Parsing Mathematical Constructs: Results from a Preliminary Eye Tracking Study

Michael Peters

Learning Development Centre, Aston University.

This preliminary research showed that the use of an eye tracker gave an insight into how mathematics is parsed. The results indicated that an expert mathematician is able to identify and process relevant information quickly compared to non-experts. The fixation times of the expert support the idea of an asemantic processing mechanism whereas the fixation times of the non-experts indicate the need for explicit semantic processing. The fixation times and gaze trail data support the notion of a first parse to identify relevant information and subsequent parsing to encode the information into working memory.

Keywords: Parsing, working memory, fixation times, gaze trails.

Introduction

The basis for this research was the work carried by the author on the development of a semantic model for the interpretation of mathematics (Peters 2008). The semantic model highlighted the importance of the parsing process in the learning of mathematics. In order to investigate how learners deconstruct mathematics it is important to gain an understanding of how they read and interpret mathematics. The first stage of this process, the reading of mathematics, was investigated using an eye tracker. An eye tracker was deemed appropriate since it enabled the researcher to determine where the participants were looking (fixation points) and the length of time they spent looking at these points (fixation times). This preliminary study used an expert mathematician, a post doctoral student, a mathematics graduate, a final year mathematics student and two final year psychology students. By using this heterogeneous group it was possible to investigate the differences in the levels of their mathematical competency. The purpose of the preliminary study was firstly to determine if using an eye tracker would yield the required data and, secondly to determine the necessary parameters for a main study.

Equipment

An Eye Response Technologies, GazeTracker was used with the sampling rate of 50ms. A fixation was defined to be a position where the subject looked at a point for 10ms or more. Before the start of the session the eye tracker was calibrated using a pattern of dots presented to the learner. The participant sat in a chair with their heads strapped into a frame so that head movement was minimised. The eye tracking software was capable of recording the eye movements to video and exporting the data to a Microsoft Excel workbook.

Method

The questions were single answer multiple choice presented on a web page. In order to ensure the participant's gaze started from the centre of the screen, a smiley face was presented at the start and subsequently between questions. The instructions given to the participant's were for them to click on the smiley face when they were ready to proceed. This gave the participant control over the rate at which the questions were presented.

The first question was the English sentence: The quick brown fox jumps over the lazy dog. By using this sentence the learners would experience how the questions would be presented and it also gave an indication of how they would read a non-ambiguous, simple sentence. The learners were then presented with 10 mathematical questions in sequence. They were not ordered in any way and were designed to test conceptual knowledge rather than procedural knowledge. Finally, an ambiguous relative clause sentence was presented. This was done so that a comparison could be made between their reading of mathematics and an English sentence that required some analysis. The data from the first 1000ms for each question was collated into a spreadsheet for analysis. In order to give an overall picture for the learners' behaviour the results from each question were collated and analysed. To give a visual representation, bar graphs were produced showing the number of fixations against fixation times.

Theoretical Background.

Eye tracking has been used in investigations into how text is read and scene perception (Duchowski 2007; Findley and Gilchrist 2003; Raynor 1998). It was also used by Andrà et al (2009) in their study of how students read mathematical representations. Ferrara et al (2005) used eye tracking equipment to connect the talk, gesture and eye motion of a graduate student who had to produce a position vs time graph for a given 'motion story'. Epelboim and Suppes (2001) used an eye tracker in their study of the differences between experts and novices in solving geometry problems.

It is often assumed that problem solving is a systematic process of matching items in declarative memory with rules. This process is assumed to continue until the goal is achieved and the problem solved. Anderson's (2007) ACT-R family of models is the most highly developed of such systems. Problem solving was shown to be more probabilistic in nature by Suppes and Sheehan (1981) using computer based proofs in set theory. Suppes et al (1983) also found that when learners were doing column arithmetic exercises they did not follow any particular algorithm.

