Different countries, different classrooms: an attempt to characterize 'mathematical cultures'.

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As part of an attempt to track the influence of culture on mathematical activity Paul Ernest's two categories of 'traditionalist' and 'fallibilist' images of mathematics was used to analyse students' perceptions of mathematics. Perceptions were gauged from written answers to a questionnaire from groups of students in England, South Africa and Botswana; these were mapped to the characteristics identified by Ernest. To reflect the complexity of responses the various characteristics were split into groups and the students' views were mapped onto a number of grids. I suggest that the mapping enables some patterns to be identified as well as illustrating the variety of views within classrooms. While the responses indicate implications for my own teaching, improvements are needed to this first attempt so that these tools might provide a robust means to clarify the different 'mathematical cultures' that we each, whether student or teacher, adhere to and contend with.

Keywords: culture; images of mathematics

Introduction

Since working in a secondary school in Botswana some years ago, and reflecting on the value of my work there, I have puzzled over the difficulties of teaching mathematics in a culture different to ones own. As part of my study for an MA in Mathematics Education I took the opportunity to try to understand better the influences of culture on the mathematics curriculum. This report focuses on peoples' images of mathematics.

Theoretical framework

Paul Ernest identifies the emergence of a new philosophy of mathematics in the last 50 years and sees implications for mathematics education. He contrasts the traditional with the new, humanistic images, drawing out the potential impact on teaching and learning. A traditional 'absolutist' image views mathematics as 'an objective, absolute, certain and incorrigible body of knowledge', where, while new truths may be discovered, they are universal in nature and exist in some superhuman, ahistorical body of knowledge which is both value- and culture-free (Ernest 2009, 51-52). The trajectory of the historical development of mathematics confirms mathematics as fundamentally a European creation, from the achievement of the Greeks in first establishing 'real' mathematics (Ernest 2009, 60) through their development of theorems and proof, to the progression through the Renaissance (Hodgkin 2005, 12-13) and Enlightenment and after to an ever more powerful and abstract intellectual discipline, underlining the superior nature of European civilization, or eurocentrism (Hodgkin 2005, 12). Mathematics, due to its very nature being 'value-neutral ..... based solely on logic', is 'free of ethical, human and any other values' (Ernest, 2009, 57) and thus 'all discussion of the values of mathematics is delegitimized within mathematics' (Ernest 2009, 57).

In contrast the humanistic image views mathematics as 'human, corrigible, historical, and forever changing' (Ernest 2009, 52). In the light of issues in the foundations of mathematics, such as Gödel's incompleteness theorems, mathematics is seen as fallible with a consequent 'loss of certainty' (Kline 1980 in Ernest 2009, 52). Mathematics has developed in different societies, at different times in different ways; it is socially constructed. Concepts and methods have developed in response to puzzles and everyday problems; consequently mathematics should be taught in context, not reduced to largely algorithmic procedures. Critical and independent thinking are essential to this process as against the authoritarianism of the absolutist approach; thus mathematics education has democratic values embedded within it. Further, applications, models and problems relevant to and arising from the cultural contexts of the students are vital (Ernest 2009, 53).

Ernest identifies ten aspects of popular images of Mathematics and describes their critical characteristics for each of the traditional and humanistic images (2009, 47).

Data Collection

The focus of this part of the study was to compare the different perceptions of mathematics of pupils in three different countries.

A short questionnaire was devised with open questions to elicit peoples' opinions about various aspects of mathematics. Two questions regarding geometry were included as I thought this area in particular might show cultural differences. The questions were:

1. How would you define Mathematics?
2. What are the characteristics of a good mathematician?
3. What is your favourite piece of Mathematics? Why?
4. What reasons would you put forward for studying Mathematics at school?
5. How would you describe what Geometry consists of?
6. In your opinion what are the benefits of studying Geometry?

The questionnaires were given to the following four groups of students, being those accessible to me:

Group 1: England: 16 – 17 year olds at sixth form college; these 13 students had just completed their AS Mathematics course, including Statistics. Some were planning to drop mathematics, some to continue.

