What maths do you need for university?

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Abstract: An exploratory investigation is reported, aimed at discovering possible evidence relevant to reform of A-level mathematics content. The investigation focussed on the many undergraduates in quantitative subject courses with only GCSE maths. Typically they get a pure maths "top-up" module in their first year to build up their foundational maths knowledge. Comparing its content with A-level indicates how well these students would have been prepared if they had actually taken A-level maths. The comparison covers top-up in three subjects at two differently ranked universities. Very clear patterns are evident: economics, business and finance top-ups overlap AS-level content moderately well. However, Matrices are important in all these courses and are not covered in A-level. The most striking result however is that computer science top-up is entirely in the area of Discrete Mathematics with no content overlap with A-level or with the other three subjects. These clear patterns seem to validate the method of investigation, which consumed only modest resources, and justify further investigation. Possible implications of the patterns are discussed.

Keywords: A-level Mathematics, Quantitative Subjects, maths top-up, matrices, Discrete Mathematics.

Introduction

Reform of the post-16 maths curriculum from 2012 was to have followed consultations in 2008-2009 by ACME and QCA. Reform is presumed suspended following election of a new government. This paper describes an exploratory investigation, conducted from the viewpoint that the purpose of A-level maths is to provide the mathematical foundations needed in undergraduate courses in Quantitative Subjects (QS). The aim of the investigation is to discover whether there is evidence to indicate that reform of A-level maths content is in fact needed. A previous paper (Osmon 2009) reported the maths qualifications required for entry, in 2008, to courses across a range of QS and tabulated these for a sample of ten universities. This work established that, essentially, just two maths qualifications are recognised by universities: GCSE and full A-level, with AS-level effectively ignored. It also established that, for a particular set of subjects (Maths, Physics, Engineering), here called Block I subjects (admitting about 25,000 students in 2008), A-level maths is essentially a mandatory entry requirement across all universities, and for another set of subjects (including Chemistry, Computer Science, Economics, Business and Finance), here called Block II subjects (admitting more than 50,000 students in 2008), A-level maths is required at very high ranking universities but otherwise GCSE maths is accepted for courses in these subjects.

If the purpose of A-level maths is to lay the maths foundations for quantitative subjects then any case for reform of A-level maths needs answers to the following:

(a) For Block I, how well does A-level content match the course requirements- taking account of possible variations across subjects? Is the students' maths knowledge "topped-up" in the first undergraduate year and how common is the top-up subject matter across courses?
(b) For Block II, at those institutions where A-level is required, how well does A-level content match the course requirements- taking account of possible variations across subjects? Is the students' maths knowledge topped-up in the first year and how common is the top-up subject matter across courses?
(c) For Block II subjects, where only GCSE is required, what maths top-up is provided during the first year to raise the students' maths knowledge level, and how does this vary with institution rank and across subjects?
(d) Why do universities virtually ignore AS-level maths as an entry qualification?

Considerable resources would be required to get detailed and reliable answers to these questions and so it would be interesting to know in advance, by doing some probing, whether the answers are likely to be helpful and hence worth the price! The investigation reported in this paper probes (c), aiming to find out whether answers to it, and the other questions, are likely to be worth the cost of getting them.

The investigation focuses on the large number of students beginning courses in quantitative subjects with only GCSE maths. Typically they take a pure maths "top-up" module in their first undergraduate year to build up their foundational maths knowledge. (NB "top-up" implies first teaching- it does not mean "remedial".) Comparing the content of these modules with A-level content indicates how well these students would have been prepared for their courses if they had actually taken A-level maths. (Quantitatively top-up should be compared just with AS-level since the 12.5% of first undergraduate year typically allotted is similar to the study time for an AS-level.)

Method

Whereas national information on student numbers and course entry requirements is readily available (UCAS data and on-line university prospectuses), the information universities publish about the top-up maths in their Block II courses is variable and often sparse. Therefore, since this is essentially a pilot investigation with the aim of discovering whether there are strong indications of interesting trends, it was eventually decided to look at only the three biggest subject groups in Block II (Computer Science: about 12,000 students, Economics: 7,000 students, and Business and Finance: 19,000 students, in 2008) at just two rather different institutions (University of Manchester and City University London) where particularly detailed information on top-up maths provision was available: Manchester's on-line prospectus (Manchester 2010) specifies module contents in fair detail, and City University (City 2010) kindly provided the author with maths module specifications and course text information for all its Block II courses.

