A study of the effectiveness of a Dynamic Geometry Program to support the learning of geometrical concepts of 2D shapes

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The study described in this paper aims to investigate 12-13 year old students' geometrical reasoning in a Dynamic Geometry environment in order to answer the question: "In what ways do the affordances of Dynamic Geometry Software act to mediate the learning of geometrical concepts of 2 dimensional shapes?" Two theories which shed light on how humans conceptualise geometric figures and how this sometimes leads to misconceptions will be described. Dynamic Geometry Software may hold some of the answers to the problems students have with conceptualising shapes in geometry, and support the development of geometrical reasoning and the correct use of mathematical language. The importance of designing a task which is seen by the students as having a purpose is also mentioned. The research project will be described and findings from early data will be discussed.

Keywords: Geometry, Dynamic Geometry Software, Dragging, Measuring

Introduction

The context of the study, being undertaken for a PhD, is the mathematics classroom in England where Geometry has a low profile in the Mathematics curriculum compared to Number and Algebra. In national tests of fourteen year olds the questions on the properties of 2D shapes require a low level of reasoning and yet are poorly answered by most students (QCDA, 2004). I hope, through a task devised for use with Dynamic Geometry Software (DGS), to observe how students reason about geometric shapes and whether the task can help to develop the students' geometrical reasoning. As Noss and Hoyles (1996) famously observed; computers provide a window through which we may observe children's learning and help us to see what we would not normally be able to see.

How does the human mind conceptualise geometrical figures?

At first glance geometry appears to be a practical area of mathematics but it is actually very abstract. Consider a 2D shape such as a kite. Researchers have suggested that problems in geometry arise because students find it hard to appreciate the difference between the actual figure (of a kite) on paper and the theoretical object that it represents (Battista 2007). In fact Love (1995) suggests that geometry is the only area of mathematics where the physical geometrical image is both the actual object and its representation.

The Figural Concept

Fischbein (1993) described students' mental images of 2D shapes in three ways. Firstly there is the image of a figure based on the **material representation** of it such as a diagram or a plastic tile. Secondly humans conceptualise shapes by their **definitions**. A definition is a minimum set of properties which describe the shape. In the example of a kite a common definition is that of two pairs of adjacent equal sides. Finally Fischbein described the **figural concept** which is the mental reality students manipulate when they reason or solve problems in geometry, the ideal perfect shape imagined in the mind and that a diagram merely represents. When students operate on a geometrical figure they are operating on its figural concept, the idealised shape, ignoring any other qualities its diagrammatic representation might have eg line thickness, colour and even orientation. By operating on a geometrical figure students are fusing the conceptual properties (definition) of the figure with its figural concept which is a highly intellectual activity using the axioms of geometry.
Students often have difficulty working with the figural concept because they are directly aware of the image which is based upon their experience of material representation: drawings, tiles, etc. Fischbein argues that the ability to work with figural concepts needs to be specifically developed.

Cognitive Apprehension

Another way of analysing how students conceptualise 2D shapes is described by Duval, (1995) which he called 'cognitive apprehension.' It has four aspects.

Perceptual apprehension is made up of the understanding of the properties of a figure with its visual aspects. Difficulties arise when students perceive that specific aspects of the drawing of the figure, such as the way the figure is orientated, are included in the properties of the figure.

Sequential apprehension is concerned with the understanding of the process used to construct the figure and this process is dependent on the tools used. Gomes and Vergnaud (2004) showed that students who constructed geometrical objects on paper, using pencil and compasses, used a different set of geometric properties than when they constructed them using DGS. Gomes and Vergnaud concluded that the learning and conceptualisation of geometry is different when using ruler and compasses and when using DGS.

Discursive apprehension relates to the way that students think about and describe the figure. **Operative apprehension** describes the way students may gain insight into a figure by operating on it in some way, for example, dividing the figure into smaller figures, performing a transformation on it, etc.

Duval goes on to say that the different types of apprehension overlap, that operative apprehension always overlaps with one of the others and that the different kinds of cognitive apprehension need to be taught separately to students. Finally he suggests that while it is obvious that computer geometry supports sequential apprehension it may also support the development of operative apprehension.