Group 2: England: 16 – 17 year olds at sixth form college; these 13 students had just completed their AS Mathematics course, including Mechanics. Again some were planning to drop mathematics, some to continue.

Group 3: South Africa: A group of seven students in Grade 11, their penultimate year, at a township school; around 17 years of age. This group are studying Mathematics rather than Mathematics Literacy; they voluntarily work together after school with the guidance of a Grade 12 student.

Group 4: Botswana: A group of 20 students from a rural boarding school; Form 4 and 5, ages 16 – 19 years. They take the equivalent of GCSE at the end of Form 5.

Data Analysis

I reviewed the answers on the questionnaire with the aim of plotting their position on a linear scale, from traditional absolutist to fallibilist humanistic, using Ernest's categorisations (2009, 47). As I analysed the responses it became evident that this was too simplistic; responses were more complex and I needed some form of multidimensional structure to cater for the various combinations of positions across the characteristics that people took. Restricting myself to two dimensional structures I grouped Ernest's characteristics into three subsets of associated factors under the headings methods/processes, application/aesthetic and affective. I also created a grid for content to capture students' views as to what objects mathematics worked on. The construction of the four grids is as follows:

Content: at the bottom of the triangle this starts from the concrete, number and shape, and moves up through levels of abstraction, seeing structure and pattern as mathematics' essence. I based this on the idea of moving through levels of representation, from the concrete through to iconic to symbolic (Bruner 1966, 44-45); at the highest level the student becomes aware of structure and pattern – 'the formal or abstract properties of the things he is dealing with' (Bruner 1966, 68).

Methods/processes: this compares the following of strict rules to creativity and flexibility in solving problems; the seeking of unique solutions using prescribed methods as against looking for alternative methods, multiple solutions.

Application/aesthetic: is mathematics seen as applicable to the everyday, useful in a variety of subjects and jobs, and/or highly descriptive of the world; or is mathematics irrelevant. Is mathematics seen as purely a tool, does it give insight as to how the mind works, is there a beauty and elegance about it.

Affective: how do individuals feel about mathematics, do they love it or hate it; is it something for everybody or is it just for the elite. Views on this were elicited in answer to questions 3 and 4.

I then plotted on each diagram where I felt the respondent's view lay. An example is given in Figure 1.

