Prospective elementary teachers' beliefs about problem solving: A comparison of Cypriot and English undergraduates at the commencement of their courses

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The research reported in this paper draws on semi-structured interviews conducted with first-year undergraduate teacher education students, in the first weeks of their course at one university in Cyprus and one in England. The interviews, focused on students' conceptions of mathematical problems and problem solving yielded substantial, culturally-located variation in students' responses highlighting continuing inconsistencies in the operationalisation of this key concept around the world. Some implications for teacher education and further research in the problem solving field are discussed.

Introduction

Teachers' beliefs have been the subject of extensive research, based on the assumption that what teachers believe is a significant determinant of what gets taught, how it gets taught and what gets learned in the classroom (Middleton 1999; Wilson and Cooney 2002). According to Aguirre and Speer (2000, 327), "being able to identify and describe the mechanisms underlying the influence of beliefs on instructional interactions would deepen and enrich our understanding of the teaching process". Older and recent studies highlight the importance of examining, analysing and changing teachers' beliefs in order to implement successfully mathematics curricula reforms (Ernest 1989; Handal and Herrington 2003), as without a challenge to teachers' underlying beliefs, teachers may exploit new resources or modify practice inappropriately (Handal and Herrington 2003). Reform classrooms are characterised by an emphasis on problem solving (Peterson et al. 1989; Cady et al. 2007) and yet many international comparisons have fallen into the trap of assuming that things with the same name must have the same function in every culture (Grant, 2000). Our aim here is to address some cultural similarities and differences of prospective elementary teachers' beliefs about mathematical problem solving in Cyprus and England.

Teachers' beliefs

Beliefs have been defined as "conceptions, personal ideologies, world views and values that shape practice and orient knowledge" (Aguirre and Speer 2000, 328). Moreover, recent beliefs-related research, continuing to draw on Ernest's (1989) triadic model, has focused primarily on how teachers think about the nature of mathematics, its teaching and its learning. In addition, drawing upon Bandura's early work in the field (Bandura 1977), research has highlighted the role of teacher self efficacy in general (Wolters and Daugherty 2007) and mathematics teaching self-efficacy in particular (Charalambous et al 2008). Thus, we argue, Ernest's original model is appropriately augmented by such additional dimensions.

Research has shown that the relation between beliefs and instructional practices is complex and cannot be described simply in terms of cause-and-effect. For example, while a number of studies have highlighted substantial disparities between espoused and enacted beliefs (Thompson 1984; Beswick 2005; Raymond 1997), others have indicated that both beliefs and actions are contingent on the changing nature of the classroom context (Schoenfeld 2000; Skott 2001). Thus, acknowledging this problem, we have examined the mathematics-related beliefs of beginning undergraduate primary teacher education students to determine the extent to which they reflect similar, culturally embedded, perspectives to their peers. In this respect, results from comparative studies of serving teachers' mathematics related beliefs (Andrews and Hatch, 2000; Santagata 2004; Correa et al. 2008) and practices (Givvin et al. 2005; Andrews 2007a) indicate that culture plays a key determinant role in both their formation and manifestation.

Curricula reforms, teachers' beliefs and problem solving

A number of papers have focused on the mathematics instruction-related problem-solving beliefs of prospective teachers, including those of Verschaffel et al. (1997) in Flanders and Timmerman (2004) in the USA. In many such studies, prospective teachers' cultural location remained, as a significant influencing variable, essentially unacknowledged. Moreover, a collective definition of *problem-solving-oriented instruction* has been assumed and, it is our contention, although this paper is not the place for a lengthy elaboration, that this assumption has little basis. For instance, as a consequence of the role of the National Council of Teachers of Mathematics (NCTM) in the framing of reform curricula in the US, much problem solving research has been undertaken in that country. The results of these studies have influenced curriculum development in many countries. Nevertheless, variation in definition and implementation can be seen, for example, in a 2007 special edition of Zentralblatt für Didaktik der Mathematik (ZDM) on problem solving around the world that includes articles highlighting the role of problem solving in the curricula of Israel, France, Italy, the UK, the Netherlands, Portugal, Germany, Hungary, China, Australia, Singapore, Japan, Brazil, Mexico, and the US respectively. In other words, despite US influence in the field, both problem solving as an activity and problem solving research continue to mean different things in different countries (Torner et al. 2007), to the extent that problem solving, according to where and by whom the term is used, can mean a goal, a process, a basic skill, a mode of inquiry, a form of mathematical thinking, and a teaching approach (Chapman 1997).

