The emergence of rhythmic patterns as a way of embodied mathematical knowing

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This qualitative study is part of a 2-year longitudinal research focused on theorizing the role of the body in elementary students' mathematical learning and understanding. More specifically, we are interested in exploring how children's transactions with geometric objects give rise to ways of embodied knowing. Drawing on an exemplary episode in which a group of 3rd-graders were classifying three-dimensional objects, our purpose is to illustrate a way of embodiment not addressed in previous investigations, namely "embodied rhythm". A rhythmic choreography emerged during the transaction between one of the students with a cream carton. The rhythm of that choreography was especially clear in the beats the student performed on the object's faces. Other two embodied dimensions produced the same rhythmic pattern: object orientation and body position. Based on a detailed micro-analysis, embodied rhythm is conceptualized as a way of mathematical knowing, as well as a means of objectification of mathematical understanding.

Embodied Mathematics, Geometry, Rhythm, Elementary Students, Ethnography

Introduction

Nowadays, there is a high consensus among educational and psychology researchers, mathematics educators and mathematicians that geometry and spatial visualization deserve a prominent role in school mathematics (e.g., Lehrer and Chazan, 1998). In contrast to traditional approaches in which these topics were pushed into the very background, today their importance is understood to be absolutely central. Curriculum writers from many countries have also acknowledged the relevance of geometry within reform recommendations. Certainly, during the last two decades, a number of educational teaching organizations have suggested that geometric and spatial reasoning can and should be integrated with other mathematical topics at all levels of K-12 curriculum, starting from the primary levels (e.g., see *Curriculum focal points for prekindergarten through grade 8 mathematics*, by National Council of Teachers of Mathematics, 2006).

Our research project is focused on elementary level students. According to the above-cited curriculum, geometry instruction in Grades K-4 needs to be built on children's informal knowledge of their environment. Thus, strong emphasis is placed on the development of abilities such as a) identification of geometric ideas in mundane everyday world, b) description, comparison and classification of shapes according to their geometric properties, and c) analysis, measurement and investigation of bi- and tri-dimensional figures. The episode analyzed in this study occurred during a task concerned with the creation of classification systems.

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Over the last two decades, a considerable number of authors have acknowledged the relevance of the human body in mathematical cognition (e.g., Lakoff and Núñez 2000; Roth 2009). Rather than conceiving mathematics as a corpus of abstract, transcendental and decontextualized ideas, these authors have characterized mathematics as an essentially embodied phenomenon. This is the theoretical perspective adopted in this study. We think that mathematics is a concrete, mundane and contextualized socio-cultural practice, absolutely dependent of our physical/sensual experience. Our body is, from our theoretical viewpoint, the location of mathematics (Thom, Roth and Bautista 2010). Indeed, we think that the body is not a structure through which we learn mathematics, but a structure that learns mathematics. In consequence, students' bodily actions cannot be simply interpreted as external "demonstrations" of their internal (mental) mathematical understanding. Rather, we consider bodily action *as* understanding.

The present study is part of a 2-year longitudinal research project focused on theorizing the role of the body in elementary students' geometry. Our project was designed to explore, analyze and theorize the growth of children's ways of mathematical learning and understanding as it emerges and evolves from their bodily transactions with physical objects. More specifically, our general aim is to investigate the emergence and development of bodily embodied ways of geometric knowing during 2nd and 3rd grades (Maheux, Thom and Roth 2009; Roth and Thom 2009a, 2009b; Thom et al. 2010).

The purpose of this study is to describe a way of embodiment not addressed in previous investigations, namely "embodied rhythm" (see Radford, Bardini and Sabena 2006). Humans are rhythmic beings in nature. Research has shown that babies are born genetically prepared to perceive, process, and respond to rhythmic stimuli. On the basis of these innate competences, a broad range of increasingly sophisticated rhythmic abilities are developed in the course of our life span. Thus, we think that rhythm might be postulated as a further way of embodied cognition. Based on that working hypothesis, we decided to set two specific research aims for this study: 1) to investigate whether embodied rhythmic patterns actually emerge in children's transactions with geometric objects; and 2) in the case of identifying said patterns, to describe in detail which dimension(s) of students' bodily action produce and/or give rise to the phenomenon of embodied rhythm in geometry. To the best of our knowledge, researchers interested in the embodiment of mathematics have not addressed this phenomenon yet.