Dynamic Geometry Software (DGS) as a mediator for the figural concept

I believe that DGS may act as the mediator for the figural concept and as such it has potential for supporting students' learning of geometrical concepts. Using DGS can provide students with a means to understand the properties of geometrical figures. In ordinary school geometry, theoretical objects (figural concepts in Fischbein's terms) are mediated by their material representations on paper (Laborde, 1993). These representations are imperfect, for example, the lines have width in the drawing. However students are expected to ignore the imperfections and work on the drawings as if they were the idealised geometrical object. Laborde explains that the introduction of DGS enables us to redefine the distinction between the theoretical object and its material representation. There is now a figure on the screen and this figure is a new kind of mediator for the theoretical object. It is different from a paper drawing in that it is dynamic (can be dragged on the screen) and its behaviour when dragged is determined by the method used to construct it, that is the geometrical properties designed into its construction.

Mariotti (1995) extends this point by claiming that drawings act as mediators between concrete and theoretical objects. Screen images represent the external version of the figural concept. To construct an object in a dynamic geometry environment the conceptual and figural aspects must be made explicit in the construction process. In this way working in a dynamic geometry environment is useful to develop the correct interaction between the figural and conceptual aspects of geometrical reasoning. The internal logic of the geometrical figure becomes apparent when it is dragged since the geometrical relationships it has been defined by remain constant under dragging.

Hollebrands (2007) argues that computer environments can help to mediate students' understandings of geometrical transformations when they construct objects in DGS and operate on these objects. However, in order to benefit from working in the DGS environment the students need to actively reason about the abstract geometrical objects whilst they are interacting with their representations on the computer screen.

Students can use two important affordances of DGS: the drag mode and the measure facility, to move between practical and theoretical geometry

An important affordance of DGS is the dragging mode whereby objects on the screen can be manipulated whilst keeping all constructed properties constant. This helps students to make the link between geometrical concepts and geometric constructions and indicates that DGS has accomplished the link between geometry and the experimental field of geometric constructions (Laborde,1993).

The Measure menu is another affordance of DGS which allows students to measure objects such as lengths and angles. As the figure is dragged the measures continually update. The use of dragging together with measuring has been observed to enable students to move between the **spatio-graphical field**, the experimental or practical side of geometry where they felt most comfortable, and the **theoretical field**, the area of geometrical theory, concepts and understanding (Olivero and Robutti, 2007). This can help to support the students as they develop more sophisticated levels of geometrical reasoning.

Hollebrands (2007) observed students using the drag mode to investigate geometrical transformations of figures on the screen. She noted that they used the Measure facility with dragging to explore relationships, create and verify conjectures and check the correctness of constructions. Students can do this in a **reactive** way by dragging in a fairly random fashion in order to see what happens or in a **proactive** way when then they are able to predict the outcome of their actions before carrying them out. Encouraging students to use strategies that are more proactive may be achieved by asking students to explain and justify what happens on the computer screen in terms of geometrical properties. However Hollebrands found that the dragging mode is only useful for learning if students know and understand how to use it to investigate the variance and invariance in geometrical objects constructed in DGS.

Working in a DGS environment can help to develop students' use of mathematical language and terminology.

Mathematical language development can be seen as an important aspect of learning mathematics. Certain words or expressions in mathematics convey a complex web of ideas which form a mathematical concept (Lee, 2006). For example the word 'rectangle' , which describes a very simple figure, contains the ideas of 2D figure, 4 right angles, two pairs of equal and opposite parallel sides and so on. When children learn how to use the mathematics vocabulary and notation they become able to articulate their mathematical ideas effectively.

Working with computer software can mediate learning through the mathematical language and notational system that is designed into the program (Hollebrands, 2007). When the teacher and student interact while using DGS they both adopt the language of the software, a language they then use to communicate with each other. They can also communicate mathematical ideas through discussing the visual images on the screen

Jones (2000), in a study with thirteen year old students, observed how the use of their mathematical language developed while using DGS; from simply describing what was happening on the computer screen to being able to give explanations relating to the mathematical content using more mathematical terminology.

Designing tasks that enhance students' learning in a computer environment

In this research project I aim to design a task which will be effective in allowing me to observe how children reason geometrically. Therefore this task needs to be carefully considered. The computer offers ways of working that helps students to access approaches and solutions which would not be available to them using pencil and paper (Hoyles and Noss, 1992). However students will not necessarily appreciate the intended mathematical ideas just because they are interacting in a particular computer environment. Tasks need to be designed with the pedagogical principles built into them.

