The influence of teacher candidates' spatial visualization ability on the use of multiple representations in problem solving of definite integrals: A qualitative analysis

Eyup Sevimli and Ali Delice

University of Marmara

This study aims to investigate the influence of spatial visualization ability in representations used in definite integral subject. In this sense, a case study has been carried out on 45 mathematics teacher candidates. Multi- method approach was adopted by using more than one research techniques. Tests, Document analysis and semi-structured interviews are the research instruments and inferential & descriptive statistics are used for the data analysis. Findings showed that spatial visualization ability of the teacher candidates is low. In parallel to these findings, it was determined that the candidates, who have low spatial visualization ability, used predominantly algebraic representation. The development of spatial visualization ability, which may influence the relationship between graphical representations and the other representations, increases the performance of solving definite integral problems. Moreover, the candidates are advised to develop their spatial visualization abilities to improve their abilities of interpretation of visual information.

Keywords: spatial visualization ability; multiple representations word; definite integral

Introduction

Some studies conducted in recent times have shown that spatial thinking has a powerful and positive relationship with mathematical thought and the development of spatial ability will contribute much in learning mathematics (Haciomeroglu, 2007; Arcavi, 2003). Definite integral is a topic in mathematics that merely some students may end up having conceptual comprehending abilities at the end of the course, so it is believed that the visualization abilities and activities used in definite integral courses can make the comprehension process easier for students. In this sense, definite integral subject possessing geometric and graphical comments is influenced by spatial visualization ability. Utilizing visual elements in the definition of Riemann sum helps to concretize the process (Sealey, 2008). Visual spatial abilities particularly have a crucial place in finding the area under and between the curves and also finding the volume of the objects which are formed by the curves have been turned (Sevimli, 2009).

Visual spatial abilities are linked with the usage of multiple representations (Gutierrez, 1996). Bishop (1980) defines interpretation of figural information as the ability of using and interpreting a data given in graphics, tables and diagrams in solving a problem. This ability of interpreting may affect multiple representations usage. Multiple representations are accepted as an advantage by many educators and NCTM (2000) due to the reasons that students approach to solving problems from different perspectives and form a cognitive relationship (Keller & Hirsch, 1998). Many different multiple representations can be used in definite integral to demonstrate concepts and relations between them, however, the results based upon routine procedural approach and the lack of conceptual interpretation should be discussed. The concept of multiple representations in this study means the use of graphical, algebraic and numerical external representations in solving definite integral problems (Kendal & Stacey, 2003). Multiple representations method has been one of the most highlighted elements for class activities after the curriculum reforms/changes in some countries. The studies with a focus on multiple representations in the Calculus topics have shown the fact that the students relying on one representation were not successful; however, the ones who were able to make transitions between different representations were successful (Goerdt, 2007). The graphical representation of definite integrals are generally used in calculations involving areas under a curve or volumes of revolving objects, whereas numerical representations are of use in Riemann's cumulative addition problems (Sealey, 2008). The relationship between the representations used in definite integral and teacher candidates' spatial visualization ability is examined in this study. The questions in this study are constructed as following; what kind of representation do mathematics teacher candidates use in the process of solving definite integral problems? What are the levels of spatial visualization abilities of mathematics teacher candidates? What kind of relationship is there between the representations mathematics teacher candidates use of and their level of spatial visualization in the process of solving definite integral problems? Use BSRLM Body Text for the main text of your article

Methodology

Research design and participants

Multi- method approach was adopted by using more than one research techniques. Quantitative techniques are used to get supportive data in this study which consisted of predominantly qualitative techniques. Since the relation between spatial visual abilities and multiple representations are deeply examined, case study (Yin, 1994), is the main strategy of the study. The participants are 45 second year mathematics student teachers trained in a state university in the spring semester of the 2008-2009.

