

What role can enrichment workshops play in student learning?

Martin Griffiths

University of Manchester

We consider here the use of a particular type of mathematics workshop in some schools, colleges and universities. This is a preliminary qualitative study, focusing largely on the factors that would appear to have an influence on the success, or otherwise, of such events. Several examples of these workshops are provided, and a number of the perceived educational benefits that arise as a result of running them, particularly the impact on student learning, are highlighted. We also discuss some of the issues that may be encountered. Finally, potential research questions in this regard are mooted.

Introduction

In this paper we provide an informal qualitative account of the use of *enrichment workshops* in several schools, colleges and universities in the UK. The mathematical content of the activities being considered here is thus appropriate for students in the 16–21 age range. Our preliminary findings are based on detailed observations of a small number of such sessions, post-workshop discussions with both students and facilitators, and indeed the author's own experiences of leading such events. At present, we claim to give no more than a snapshot of the national picture in this regard.

Mathematics workshops come in many forms, so it is important initially to describe carefully what is meant here by an 'enrichment workshop' in order to be able to contrast it with other varieties. The first point to make is that such events are, if at all, usually staged outside the normal timetabling schedule of the institution, and the mathematical content generally goes beyond the official curriculum. They may thus in some sense be regarded as extra-curricular activities. As a consequence, attendance of enrichment workshops is often on a voluntary basis. An exception to this might occur when, for example, the further-mathematics students at a particular school or college are taken off timetable for a morning in order to participate in such a session. The workshops are initiated via an intriguing yet accessible mathematical challenge, frequently involving some sort of practical demonstration in order to motivate the activity; specific examples of these tasks will be considered in due course. Once the scene has been set, there is very little instruction from the teacher, although a number of appropriate resources are provided, and discussions continue throughout the course of the session. The format of the workshop promotes interaction and the exchange of ideas and information amongst the participants. Furthermore, there is scope for generative learning to take place and for individual creativity. Indeed, although the starting point might be firmly grounded in the mathematics of the curriculum, the nature of the workshop provides students with

both the opportunity and the motive to explore more sophisticated ideas and to establish connections between hitherto seemingly unrelated areas of mathematics.

In what follows, it is necessary to make the distinction between enrichment workshops and other, possibly more common, types of workshop activity. Bowers (1994) describes *drop-in workshops*, which students may attend if they require help from a teacher or lecturer on a particular aspect of mathematics. In some institutions, one or more mathematics courses might be delivered solely through a *total-workshop* approach, whereby students are timetabled to attend a learning-resource centre staffed by general mathematics tutors. Neither is an enrichment workshop necessarily a form of *masterclass*. As the name would imply, a masterclass generally targets exceptionally able students. However, both the nature of the tasks chosen for, and the strong collaborative element of, enrichment workshops make provision for weaker students to participate. Furthermore, many masterclasses have a strong didactic element to them in addition to rather prescribed learning outcomes.

Our purpose here is manifold. First, from a practical point of view, we describe some examples of enrichment workshops, with the intention both of disseminating good practice and of highlighting potential pitfalls. Second, we consider the types of learning that take place and, as a consequence, the possible educational benefits to be accrued from running such activities, as well as the plethora of issues that can arise. A further goal is to promote dialogue in this regard amongst educators, teachers, lecturers and those charged with planning the curriculum. We go on to examine what it is that makes such workshops successful or otherwise, and give some thought to assessment. Finally, although there are many interesting questions that we may ask ourselves in connection with enrichment workshops, this does seem to be a largely-ignored area with regard to research. The possibility of engaging in further scholarly activity in this field is therefore considered.

Examples of enrichment workshops

In this section we describe one workshop in detail and offer brief summaries of several others. Some ran for a series of sessions while others were one-off activities. Attendance at the majority of these workshops was voluntary, so there was generally the necessity to recruit students. This was achieved by a combination of announcements in lectures and messages via electronic mail.

The first workshop, held at a university in the UK, was open to mathematics undergraduates who were studying at that university and had a particular interest in statistics. In all, 22 students attended the one-off session of approximately three hours. The problem concerned an application of continuous probability distributions, an aspect of statistics the students had encountered only superficially thus far on their course, and was framed in the context of a night-time hike; see (Griffiths, 2010a):

A group of friends are away camping for the weekend and decide to go for a walk after dark. It is so dark in fact that each of them brings a torch. They all turn their torches on at the start of the walk, head out to some point and then retrace their steps to get back to the tents. Some torches die on the way out, others die on the way back, while some are still shining at the end of the walk. A non-trivial and intriguing question is: How long should their walk be in order that the expected number of torches dying out on the return journey is maximised?