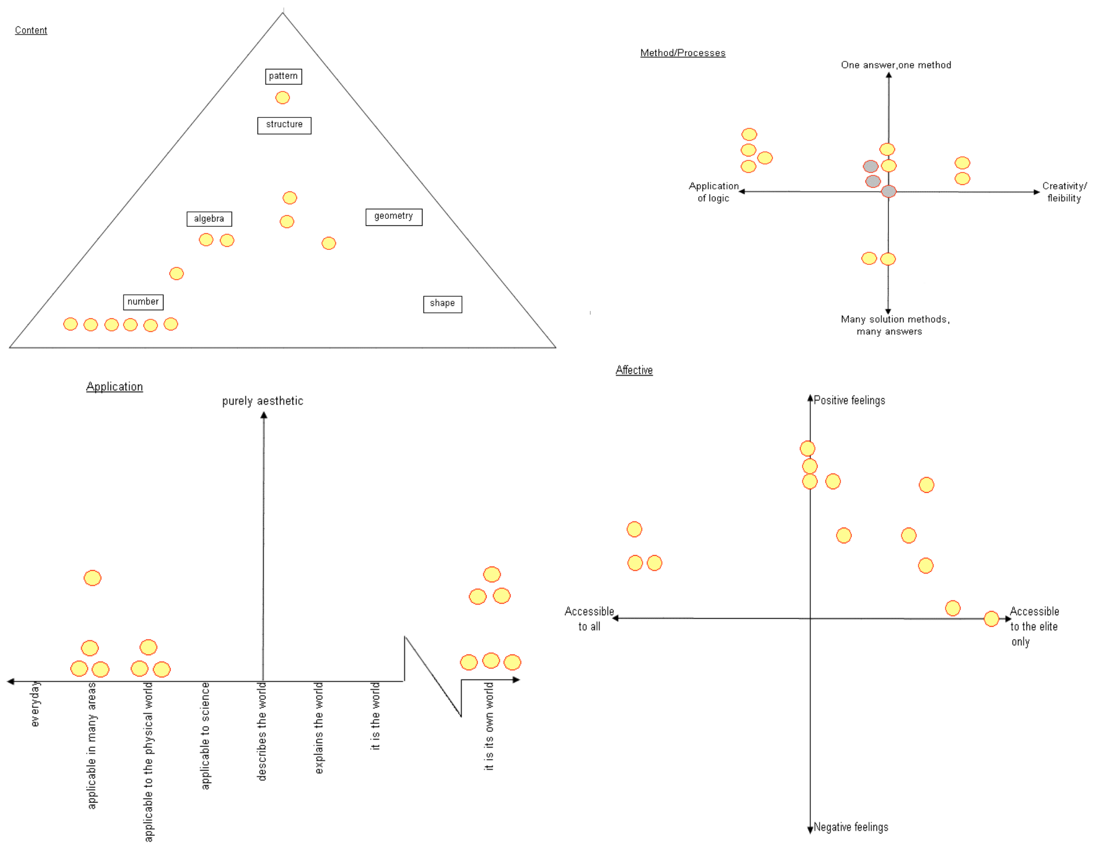

Figure 1. Grid for Group 2: England: 16 – 17 year olds at sixth form college

As an example of how these were plotted for the 'Content' grid, one student defined Mathematics simply as 'Numbers' and there was little other information about the content from the rest of the answers; so this was plotted bottom left. Another student answered 'Theories and patterns' to the question regarding 'your favourite piece of Mathematics' and so was plotted towards the top. Another student defined Mathematics as 'The theoretical system for resolving problems which include different amounts of things, either physical or theoretical' and specified their favourite piece of Mathematics as 'Geometry and shapes, because it is very physical and can imagine it easily'. I judge this to sit fairly centrally but towards and just below the Geometry label; the student recognised both number and shape and an element of abstraction but very much grounded in the physical.

As an illustration of the results across the four different groups the 'Content' grid is shown in Figures 2 and 3.

