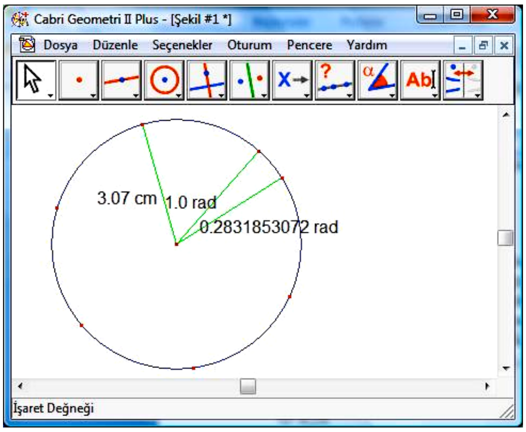


Figure 4. Mutlu's activity with Cabri Geometry during her second lesson

*Recognising π as equal to 180 degrees*

Before the workshop, neither of prospective teachers considered this difficulty in their planning. During their first micro-teaching lessons, both Gamze and Mutlu constructed the formula by direct proportion (If 2π radians is equal to 360 degrees, then how many radians do 30 degrees equal to?). They wrote 360=2π on the board. However, this might lead students to think that π is equal to 180. They also emphasised π as 3.14 as well. In other words, they focused on two different images of π: π as an angle and π as a number. However, they did not relate these two images in their first lessons. They used a similar approach during their second micro-teaching lessons but related two different images of π by emphasising that the length of the arc on a unit circle is two times 3.14.

**Discussion**

As the data indicated, the way participants integrate technology into their lessons to address student difficulties with radian has improved considerably. The workshop conducted by prospective teachers during the course was helpful for participants to become aware of student difficulties with radian which were reported in the literature. Participants tried to integrate technology into their lessons during their micro-teaching sessions before the workshop. However, the way they integrate technology was more effective during their second micro-teaching lesson. This situation revealed itself especially during Mutlu's lessons. Before the workshop, she used Cabri Geometry to show that there are six 57.3's and one 16.2 degrees in a round angle using the fact that one radian is 57.3 degrees. She evaluated 16.2/57.3 nearly as 0.28 and concluded that there are approximately 6.28 radians in a round angle. Although she expressed radian as a real number, she reached that conclusion using the degree measurement. On the other hand, during her second-micro teaching lesson after the workshop, she used Cabri Geometry software to find the measure of the angle in radian and express it as a real number by finding the ratio of the length of the arc and the length of the radius. This way, she used the potential of the software to address students' difficulties with recognising real numbers as radian measure.

From the data, it can be concluded that content specific coursework had a strong impact on prospective teachers' TPCK with regard to student difficulties. Effectiveness of content specific teacher preparation is also reported by Fennema and Franke (1992). In their study elementary teachers were able to gain knowledge about their students' thinking and this knowledge favourably influenced their teaching and the students' learning. For prospective teachers, since they do not have much experience with students, one of the ways of gaining this knowledge is to examine the research findings on students' difficulties. This study indicated that this could be a useful method for helping prospective teachers gain the knowledge of students’ difficulties. Furthermore, this knowledge helped prospective teachers to integrate technology into their lessons more effectively.

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