While some single-national studies of teachers' problem solving beliefs and practices have been undertaken in, for example, Australia (Anderson et al. 2004), few cross-national studies have been undertaken in this area. From the perspective of serving teachers' beliefs about mathematics, Andrews (2007b) concludes that English teachers tended to view mathematics as applicable number and the means by which learners are prepared for a world beyond school, while Hungarian teachers perceived mathematics as *problem solving* and independent of a world beyond school. Such findings seem to confirm that the *teachers' perspective* is a neglected dimension in comparative studies of problem solving.

Mathematical problems

Despite the notion of mathematical problem having been used differently by different scholars, not all research in the field shows a lack of consensus. There is much agreement as to the nature of a problem. One key characteristic is that a problem lies with the person seeking the solution and not the problem itself. As Schoenfeld (1985, 74) notes, "being a 'problem' is not a property inherent in a mathematical task.

Rather, it is a particular relationship between the individual and the task that makes the task a problem for that person”. That is, a problem for one person may not be for another (Borasi, 1986; Nesher et al. 2003). Such insights, and our interpretation of the work of these scholars, are helpful in framing our study, not least because they allude to three key criteria for defining the relationship between problem and problem solver. Firstly, problem solvers must have encountered a block and see no immediate and obvious way forward. Secondly, they must actively explore a variety of plausible approaches to the problem. Thirdly, they must accept that the search for a solution necessitates an engagement with the problem. With regards to the context of problems, Blum and Niss (1991) distinguish between the problems embedded “in some mathematical universe” (p. 38) and those related to real world. In other words, problems are seen to be either purely mathematical or applied. These perspectives frame the study we report below.

Method

As stated above, in this paper we report on the problem and problem solving beliefs of prospective elementary teachers from Cyprus and England. Participants were in the first weeks of an undergraduate teacher preparation programme at a one reputable, as measured by systemic measures of teacher education accountability, university in each country. Data were collected by means of semi-structured interviews at the beginning of the academic year 2008-2009. The interview questions regarded students’ definitions of mathematical problems and problem solving, their beliefs about pedagogy (teaching/learning), and their self-efficacy beliefs both as problem solvers and as potential teachers of problem solving. The comparative dimension is important in helping to identify and understand the ways in which culturally located experiences inform the construction of students’ beliefs.

The Cypriot cohort comprised thirteen students (twelve female, one male), while the English comprised fourteen (thirteen female, one male). At the time of the interviews participants had received no problem solving-related university instruction. Therefore, they were seen as products of the school rather than university systems of their countries. Analyses were focused on students' *meaning* (see Kvale and Brinkmann 2009) and drew on both theory-driven and data-driven approaches (Boyatzis 1998; Kvale and Brinkmann 2009). In this paper, due to space limitations, we report on three themes identified by the data-driven analyses. These are students' perspectives on the nature of mathematical problems, the nature of mathematical problem solving, and the characteristics of effective problem solvers. The results are presented alphabetically by nationality, Cyprus then England.