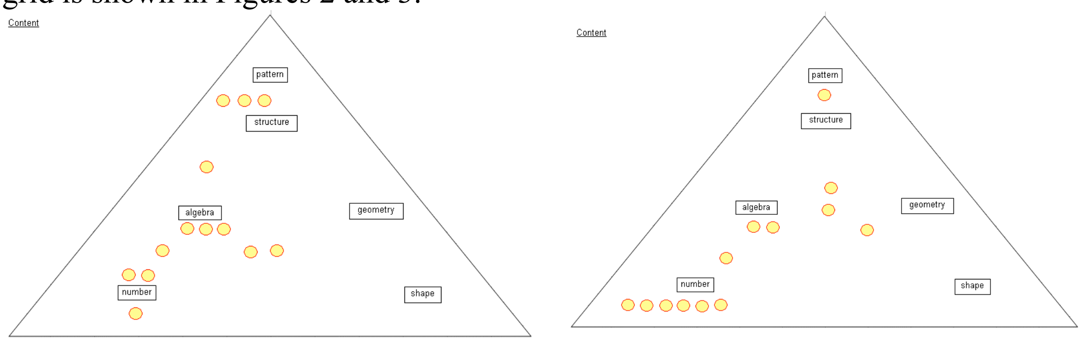

Figure 2. Group 1: England AS level and Group 2: England AS level

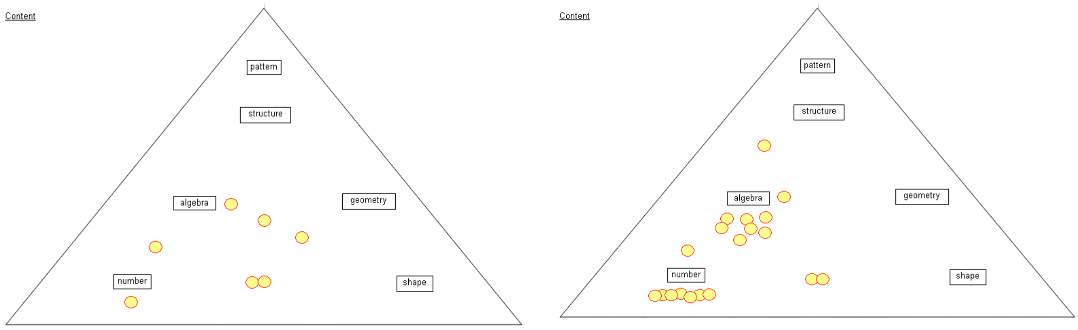

Figure 3. Group 3: South Africa Grade 11 and Group 4: Botswana Form 4 and 5.

Comparing the similarities across the groups, it appears that number is the most commonly associated concept; measurement a little more, but shape and geometry are not commonly picked out. Pattern, structure and relationships in all groups were occasionally but rarely noted. From the other grids, in all groups mathematics is commonly seen in utilitarian terms, necessary for other courses and for getting a good job; few saw it as a modelling tool for the world – though notably one student, from South Africa, described mathematics as providing the building blocks for science and technology. Across the groups some students saw other side-benefits of studying mathematics – such as increasing thinking capacity. In each group there tended to be a split between those who considered the subject hard and those who considered it accessible. In general students were positive about the subject though a few notable negative comments stood out.

The group from Botswana appeared to mainly see mathematics in a neutral mode – another subject that one needs to learn, in the same way one needs to learn English perhaps, in order to get on in the world. That may be because they had a year or two less schooling than the other groups and had not had a chance to make any choice about mathematics options. Both the groups in England saw characteristics such as being imaginative, able to consider a range of techniques, and looking for different ways to solve problems as typical of a good mathematician; this was not evident in the groups in South Africa and Botswana.

Students see mathematics as a means for helping in everyday life and for getting better qualifications and a good job.

Conclusions

- Across the countries
  - Students perceive mathematics at the concrete level rather than in terms of patterns and relationships
  - There is a strong focus on number
  - Mathematics is seen as necessary for other courses and for getting a good job
  - Some students see other benefits – e.g. increasing thinking capacity
  - Few see mathematics as a modelling tool
  - There is a split within each group between those who see the subject as hard and those who see it as accessible

- Botswana
  - Mathematics is seen as just one more subject
- England
  - Characteristics include imagination, range of techniques; not apparent in South Africa and Botswana
  - Students have a restricted comprehension of what comprises geometry and its application

Ernest argues that images of mathematics lead to different perceptions of how mathematics education is to be carried out. The questionnaires showed that within the same group students can have quite different perceptions of what mathematics is, what it can do, and have quite different experiences. Reflecting on my own practice I can see that my classes on the whole have a restricted view of the applicability of mathematics and lack an all-round appreciation of its characteristics both in terms of content and methods. Ernest's characteristics may help me direct my teaching even while I become more aware of constraints highlighted by other cultural factors. It also highlights that students are moving between different cultures, whether promulgated by different schools and systems – primary, secondary, sixth form college – or different teachers, and arrive in my class with a view of mathematics fashioned by various influences. Interestingly, all bar one of the South African students mentioned mathematics as advancing their career prospects, a significant contrast to previous government's planned aspirations through apartheid education.

The process is open to a number of criticisms such as interpretation of responses, my own consistency of judgement, variation in interpretation by others of both the criteria and responses and dependence on the English comprehension and writing skills of the respondent. I do feel, however, that it is worthwhile as a first attempt in trying to gain insight into peoples' views, with the further intention of reviewing the process and making good its weaknesses.

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