Data collection tool

Purdue Spatial-Visualization Ability Test (PSVT), Representation Preference and Transition Test (RPTT) are used with respect to the nature of the research questions in the study to collect data. Firstly, the researchers adapted PSVT, which was developed by Guay (1977) and used in many studies before (Ferrini- Mundy, 1987; Haciomeroglu, 2007), to Turkish. Then PSVT was administered to the teacher candidates in the second week of the spring period. PSVT consists of developments, rotations, views parts each of which has 12 items. PSVT is a speed test which is aimed to be conducted in 24 minutes; it is a multiple choice exam of 36 items with only one correct answer; it should be answered by paper and pencil. Construct and content validity of PSVT adaptation (Sevimli, 2009) is also found parallel to the validity of PSVT found by Guay (1977). Internal consistency reliability coefficient is calculated as 0.87 by Kuder-Richardson (KR-20) reliability test, reliability coefficient is determined as alpha=0.82 by split- half method, r=0.23 is found as correlation coefficient between the forms and reliability coefficient determined by parallel- test method is found as alpha= 0.88.

RPTT test developed by the researchers consists of nine items each of which represent a different objective of the course. Each question contains input representations defining the givens of the problem and output representations which the solution of the problem includes. In abbreviations of characteristics, input and output representations are presented with capital and small letters respectively as N or n (numerical representations), G or g (graphical representations), A or a (algebraic representations).

The test has two sub-categories: 'Transition within representations' sub-category contains question types in which input and expected output representations are the same. 'Transition between representations' sub-category contains question types in which input and expected output representations are different. The test was found to have construct and content validity after the analysis made by five experts in the area.

Data analysis

To begin with, the level of teacher candidates' spatial visualization abilities is determined to analyse the data. The studies using PSVT took the correct answers into account and the criteria of spatial visualization ability was constructed by the addition and subtraction of standard deviation to the average of the numbers of correct answers (Guay, 1977; Haciomeroglu, 2007). According to PSVT, the maximum number of correct answers is 36; the maximum number of correct answers for each subsection of this test is 12. Secondly, the representations respondents used were coded as numerical, graphical, and algebraic and mixed. A mixed representation is said to exist when, more than one representation are used interrelatedly for the same question. The relation between teacher canditates' representations and success in spatial ability were investigated. Findings of interview were analysed using common categorizations and statistical analyses were done using statistical software.

Findings

The answers given to PSVT are shown in Table 1 which also includes the points in the subparts of the test. Aside from the fact that there is not a distinctive difference between the subparts of PSVT in terms of the number of correct answers, the subpart which has the lowest correct answer average is "views" with a rate of answers, 5.91. It is seen in the results of PSVT that the maximum number of correct answer is 32; the minimum number of correct answer is 6 and the average of the number of answers is around 19.

	Min.	Max	Average	Standard Deviation
Developments	2	11	6.51	2.72
Rotations	2	12	6.75	2.93
Views	1	10	5.91	2.31
Total	6	32	19.7	6.81

Table 1: PSVT answers' average points

The levels of spatial visualization ability of the candidates is determined by adding and subtracting a standard deviation point to the average by taking the number of maximum and minimum correct answers into account. By adding to a standard deviation point to the average, the teacher candidates with 26 correct answers and more is coded as "High Spatial Visualization Ability (HSVA), by subtracting a standard deviation point from the average, the candidates with 12 correct answers and less is coded as "Low Spatial Visualization Ability" (LVSA) teacher candidates. As a result of PSVT analysis, the candidates with a number of correct answers between 13 and 25 were classified and coded as "Average Spatial Visualization Ability (ASVA) teacher candidates. Consequently, the results revealed that 22% of teacher candidates are HSVA, 60% of them are ASVA and 18% of them are LSVA.

Representations used in definite integral problem solving process

Findings indicate that the highest proportion of use of representation (93.3%) is observed in a problem with an Aa characteristics (RFTT/2) in which an algebraic representation was used. Numerical and graphical representations were not used in problems having Gg, Ga and Aa characteristics. The high proportion of use of algebraic representations in all of the RFTT problems is another noteworthy finding. In addition, contrary to the expectation of a high proportion of use of graphical representation in the solution of RFTT/5, only 15.6% seemed to have used it (Table 2). Another important finding is related to the types of problems which were left undone. RFTT/6, for which a Ga transition was expected, was the problem with the highest proportion of blank responses. Besides, it is seen that the problems for which either numerical or graphical representations were expected had the higher blank proportion than the other questions.