In order to afford further motivation, the above challenge was introduced via a short practical activity in which all those present participated. Scaled-down versions of the hike were recreated in the classroom, and computer simulations were utilised in order

to determine the points on the journey at which each the students' torches died out. The aims of this workshop, as presented to the students, were as follows:

- To nominate, with justification, a suitable probability model for the longevity of the torch.
- To select appropriate parameter values and hence calculate the length of the walk such that the expected number of torches dying out on the return journey is maximized.

To this end, the students were put into groups and provided with appropriate resources, including statistical software and internet access. The lecturer, although playing a minimal role with regard to determining the course of events, was always on hand in order both to provide constructive feedback and to facilitate discussion and collaboration.

The workshop culminated in a series of presentations by each of the groups, during which they explained their approach to the problem, justified the choices and assumptions that were made, and interpreted their analytic results. Students were encouraged to challenge each others' findings if they remained unconvinced by particular methods or arguments. A considerably more detailed account of this workshop appears in (Griffiths, 2010a).

Enrichment workshops also allow for the possibility of exploring some fixed mathematical theme. An example of this is given in Griffiths (2010b), where the students built up a whole series of relatively sophisticated results associated with the combinatorics of prime factorisations. Indeed, this theme-driven workshop proved to be a veritable cauldron of generative activity. Furthermore, although the workshop ran over several sessions, the levels of attendance were maintained throughout.

The same cannot be said, however, of a recent workshop concerning Dirichlet's theorem on primes in arithmetic progressions. It was overambitious certainly, but there were a number of additional factors that contributed to the falling numbers each week and to the gradual dilution of the original aims of the project. The salutary lessons learned here will be discussed in following sections.

Some of the more successful enrichment workshops that we have either observed or facilitated in schools have been those which, through the initial mathematical challenge and any subsequent practical activity or visual imagery, are able to reveal some beautiful underlying mathematical structures; indeed, Sinclair (2004) notes the important role played by the aesthetic in mathematical enquiry. A good example in this regard concerns the number of ways of distributing those present at the workshop amongst the various tables in the classroom. Under appropriate conditioning this leads to the fascinating mathematics associated with Stirling and Bell numbers.

The characteristics of an effective enrichment workshop

When coming to some sort of judgement over the success, or otherwise, of a particular workshop, it is important to appreciate that there are a wide variety of factors to take into account. Most facilitators agree that it is not simply a matter of deciding whether or not the initial aims, as presented to the students, have been achieved; indeed, many see these aims as merely providing some initial focus for the participants and, as such, deem them as playing a somewhat perfunctory role. In such flexible learning environments it is necessary to allow for different, though equally valid, outcomes and to consider carefully what counts as 'competence' in a particular setting; see (Gresalfi *et al.*, 2009). To some extent, the success of a workshop might

be judged both on the nature of the mathematical activity that ensues and on the longer-term impact it has on the participants. We discuss the first of these here, and the second, along with the potential issues that may arise, in the following section.

Although the workshops observed by us were diverse in nature, we came to recognise a common thread running through those that seemed to be particularly effective in terms of promoting student engagement, collaborative activity, generative learning (see Grabowski, 1997 and Wittrock, 1974a and 1974b), independence of thought and creativity. First, most were initiated by way of some immediately-intriguing yet accessible problem, often introduced through a practical activity or an interesting mathematical challenge. In fact, many of these problems had the attributes of so-called *rich tasks*; see (Griffiths, 2009). Not surprisingly perhaps, both the choice of material and the way the task was introduced proved to be absolutely crucial; indeed, it appeared that any student not engaged right from the very start tended often to drift aimlessly through the session.

Furthermore, it seemed important that the teacher or lecturer was prepared to relinquish some control over the mathematical activity that was taking place in the classroom. It is interesting that this is in direct conflict with the natural instinct of many in the teaching profession. Successful facilitators were each very aware of all that was happening around them, but only intervened perhaps to initiate a discussion amongst a group that were clearly floundering, to encourage or reign in, as appropriate, students that were looking to veer off at tangents or to provide some constructive feedback to those pursuing a promising line of enquiry. Indeed, the importance of sensitive feedback in such situations cannot be overestimated; see (Angelo and Cross, 1993). In taking a step back, the teacher gives the students a degree of autonomy over their learning, which in turn allows them to experience some sort of genuine ownership of the work they produce.