Central to any cognitive activity is the role of memory. Memory can be divided into sensory, working and long term systems. The sensory memory holds a large amount of data which has not been processed. In particular the sensory store associated with visual data is known as iconic memory (Purves et al., 2008). Most of the data held in this store is quickly lost. Baddeley (1986) proposed a model of working memory that consisted of a central executive, a phonological loop and a visuospatial sketchpad. Working memory holds relatively small amounts of information for several tens of seconds. It is generally accepted that 5-9 pieces of information can be stored at any one time but this is dependent upon the size of the pieces of information being stored. To reduce the cognitive load on working memory, smaller objects can be combined or 'chunked' to form larger single entities. The semantic model developed by Peters (2008) proposed that experts were people who had, over a period of time, combined concepts to form super-concepts and therefore where able to process information asemantically (Dehaene 1992). In terms of working memory, the formation of super-concepts is the process of chunking and can be used to explain why experts tend to have better short term memory skills than novices. The contents of working memory need to be constantly refreshed or else the information is lost. Working memory operates in three phases: encoding, delay and response. Encoding is where one or more pieces of data are incorporated into working memory, the delay phase ensures the encoded information is maintained in working memory and, the response phase is where an action is executed on the basis of the maintained information. Since mathematical problem solving involves many different symbols, according to the Baddeley model, the central executive and visuospatial sketchpad would be the most active components. It is assumed that fixation times indicate some form of cognitive activity. This could be linked to the encoding phase of working memory and retrieval of information from long term memory. Lower fixation times could indicate the speed at which information is processed and retrieved. This seems a reasonable assumption given the notions of chunking and the formation of super-concepts. Back tracking, the process of re-reading information indicates that the learner could be clarifying, refreshing or confirming the validity of information held in working memory. In the case of word problems the learner has to associate variables with entities (eg. W with wages) and store these associations for future use. This implies that the longer the problem and the position in which the process of assigning variables to entities occurs have a direct impact on cognitive load.

Experts, according to the Haidr and Frensch's (1999) information-reduction hypothesis, have learnt to distinguish between relevant and irrelevant information. This skill effectively reduces the cognitive load thus enabling an increase in processing speeds.

Example Graphs of Fixation Times.

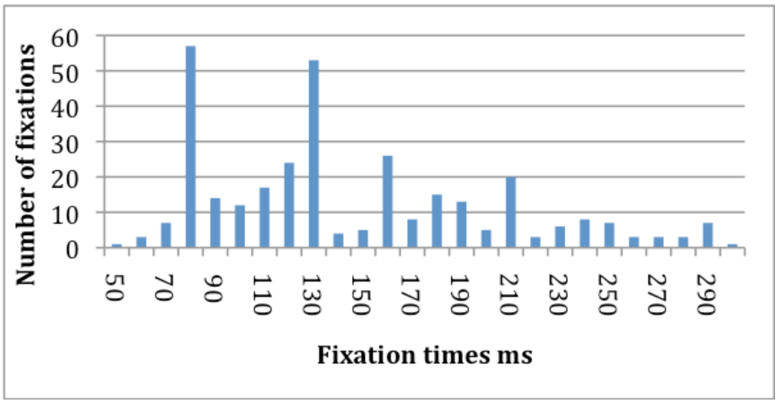

Figure 1, Expert Mathematician's Fixation Times Graph

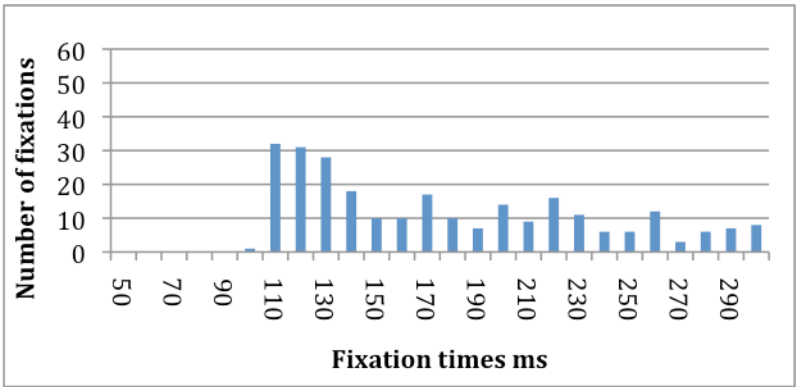

Figure 2, Non-expert's Fixation Times Graph

Figures 1 and 2 show the fixation times of an expert mathematician and a learner who was not a mathematics specialist.

Example Gaze Trails

Question 7

Mary's basic wage is £200 per week. She is also paid another £4 for each hour of overtime that she works.

If *h* stands for the number of hours of overtime that she works, and if *W* stands for her total wage (in £s).