Cypriot students’ perspectives on mathematical problem

Eleven students indicated that mathematical problems, usually embedded in text, should be clearly presented with adequate information and data so that solvers can easily attempt a solution. Panayiota’s comments were typical of others. She said that a mathematical problem comprises “mathematics related sentences, which include information, data and a desired outcome. We have to think about the data, to process them and get the answer”. Eight students commented on the significance of difficulty in defining a problem. For some as reflected in Sofia's comment, a problem by definition “has difficulty and unknown factors within it”. Others indicated that notions of difficulty lay, essentially, with the problem solver and not of itself, the problem. Such a perspective could be seen in Demetra's comment that “problems and their difficulty are connected to certain age groups”, while Haroula added that “the criterion is school, if it is primary, gymnasium or lyceum. In primary school, problems are very easy, in gymnasium they are more complicated, and in lyceum you can find the hardest”. All thirteen students implied that mathematical problems are contextualised within a real-world framework. Christina's comment was not atypical. She said,

> at the first grade, problems were like “I have two apples, my grandmother gave me two more, how many do I have now?” Later on, at the sixth grade, problems were more complex, let’s say something about how many square metres of a wall surface could someone paint with so many litres of paint. In gymnasium, they might be something like “how much it cost to paint a surface”, which had to do with area and volume. In lyceum, they were more or less the same.

Cypriot students’ perspectives on mathematical problem solving (MPS)

Eleven students indicated, either directly or indirectly, that MPS is a process. Of the four who used the word process explicitly, Pantelis' comments were typical. He said that “mathematical problem solving is a process, the process towards what we are asked to find. It is the process during which you use the given data in order to find the answer to a problem”. Of the others, Demetra's comment was typical. She said that “mathematical problem solving is the use of the data in order to find what you are asked to”. A recurrent theme in these students' responses was the need to read the problem repeatedly. Panayiota’s comment was typical. She said,

> You have to read the problem two-three times, underline some key points, because you know, sometimes problems have unnecessary things in them, you have to find what is important, then start processing all these in your mind, read two-three more times, write down your data and what you want to find, do a shape if it’s needed and then do the algorithms.

Cypriot students' perspectives on what makes a good problem solver

Eight students suggested, along the lines of Martha's comments, that good problem solvers “have the skills for organising the given information quickly. They tidy up the data, the questions. Those who are not good don’t structure their work”. Sofia presented a typical response in respect of distinguishing the expert from the novice. She said that

> there is a big difference. Someone who is a good solver, as soon as he sees the problem, he has a clear picture in his mind about what has to be done, directly. Someone who is less good will have difficulties in finding which way to follow for solving the problem.

Several students added that problem solving requires concentration, as seen in Demetra's comments that “solvers who concentrate when they encounter a mathematical problem perceive what has to be done quickly and manage to resolve it”.

Eight students also suggested that, with practice, problem solving competence can be acquired. For example, Angeliki commented that a novice “could spend more time on practice... to develop his mathematical thinking, learn about different types of problems, and develop a clearer idea around mathematical problems”. However, the remaining five students indicated that being a good problem solver was natural. As Panayiota noted, it all “depends on the individual, biologically, I think some people are born with it; it’s their talent, either you have it or not”.

English students' perspectives on mathematical problems

In respect of their conception of mathematical problems, the English students presented a range of perspectives. Common to ten was an explicit invocation of number operations, as seen in Victoria's comment, that a problem was "anything, from adding, dividing, subtracting, timesing, or arrange them and then put together". However, the context in which they described their perspectives varied. For four students problems were essentially mathematical in nature, as seen in Daniel's slightly recollection of his school experiences. He said

> Pythagoras' theorem, if you have the length of the hypotenuse and you need to work out,.. I can't remember how it was. Like we've got sin, cosine and tangent and you need to work out the other two. Like you've got one of the angles and you need to work out another angle or length. That's one which sticks in my head.

Two students indicated that mathematical problems were related to the real world and every-day life, as in Laura's comment that they had to do "with money, we did how, if apples cost 10p each, how much money do I need to buy six apples? Which is 60. Six times 10p". However, the majority of the group, seven students, implied that problems could be construed as either mathematical or real world. Melanie's comment reflected those of others. She said that a "mathematical problem could mean lots of things. It can be a standard two plus two on a piece of paper or how much money I need to go for shopping. (…) Something that uses numbers to come up with 'a' answer, or a series of answers".