Problem no	Input	Output	Subcategories	Type of representation				
				Numerical	Graphical	Algebraic	Mixed	Empty
				%	%	%	%	%
3	N	n	Transition within	31.1	37.8	-	13.3	17.8
8	G	g		-	-	48.9	51.1	-
2	A	a		-	-	93.3	6.7	-
9	N	g	Transition between	6.7	57.8	13.3	13.3	8.9
7	N	a		15.6	15.6	40	20	8.9
1	G	n		35.6	22.2	20	-	22.2
6	G	a		-	-	62.2	13.3	24.4
4	A	n		17.8	4.4	64.4	2.2	6.7
5	A	g		-	15.6	37.8	42.2	4.4

Table 2: Frequency of representations used in RPTT

When all answers given to RPTT are considered, the proportion of using algebraic representation is 46% and the graphical representation is 17%. The difference between percentages of the most widely used two representations algebraic and graphical suggested a dominance of relying on a single representation. The least preferred type of representation is the numerical representation; which is preceded by mixed representations. The findings of RPTT's subsections suggests the domination of algebra representation in the within and between transition questions.

The relationship between representation types and levels of spatial visualization ability

Teacher candidates were classified in three groups as "low, average and high" with respect to their spatial visualization ability. In the light of RPTT analysis, it was seen that 34% of HSVA candidates used algebraic representation. It was followed by 27%, of the ones who used two representations together and 20% of those who used graphical representations. It was found that the teacher candidates in the level of ASVA have predominantly used algebraic representation and this is followed by graphical and mixed representations respectively (Table 3). In terms of the answers given to PSVT, it was revealed that the ones belonging to the group of low spatial visualization ability also used mostly algebraic representations which are followed by the findings of mixed and graphical representations respectively. In all groups, numerical representation was seen to be the least used representation type.

%	Numerical	Graphical	Algebraic	Mixed
HSVA	11	20	34	27
ASVA	14	23	46	17
LSVA	16	16	53	15

Table 3: The relation between spatial abilities and representations

Moreover, meaningful differences were obtained through spatial visualization ability and algebraic representations among the representation types used. It was seen that as spatial visualization ability increases, the percentage of usage of algebraic representation decreases and there is a balance between with the frequency of usage of mixed representations and spatial visualization ability, as well. While a direct relationship between spatial visualization ability and usage of graphical representation is not seen, the increase of spatial visualization ability has increased the frequency of usage of graphical representation because graphical representations and the other representations are mostly linked in mixed representation group. It was seen that LSVA group was the one used graphical representations the least.

Discussion and conclusion

The candidates, who did not use graphical representations, experienced difficulties in the process of solution were algebraic-based thinking, misuse of graphical data and interpretation (Kendal & Stacey, 2003). The candidates with high spatial visualization ability use of graphical representations in high percentages along with algebraic representations and linking these two representations lead to the success of problem solving. Especially, in the problems showing Ag and Ga characteristics in which most of the candidates had difficulties, it was observed that the candidates with high spatial visualization ability tried to solve the problem by the graphical interpretation and by considering different definitions of definite integral. The development of the spatial visualization ability (Arcavi, 2003), which is needed for the success in teaching mathematics and geometry is helpful for benefiting from different representations in solving definite integral problems. The development of spatial visualization ability, which may influence the relationship between graphical representations and the other representations, increases the performance of solving definite integral problems (Haciomeroglu, 2007).

Overall, it is found that the teacher candidates do not have enough level of spatial visualization ability and representation transition ability. The failure in these abilities implicitly influenced the performances in definite integral. The teacher candidates with high spatial visualization ability seem also to be successful in representation transition and in using multiple representations. It is observed that the teacher candidates, who particularly used two representations as mixed representations in solving the definite integral problems, have high spatial representation abilities; the candidates with low spatial representation ability predominantly used algebraic representations even in problems which could be solved through different representations. Possible reasons for this tendency include lecturers' reliance of a single representation in their teaching, Lecturers' reluctance to show textbook examples that enable the use of multiple representations leads to students' lack of knowledge of alternative definitions. Moreover, the candidates are advised to develop their spatial visualization abilities to improve their abilities of graphical and table interpretations.

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