Method

The participants of the study were three students: Owen (grade 3), Nadia and Elisha (grades 2 and 3, respectively). Their school, located near the University of Victoria (British Columbia, Canada), serves a socioeconomically and ethnically diverse population of students. When the data was gathered, the three children met the provincially set curricular expectations. None of them had recognized special educational needs.

The second author of this report (JST), in collaboration with a research assistant and the students' teachers, conducted a three-week three-lesson/week study of these three children. The lessons consisted of sets of activities specially planned to assist the students in expanding their knowledge and understanding about three-dimensional figures. Besides of triggering forms of verbal interaction among the students, the activities were aimed at eliciting as many forms of physical transaction as possible. For this reason, most of the activities were manipulative in nature (i.e., hands-on activities). JST acted as the teacher in all nine lessons. The entire units were videotaped using two digital cameras.

In the lesson analyzed here (second lesson of the curriculum), the children were asked to sort a set of objects. It was composed of both conventional geometric figures (e.g., cubes, cones, pyramids, different types of prisms) and everyday objects (e.g., cream carton, binder clip, dies). By taking turns, their tasks were 1) to classify the figures according to their similarities/differences, either adding them to existing groups or creating new groups; 2) to explain and justify their reason/s; and 3) to discuss their decisions and reach a consensus with the rest of their partners.

The exemplary episode analyzed below exhibits features that were repeatedly observed throughout the whole database. It is important to emphasize that we found many similar examples that could have been chosen instead as they presented similar characteristics.

Analysis of an exemplary episode

This episode occurred during the 8th turn of the classification task. It is Owen's turn. He is to choose an object and place it with one of the existing collections, or on the empty sheet that the teacher has just placed on the table. At that time, there were 20 objects distributed in 12 collections. Owen decides to sort a cream carton composed of eleven sides: "Square" (base), "Four big oblongs" (main body), and "Two triangular shapes", "Two medium oblongs" and "Two small oblongs" (top part).

01 Teacher: Who's next? Owen, your turn.

02 Owen: Milk carton.

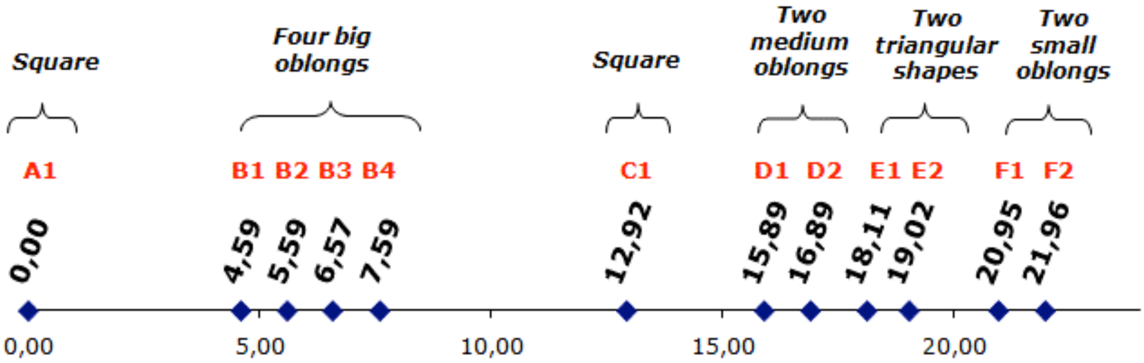
03 Teacher: Okay? Cream carton? Okay. Well, tell us where you think that one belongs.