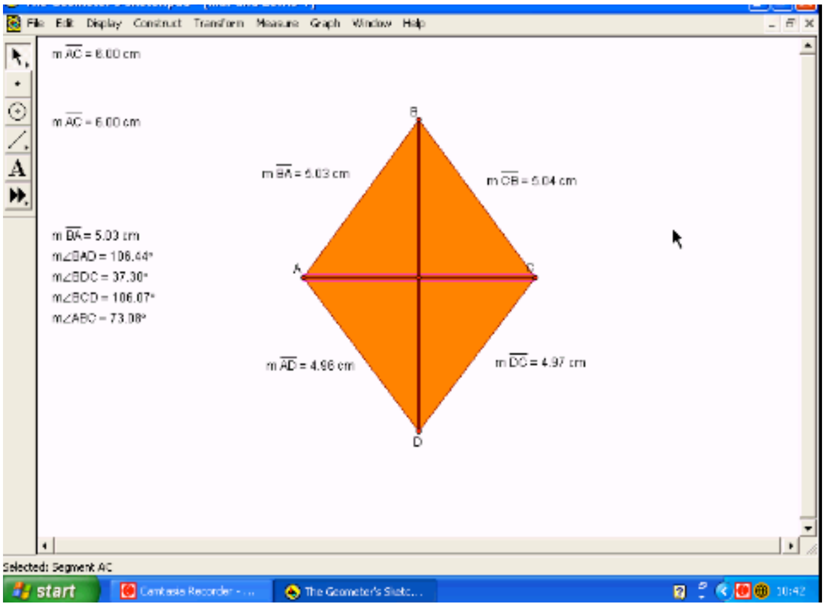
Ainley, Pratt and Hansen (2006) and De Villiers (1994) argue that is necessary to address both the mathematical learning and the motivation issues when designing a task. The task needs to have a purpose and it is this purpose which helps the students to appreciate the utility of the mathematics they need to learn.

The research experiment

The research experiment, which is in the early stages, is based around the design of tasks to support geometrical reasoning in the properties of 2D shapes. The student subjects are 13 year olds working in pairs with one computer which has been loaded with the Geometers Sketchpad (GSP) (Jackiw, 2001), in which files have been created for the tasks. The students take turn to control the computer mouse and are encouraged to work together and discuss the activity.

The students are given files containing two perpendicular bars and are told to drag one bar over the other and then join the ends of the bars to make a shape. They investigate by dragging the bars to generate different triangles and quadrilaterals. The figure below shows a screen shot taken from one of the screen recordings during the first part of the task. The students have dragged the bars to make a shape very close to a rhombus.

Figure 1.



The data which has been collected so far indicates that the students use dragging and Measures in the way that Hollebrands describes. That is they begin by using reactive dragging to investigate what shapes they can make when they drag the bars. Later in the task, with experience they use proactive dragging to drag the bars straight into the position for a specified shape. They also use a third way of dragging with Measures which I have called **dragging to adjust measures** when the students make very small adjustments of the bars whilst attempting to make various angle and line measures equal, depending on the properties of the shape which has been made. It is never usually possible to get measures exact but, as in diagram 1, the students have been able to make the measures quite close and are happy to make something which is "pretty much a rhombus" to quote the thirteen year old creators of the figure. In order to make this figure which, is close to a rhombus, the students proactively dragged the bars until they looked as if they intersected at the mid-points and then dragged to adjust measures to get the lengths of the sides as congruent as possible. As can be seen, the particular pair of students who made this rhombus placed the measures next to the sides to make it easier for them to attend to the correct length measures. Not all students do this however.

Data collection and analysis

The data is in the form of a recording of the computer screen activity using a version of Image Capture Software which records activity on the computer screen with an audio recording of the conversation. The activity on the computer screen and the dialogue will be coded in order to ascertain themes which may indicate how the students are creating meaning from their activities.

Whilst observing how the students conceptualise the shapes they generate I also aim to develop the task in order to encourage the students to build a classification of shapes which could lead to their understanding of the hierarchy of quadrilaterals. Using the Van Hiele levels (Van Hiele, 1986) I hope to assess whether the task can lead to the students gaining a more sophisticated level of geometrical reasoning.

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