Which of the following equation best describes the relationship between W and *h*.

- W=200+4h
- 200=W+h

Next Page

Figure 3, Question 7

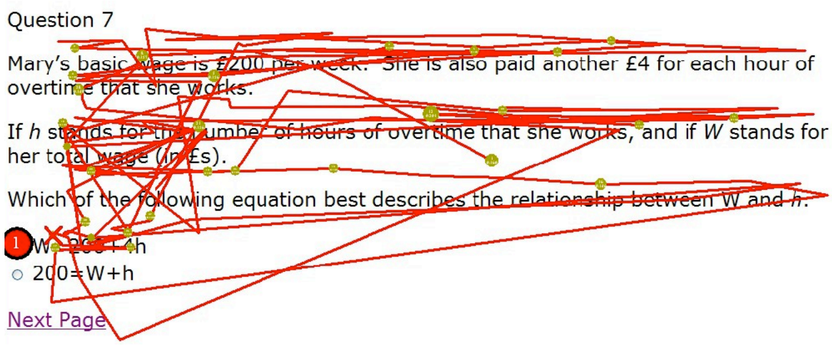

Figure 4, Non-expert's Gaze Trail for Question 7.

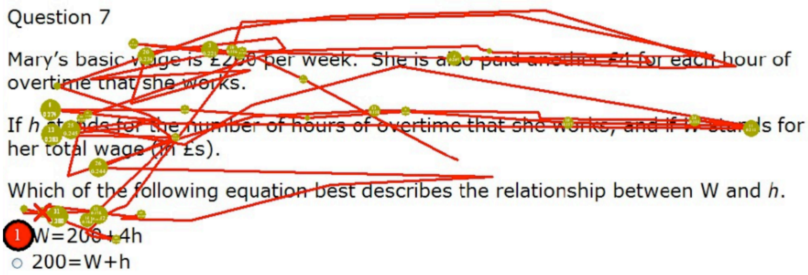

Figure 5 Expert's Gaze Trail for Question 7.

Figure 3 shows the question and Figures 4 and 5 show the gaze trails for two of the participants. The green filled circles represent the fixation points. They are numbered sequentially and the size of the circle represents the fixation time.

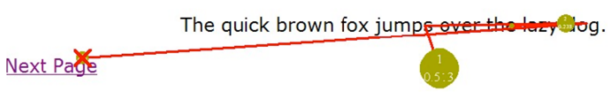

Figure 6, Unambiguous Sentence

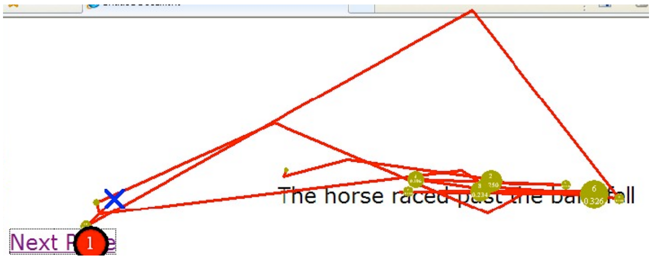

Figure 7, Ambiguous Relative Clause Sentence

Discussion

The expert's fixation time graph, Figure 1, indicates that he was able to process the information very quickly (fixation times started at approximately 50ms). The implication of this is that he had developed sufficient super-concepts to be able to process the information asemantically, whereas the non-expert had to rely upon explicit semantic processing of the information. This is reflected in the way the fixation times are distributed and that initial processing did not start until approximately 100ms. The difference in fixation times could also be the result of the expert's ability to discriminate between relevant and irrelevant information. The fact that he had a lot of short fixation times indicates that he was able to focus on the key concepts and did not spend time reading and analysing every piece of information. In comparison, the non-expert had a 'flatter' spread of fixation times indicating that she tended to fixate on more pieces of information for longer periods of time.