The resources available to the participants were also key factors in determining the success, or otherwise, of a particular workshop. It was found that, in addition to internet access, high-quality software and texts were, when appropriate, frequently able to play a motivational role.

Much of the above might be associated with what is often termed student-centred learning, which is an approach to education that focuses on the needs of students as opposed to the needs of teachers, managers and others in positions of power. Iiyoshi *et al.* (2005) state that such learning environments often place an unusual cognitive burden on the learner, and they go on to consider the use of cognitive tools in scaffolding the learning in these situations. Hockings (2009) has carried out an interesting study on the engagement and participation of students involved in student-centred learning from epistemological, sociological and approaches-to-learning perspectives.

Perceived long-term benefits and potential issues

Many students attending an enrichment workshop for the first time may never before have had the opportunity, or indeed the inclination, to engage with mathematics in this way. Interviews with such students revealed a number of common responses. The overwhelming majority felt that, however difficult things may have seemed at times, they were genuinely engaging in mathematics throughout the whole session; some said that they considered themselves to be more confident and rounded mathematicians as a result. A number claimed now to appreciate that the process of 'doing mathematics' is not simply a matter of going through a few exercises from a

textbook (this included, incidentally, three potential doctorate mathematicians). Some stated that they had been encouraged to undertake some background reading, whilst others enjoyed the collaborative element and claimed they could, as a consequence, see the need for members of the mathematical community to share ideas and engage in debate. It would appear that such activities also create opportunities for participants both to encounter aesthetic aspects of mathematics and to see the discipline from a more holistic point of view.

Of course, many will question the genuine effectiveness of a one-off session with a relatively small number of students, seeing it either as a token gesture or as some bolt-on activity sitting uneasily on the edge of the curriculum. There are a number of further issues that arise when organising these events. An initial problem is finding staff with sufficient interest and expertise to run the workshops. The delivery, choice of task and provision of appropriate resources is a difficult area, even for experienced teachers. Some lack the confidence to create their own resources, and the dearth of available sources of ideas may act as an additional deterrent. Indeed, if the work is not pitched at the correct level or is not rich enough in mathematical possibilities then it is unlikely that the workshop will be effective. Should the content be too demanding then these sessions can transform into what are essentially a series of lectures. Indeed, due to misjudgement of the material, this is exactly what occurred in the series of workshops on Dirichlet's theorem mentioned previously.

There are also the practicalities of student recruitment and timetabling to contend with. For a voluntary workshop comprising more than one session, there is the further issue of maintaining numbers. In other words, how may we attract students to the event in the first place, and then keep them coming week after week? This might raise some interesting questions associated with inclusion and identity. Will all students have equal access to the workshops or might some inadvertently be excluded from participating? What about those that are able to attend, only to find the freedom afforded by such workshops to be extremely daunting; how may we best integrate these students into the learning environment so that they become active participants?

Assessment also needs to be considered. As there are so many possible learning experiences and outcomes, this is certainly not a straightforward matter. Some argue that assessment is an unnecessary imposition on students who are undertaking a voluntary activity. However, this has to be balanced with the desire for all workshop participants to make progress mathematically. Thus, at the very least, it would seem that some use of formative assessment through individual questioning and discussion is, in order to be able to provide feedback, inevitable. There will, in any case, also be a degree of peer assessment taking place. Furthermore, any student that is engaging actively with the mathematics ought to be made to feel that they are achieving success. It is of course possible to include slightly more formal elements of assessment through presentations, whole-group discussions and written reports.

Concluding remarks

Our findings have both highlighted a range of educational benefits and raised a number of issues associated with the staging of enrichment workshops. Although our initial study did involve detailed observations and interviews, it was rather limited in scope. It would not, therefore, be sensible to attempt to extrapolate these findings in order to make any definitive statements about the national picture. A potential research question might be: What role *do* enrichment workshops play in student

learning? Any such investigation would involve a large-scale national study incorporating both qualitative and quantitative elements. Amongst many other things, we would wish to ascertain the prevalence of these events, estimate levels of participation, identify good practice and possibly establish some sort of practitioner network in this regard. Of course, one of the many difficulties with a study of this kind is the *ad hoc* nature of these workshops. From a philosophical viewpoint, we might contemplate the nature of any value judgments or ethical dilemmas that arise, while, from a more pragmatic angle, some consideration might be given as to whether or not there was any scope for embedding such workshops into the curriculum.

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