04 Owen: All right.

The transaction we are going to describe here lasted about 30 seconds only. From the very moment in which Owen picks up the object and approaches it to his body, they both initiate a dialectical relationship. Their relationship starts with a detailed sensuous exploration of the cream carton's constitutive geometrical elements. Owen orients his gaze to certain components of the figure "key" to understanding it a geometric entity (i.e., base, sides, edges). In contrast, he does not seem to pay attention to other non-geometric properties (e.g., color, side), as he does not pronounce a single utterance regarding any of these properties. The symmetrical manner in which Owen holds and manipulates the object, putting his hands at the same height of parallel sides, can be interpreted as a further semiotic means of objectification of his geometric understanding. He is transacting with a 3D object that has symmetrical properties, and vice versa.

After his very first visual/physical exploration, Owen and the object perform together a beautiful "choreography". The rhythm of that choreography is especially clear in the beats that Owen performs on the cream carton's faces. Apart from the "Square" which was beaten twice (see A1 and C1), the rest of sides were beaten only once. Diagram 1 shows on a temporal line the distance existing between all these beats. Time is measured in a fraction of a second.

Diagram 1.- Temporal distribution of the beats performed on the object's sides.



As the diagram illustrates, the beats performed on sides belonging to different categories were --in all the cases-- temporally more separated than those belonging to the same category. Besides, these beats performed on sides of the same category were quite evenly distributed, being separated by one second approximately. On the basis of these observations, we argue that Owen is using the variable time to produce "organization" in his transaction with the object, as well as to establish embodied distinctions among the sides belonging to different categories. We do not understand this use and production of rhythm as a conscious and deliberate phenomenon, but rather as embodied and unconscious.

During Owen's transaction with the object, the same rhythmic pattern can be observed in other two embodied dimensions: a) the shape and movement of his body when producing the beats (e.g., fingers, hands, arms); and b) the orientation of the object in relation to the body. Indeed, corporal (body) and material (object) distinctions are clearly established while the student manipulates the object and performs the beats on its sides. These distinctions are summarized in Table 1. The corporal and material distinctions described in the table "produce" the same rhythmic pattern presented in Diagram 1. Rhythm allows Owen to bodily enact differences according to the constitutive properties of the object (faces). He uses rhythm to create and produce organization in his embodied action, as well as to objectify (i.e., make available to Nadia, Elisha, and the teacher) his mathematical understanding of the figure.

Conclusions

Numerous instances of embodied rhythmic patterns have been identified throughout our whole database. In particular, these patterns tend to emerge when students are performing actions such as manipulating and exploring the geometric objects, counting the objects' number of sides, or comparing the objects with one another. Thus, in response to the first aim of this study, we conclude that embodied rhythmic patterns do emerge frequently during elementary students' transaction with geometric objects. In other words, children's geometry has been found to be characterized by the emergence of the phenomenon of embodied rhythm. Human production, reproduction and communication of mathematical activity, therefore, can potentially emerge and unfold according to rhythmically organized patterns.

Table 1.- Summary of the corporal/material distinctions established during Owen's transaction with the cream carton.

Time	CATEGORY	Body *Shape and movement*	Object *Orientation*
00:00	A) "Square"	Beaten with the palm of left hand (flat hand, fingers together). The shape of the hand is flat and its movement is parallel to the base.	Parallel to his chest.
04:59	B) "Four big oblongs"	First beat with right hand, and the rest with left hand. In all of them, fingers, hand and arm form a straight line. Their movement is almost parallel to the object.	Forming an angle of approx. 45º with Owen's chest (except for the fourth beat, when the orientation is almost perpendicular).
12:92	C) "Square"	Beaten with four fingers of left hand (all except for the thumb). The shape of the hand is flat and its movement is parallel to the base.	Orthogonal to his chest.
15:89	D) "Two medium oblongs"	Beaten with the tip of his fingers, first right hand and then left hand.	Sitting on the table vertically.
18:11	E) "Two triangular shapes"	The first one is beaten with the tip of his right index finger, and the second one with the tips of three fingers of this left hand (all except for his thumb and little finger).	Forming an angle of approx. 30º with Owen's chest.
20:95	F) "Two small oblongs"	Beaten with the very tip of his left index finger.	On his chest, totally next to his body.