English students' perspectives on mathematical problem solving (MPS)

Eleven students indicated that MPS is a structured process during which solvers apply prior knowledge in a structured step-by-step approach. Julia's response, typical of others. Suggested that MPS was "just basing what skills you know on trying to solve a problem in maths, so just applying the knowledge to structure it and work step by step to work out a problem", while for Laura it meant having to "take it step by step and apply things you already know". For some this step-by-step approach meant breaking down a problem into smaller pieces-tasks, working on each piece separately, and finally putting all the pieces together. Rachel, reflecting comments of others, said that "I break it down..., and then do a bit a time and then at the end I put them all together. I do that with most mathematical problems". All students saw MPS as drawing on prior knowledge. Interestingly, Melanie, was the only student who used the word process explicitly. She commented on the

> the processes that you use to solve a problem. So, the way you think, the way you work out a problem, whether you need a resource to do it or whether you do it in your head. What steps you take to come to your conclusion. You need to understand the problem, what you've been asked to find out, cause lots of problems are in a context where you have to pull out the information you need, you need to understand the processes, you need to follow out the procedure and then you need to understand your answer.

English students' perspectives on good problem solvers

Thirteen students suggested that expert problem solvers can see through problems and apply the necessary mathematical knowledge and strategies quickly and efficiently. For example, Laura indicated that "an expert knows how to answer straight away, whereas someone who is not so good does think a long time about it and mainly have one option, whereas an expert might have lots of different ways to think about it".

Expert solvers, she added, “already have the knowledge to work out what you need to do to solve the problems, whereas, otherwise they have to think what steps you need to take to get there”. Daniel summarised the nature of expertise thus, “I would say an expert is kind, they already have the formulas in their head so they can just work it out mentally”.

The same thirteen students indicated that a prerequisite of expertise was practice. Lucy's comments were typical. She said

> I think is just practice than anything else... if you’re learning a language, you have to practice it, don’t you? To learn it. So, like if they have different, if they have theories, like an example of how to solve a problem, and if they practice and do it over and over again, like just memorise it and you know how to use that theory then I think that would get better.

Discussion

Space limits the extent of our discussion, although a number of important outcomes have emerged that merit comment. Despite some within-country differences, the major variation lay between countries. Firstly, all Cypriot students saw mathematical problems as tasks related to real world, while their English colleagues fell into three camps; those of the same opinion, those who saw mathematical problems as purely mathematical and a majority who acknowledged that problems could be purely mathematical or related to the real world, along the lines of Blum and Niss (1991). Nevertheless, the Cypriot students tended to talk in general, almost abstract, terms while the English in particularities. For example, in defining a problem many Cypriot students focused on the generic characteristics of a problem while the English tended to offer examples of problem types from which properties could be inferred by the reader.

Similar issues emerged with respect to the nature of problem solving. Both groups of students attended, in some way, to process. The interesting difference lay in the sense that Cypriot students tended to view the process holistically – read, understand, collect data, analyse data and so on – while the English saw the process as one of simplification or reduction of the task to a series of small steps. Such differences are unlikely to be coincidental. Inevitably they will reflect the teaching these students received prior to going to university. Indeed, the English perspectives on the process of simplification find resonance with an earlier study of English and French curricular traditions (Jennings and Dunne 1996). Importantly, from the perspective of future teacher education programmes, these students are not mathematics majors but prospective generalist primary teachers. They are people who, one day, will teach young children mathematics. If problem solving is a key element of that country's curriculum, then English universities clearly need to understand the beliefs their undergraduates bring to their studies. The problems would appear less severe, at least as far as beliefs are concerned, for the Cypriot authorities.

In terms of their beliefs about the characteristics of effective problem solvers there was evidence of genuine similarity across the two groups, with substantial proportions comparing the approaches of effective problem solvers – the ability to see straight through the problem to a solution strategy with no apparent difficulty - with those of novice or inefficient problem solvers. Moreover, there was also a consensus that such skills could be acquired through appropriate practice. Lastly, such findings highlight the plea made earlier that those involved in research on problem solving, at all levels, need to acknowledge not only the lack of definitional consensus but also the key role played by culture in the construction of participants' construal of mathematical problem, problem solving and the characteristics of effective problem solvers.

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