The gaze trails also indicated that the expert was able to select the relevant information and fixated more on upon the important mathematical concepts. For example in question 7, Figure 3, the expert fixated upon information such as the numerical values and the variables and skimmed very quickly over the last line. The non-experts tended to read the whole question including the last line. In all cases the possible answers were looked at and parts of the question re-read. This behaviour could indicate that after initially encoding what they considered to be relevant information in working memory they were checking to see if the encoded information was actually relevant. Another noticeable behaviour was that the expert read the first two lines and then checked the answer options. Once he had looked at the answers he re-read the relevant parts of the question before selecting an answer. The non-experts tended to spend more time on the initial reading of the question on a line by line basis. It seemed they wanted to ensure that they understood each line before proceeding to the next. Once they had studied each line in detail, they checked the answer options and then re-read what they considered to be relevant parts of the question.

The visual field is divided into three areas: foveal, parafoveal and peripheral. Foveal vision occurs within a region ranging up to 1o from the central axis, parafoveal vision occurs between 1o and 5o and peripheral vision accounts for the rest (Findley and Gilchrist 2003). Figure 6 shows the gaze trail for the unambiguous English sentence. As the diagram shows the eyes quickly scan the sentence and it appears that the eye tracker software catches up part way through. This scan demonstrates that when someone is reading parafoveal vision is scanning approximately 14 characters to the right for English texts (ibid). Figure 7 shows the gaze trail when an ambiguous sentence is read. The sentence is first read including the ambiguity and then is read again in an attempt to make sense of the sentence. The gaze trail for the ambiguous sentence is similar to the ones seen for the mathematics questions. This indicates that the first parse generally involves obtaining an overall picture of the question and then reparsing to make sense of the information.

Main Study

The knowledge and skills gained from this preliminary study will be used to inform a main study. This main study will use experts and a range of engineering students. It is proposed to use approximately 10 first year undergraduate students and a similar number of second and third year undergraduate students. This should give a sufficiently large data set to be able to employ informative statistical analysis techniques.

References

Anderson, J.R. 2007 *How can the Human Mind Occur in the Physical Universe?* New York, Oxford University Press.

Andrà, C., F. Arzarello, F. Ferrara, K. Holmqvist, P. Lindstrom, O. Robutti, and Sabena, C. 2009 "How students read mathematical representations: an eye tracking study." In Proceedings of 33rd Conference of the International Group for the Psychology of Mathematics Education Eds. Tzekaki, M., Kaldrimidou, M. & Sakonidis, C. (Eds.). 123-131. Thessaloniki, Greece PME.

Baddeley, A. D. 1986 *Working memory*. Oxford, Oxford University Press.

Dehaene, S. 1992 'Varieties of numerical abilities'. *Cognition* 44: 1-42.

Duchowski, A. T. 2007 *Eye Tracking Methodology: Theory and Practice 2nd Edition*. London, Springer-Verlag London Ltd.

Epelboim, J. and P. Suppes, 2001 'A model of eye movements and visual working memory during problem solving in geometry.' *Vision Research* 41: 1561- 574.

Ferrara, F. and R. Nemirovsky, 2005 'Connecting talk, gesture, and eye motion for the microanalysis of mathematical learning',. In Proceedings of 29th Conference of the International Group for the Psychology of Mathematics Education eds. H.L.Chick and J.L.Vincent. 137-142. Melbourne, Australia.

Findley, J. M. and I.D. Gilchrist, 2003 *Active Vision: The Psychology of Looking and Seeing*. Oxford, Oxford University Press.

Haider, H. and P.A. Frensch, 1999 'Eye movement during skill acquisition: more evidence for the information-reduction hypothesis.'. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 25: 172-190.

Peters, M. D. 2008 'The Development of a Semantic Model for the Learning of Mathematics', Joubert, M. (ed). Informal Proceedings of the British Society for the Research into Learning Mathematics 28(2) 79-84.

Purves, D., E. M. Brannon, R. Cabeza, A. Scott, K. S. L. Huettel, M. L.Platt, and M. G. Woldorff, 2008 *Principles of Cognitive Neuroscience*. MA: Sinauer Associates.

Raynor, K. 1998 'Eye Movements in Reading and Information Processing: 20 years of Research'. *Psychological Bulletin*, 124: 372-422.

Suppes, P., M. Cohen, R. Laddaga and H. Floyd 1983 'A procedural theory of eye movements in doing arithmetic.'. *Journal of Mathematical Psychology*, 27: 341-369.

Suppes, P. and J. Sheehan, 1981 'CAI course in axiomatic set theory.'. In *University Level computer assisted instruction at Stanford*, ed. P.Suppes, 3-80. Standford University.