To achieve our second aim, we have provided an exemplary episode that clearly illustrates how embodied rhythm can emerge in children's geometric transactions. The excerpt demonstrates how mathematical understanding can be realized *in* and *through* the physical/sensual body, and simultaneously be socially communicated. The episode shows the first seconds of Owen's transaction with a cream carton. A dialectical relationship between subject and object is established from the very moment in which Owen picks up the cream carton and brings it to his body. As proof of his mathematical understanding, he displays a number of bodily embodied ways of geometric knowing. He grabs the cream carton, holds and moves it, performs different gestures on it, beats its faces, counts its sides, etc. During his physical/sensuous transaction, a rhythmic pattern clearly emerges from the beats that Owen performs on the cream carton's faces. That rhythm is not only available to us in an audible manner, but also in a visible manner. The rhythmic pattern was identified in three embodied dimensions of the student's action: 1) Temporal distribution of the beats performed on the sides of the object; 2) Shape and movement of the students' body as he beats the object's faces; and 3) Orientation of the object with respect to the student's body. In response to our second aim, therefore, we conclude that embodied rhythm can emerge in different embodied dimensions of students' bodily mathematical action.

Our evidence suggests that embodied rhythm plays a double function in students' geometric transactions. First, it functions as a semiotic means of objectification of mathematical understanding (Radford et al. 2006), allowing the students to make socially available their own "interpretations" or "performances" of the geometric figures. Second, embodied rhythm functions as an embodied way of mathematical knowing itself (Roth and Thom 2009b). Our data show that rhythm allows elementary students to produce organization in their bodily action, as well as to establish distinctions among the objects' faces. Thus, we think that the emergence of rhythm cannot be simply understood as an external "demonstration" of students' internal (mental) mathematical understanding. Further, we think that embodied rhythm also needs to be conceptualized as understanding itself. In a nutshell, we suggest that embodied rhythm constitutes both a *resource* and an *outcome* of geometric activity. From our theoretical perspective, this double function reflects that rhythm is expression of and exists in our own *flesh* as human beings, which for us is the location of mathematics (Thom et al. 2010).

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References

Lakoff, G., and R. Núñez. 2000. *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Lehrer, R., and D. Chazan, eds. 1998. *Designing learning environments for developing understanding of geometry and space*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.

Maheux, J. -F., J. S. Thom, and W. -M. Roth. 2009. What Makes a Cube a Cube? Contingency in Abstract, Concrete, Cultural and Bodily Mathematical Knowings. In *Mathematical representation at the interface of body and culture*, edited by W.-M. Roth. Charlotte, North Carolina: IAP.

National Council of Teachers of Mathematics. 2006. *Curriculum focal points for prekindergarten through grade 8 mathematics: a quest for coherence*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Radford, L., C. Bardini, and C. Sabena. 2006. Rhythm and the grasping of the general. In Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, edited by J. Novotná, H. Moraová, M. Krátká and N. Stehlíková. Prague: PME.

Roth, W.-M., ed. 2009. *Mathematical Representation at the Interface of Body and Culture*. Charlotte, NC: IAP, inc.

Roth, W.-M., and J. S. Thom. 2009a. The Emergence of 3D Geometry From Children's (Teacher-Guided) Classification Tasks. *The Journal of the Learning Sciences* 18:45-99.

Roth, W.-M., and J. S. Thom. 2009b. Bodily experience and mathematical conceptions: from classical views to a phenomenological reconceptualization. *Educational Studies in Mathematics* 70:175-189.

Thom, J. S., W.-M. Roth, and A. Bautista. 2010. In the Flesh: Living, Growing Conceptual Domains in a Geometry Lesson. *Complicity: An International Journal of Complexity and Education* 7 (1):77-87.