The fact that Manchester and City have different rankings was important, but it did complicate the investigation. Manchester attracts more students with A-level to its Block II courses- in fact this is a requirement for entry to Computer Science- and provides different top-up modules for Economics according as the students have GCSE or A-level maths. Another, but minor, difference is due to internal university organisation rather than ranking, and this is the provision of separate post-GCSE top-up maths for Economics and for Business and Finance at City, whereas Manchester has a common post-GCSE module for all three subjects.

So, the content of five pure maths top-up modules was available for comparison: Computer Science at City (post-GCSE) and Manchester (post A-level), Economics and Business-Finance at City and Economic-Business-Finance at Manchester (all post GCSE). These module descriptions, were compared in detail with the core pure maths content of A-level maths (Ofqual 2002) and certain patterns were apparent immediately.

Patterns of differences and similarities

The contents of the three Economics, Business and Finance modules were very similar and there was considerable overlap of these three with AS maths, but the overlap did not increase when A2 maths was included in the comparison. Comparing these with City's Computer Science maths module revealed no overlap of content at all. Comparison of the two Computer Science modules showed the City content was a sub-set of the Manchester one (COMP 10020). However it must be noted that the Manchester CS top-up is actually a double module and also post A-level.

These strong features suggested presentation of the comparison results in two ways. First, the table below shows the largely common content of the three Economics-Business-Finance top-up modules and the extent of commonality with AS. (To save space topics are listed at coarser granularity than in the original documents.)

Two clear divergences between AS and the E-B-F top-ups are evident:

[i] No trigonometry in any of the E-B-F top-ups

[ii] Matrices are included in all the E-B-F top-ups but not in AS (or A2)

Topic	AS	ME	CE	CBF
Algebra & Coordinate Geometry				
Algebraic manipulation	Y	Y	?	Y
Quadratic equations	Y	Y	?	Y
Simultaneous equations	Y	Y	Y	Y
Solution of inequalities	Y	?	?	?
Graphs of functions	Y	Y	Y	Y
Graphl. solution of equations	Y	?	?	?
Straight line	Y	Y	Y	Y
Circle	Y	N	N	N
Sequences and series				
Sequences	Y	N	Y	Y
AP and GP	Y	N	?	Y
Binomial (pos. int. n)	Y	N	N	N
Trigonometry	Y	N	N	N
Exponentials and logarithms				
Logarithms	Y	Y	Y	Y
Exponentials	N	N	Y	Y
Calculus				
Tangent as limit of chords	Y	Y	Y	Y
Derivatives	Y	Y	?	?
Differentiate powers of x	Y	?	Y	?
Gradients, maxima, minima	Y	Y	Y	Y
Partial differentiation.	N	N	Y	N
Diffn. vectors, matrices	N	N	Y	N

Inverse of differentiation	Y	N	Y	?
Definite integrals	Y	N	?	Y
Matrices				
Operations	N	Y	Y	Y
Inverse	N	Y	Y	Y
Determinants	N	Y	Y	Y
Simultaneous equations	N	Y	Y	Y
Vectors	N	N	Y	N

Abbreviations used in the table: EM is the post-GCSE maths module shared by courses in Economics, Business and Finance at Manchester (ECON 10061); EC is the post-GCSE maths module in the Economics course at City (MA1655); CBF is the maths module in City's Business and Finance courses (BS1003); Y = topic present, ? = topic implicitly present, N = topic not present.

Second, a direct comparison of the text of the City Computer Science module with one of the Economics-Business-Finance (E-B-F) modules shows there is no overlap of content. The City University Computer Science and Manchester University Economics descriptions are so succinct they are quoted in full to show their remarkable divergence.

> *Computer Science (1N1004 Mathematics for Computing, post GCSE)-*
> Set notation, properties of the main connectives, numbers as sets.
> Propositional logic.
> Set comprehension, ordered pairs, Cartesian product, relations and functions.
> Restrictions and closures of relations and functions.
> Quantifiers and first order predicate calculus.
> Proof by mathematical induction.
> Deduction proof in propositional logic.
> Graph Theory.
> Combinational Analysis.
> Probability and Statistics.
> (Statistics is anomalous in the sense that all the other topics are pure maths.)
>
> *Economics (ECON10061 Introductory maths, post GCSE)-*
> Algebra and Coordinate Geometry: Basic algebra, factors and factorising, quadratic equations, powers, logarithms, mathematical functions, graphs, linear and non-linear functions.
> Differential Calculus: Slopes of graphs between points and at a point, differentiability, rules of differentiation, marginal concepts, second derivatives, maxima and minima, point maximisation.
> Matrices: matrix transposition, addition, subtraction and multiplication, determinants and inverse matrices, solution of simultaneous equations, input/output analysis.
> (Statistics is the subject of a separate top-up module: ECON 10062.)

The reader may wonder if this comparison result is so striking because the City CS module is anomalous in some way. Its content is a subset of the (post A-level) double top-up module COMP10020 at Manchester- as it would have to be if Discrete Maths is important in Computer Science- since it is not taught within A-level pure maths.

Since this is not a straight comparison of post-GCSE modules further evidence was sought from other on-line module descriptions- but they proved generally rather vague. However, the Imperial College website (Imperial 2010) carries module descriptions (post-A-level again) for two maths top-up modules in Computer Science: Discrete Maths, which has content substantially the same as the City module, and Uses of Maths, which includes Matrices- for application in computer graphics (also contained in the Manchester double module). Imperial's distinguishing of its modules in this way indicates the generic importance of Discrete Maths to Computer Science, in contrast with the particular (application) reason for studying Matrices.

Summary and discussion

The investigation has probed courses in the Block II quantitative subjects, which admit about 50,000 students a year, compared with half this number entering Block I (traditional STEM) courses, from the viewpoint that the purpose of A-level maths is to lay mathematical foundations for quantitative subjects. The investigation focussed on top-up maths courses in the three most popular Block II subjects at two different universities, seeking evidence as to whether A-level maths may need to be reformed and comparison of top-up content with A-level pure maths content showed some strong patterns that are relevant to this issue. Broad brush results from the comparisons are:

A. Two rather different universities provide similar top-ups for their corresponding Block II courses- validating the decision to probe just two universities;
B. In the largest Block II subjects- top-ups in Economics and Business and Finance are very similar, whereas Computer Science content is entirely different;
C. AS-level maths content overlaps the E-B-F top-ups only moderately well. The latter have less coordinate geometry and no trigonometry and instead emphasise matrices (absent from the whole of A-level). These top-ups are similar in size to AS-level;
D Computer Science top-up is entirely Discrete Maths with none of the A-level pure.

The relevance of these results for possible reform of A-level mathematics is now discussed. Evidently:

1. Universities generally ignore AS-level maths as an entry qualification- requiring GCSE or A-level, with only the highest ranking universities demanding A-level;
2. The foundational maths needs of courses are characteristic of the subject and independent of the university;
3. Block II subjects need maths foundations beyond GCSE, at least to AS-level standard (evidenced by the provision of a pure maths top-up module);
4. The current AS-level content does not actually satisfy the foundational needs of any of these subjects (but there was a divergence- CS needs all Discrete Maths, E-B-F's needs are more traditional and include Matrices);

Subject to a properly resourced and more detailed study- covering more Block II subjects at more universities- confirming these findings, it may be concluded that t*he current AS-level curriculum does not meet the needs of the majority of students taking quantitative courses and ought to be reformed accordingly.* A reformed AS-level maths might well be more attractive to potential Block II students, providing universities take it seriously as an entry requirement. The numbers entering Block II courses are very large, so this implies knock-on consequences for the capacity of post-16 maths provision.

However, A-level as a whole is a mandatory requirement for entry to the Block I (traditional STEM) courses and so reform of AS-level cannot be undertaken in isolation.

Consequently, how well the current A-level is matched to the foundational needs of Block I subjects would have to be studied first. Reform of A-level, so as to substantially satisfy the foundational maths needs of all quantitative subjects, leading to the clearly defined roles for AS and A2 maths as follows, would be an attractive outcome:

AS-level should meet, as far as possible, the needs of Block II subjects;
A2 should meet, as far as possible, the additional needs of Block I subjects.

Conclusion

Probing the provision of top-up maths in the three main Block II subjects at just two universities and comparing these with AS-level maths content has produced, very economically, some striking patterns of differences and similarities. These results warrant a more wide-ranging and detailed study and, if confirmed, there are implications for the reform of A-level mathematics.

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