Conceptualising the mediation of mathematics in classrooms as textured narratives

Geoff Wake and Birgit Pepin

University of Manchester, UK & Høgskolen i Sør-Trøndelag, Norway

This paper builds on a framework that conceptualises mathematics teachers as narrators developing narratives that interweave two important strands that we identify as being focused on the mathematical and social. It is these narratives that we consider to mediate the mathematics for students in classrooms that in turn we consider as activity systems and analyse using Cultural Historical Activity Theory. We draw on case study data collected in the ethnographic tradition in colleges as part of a project funded as part of the ESRC Teaching Learning Research Programme (TLRP) to consider how implicit in such narratives are socially emergent and shared understandings of what constitutes mathematics itself and what it might mean to be a mathematician in different settings. Giving a texture to teachers' narratives, therefore, we identify factors relating to shared epistemologies and didactical contracts (Brousseau, 1997) that we find crucially important in defining what it means to study mathematics as a discipline.

Keywords: Pedagogy, Narrative; Cultural Historical Activity Theory

Introduction

The UK's Economic and Social Research Council funded research project, 'Keeping open the door to mathematically demanding courses in Further and Higher Education' involved both case study research investigating classroom cultures and pedagogic practices and individual students' narratives of identity together with quantitative analysis of measures of value added to learning outcomes in an attempt to investigate the effectiveness of two different programmes of AS mathematics for post-16 students. Here we focus on the classroom experiences of students drawing on data that was collected in the ethnographic tradition with video and audio recordings, photographs and researcher notes together with follow-up interviews with students both in small groups and individually and pre- and post- lesson interviews with the teachers involved.

In doing so we aim to problematise the often held assumption that there is only 'one story' to be told in mathematics teaching, and in mathematics classrooms. Instead, we propose 'teaching as narrating' as a way of participating in an activity system (e.g. Engström, 1999) such as a classroom. In this paper, we address a specific aspect of this system - the means by which teachers engage students with, and mediate mathematics through, their 'mathematical story'. We develop a framework to explore how teachers have their own particular 'stories' to tell, their particular ways of facilitating student participation, and consequently a particular process of 'negotiating' (or directing) what it means to 'do mathematics'. However, we also suggest that these individual narrations reflect aspects relating to epistemologies and didactical contracts that have historically evolved and are culturally bound and situated.

Perspectives/theoretical framework

In our analysis of classroom pedagogic practices in an attempt to understand the social construction of mathematics that we observed, we turned to the construct of "narrative", in the sense of Ricouer (1984), and as developed in educational settings by Bruner (1996) and others. We, therefore, conceptualise the teacher as "narrator" revealing a mathematical plot whilst drawing on a range of pedagogic practices in an attempt to engage his or her audience in different ways. We suggest that teachers in operationalising their actions therefore develop different narrative constructions that can be considered as an interweaving of two different strands:

- a mathematical strand, which is often in effect the development of a mathematical argument, and
- a social strand which comprises of social activities (which arise from the teacher's choice of pedagogic practices) and social discourse. (see fig.1).

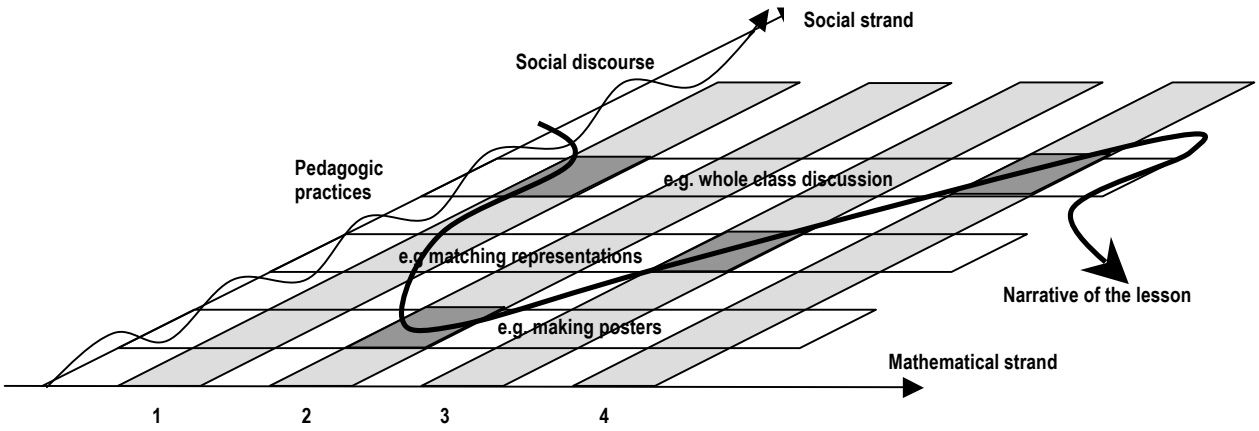

Figure 1: Schema illustrating two dimensional framework used to analyse the narrative of mathematics lessons

We build upon this two dimensional framework drawing on Cultural Historical Activity Theory to identify key factors that we suggest mediate teachers' developing narratives giving additional texture in ways that implicitly define what it means to study mathematics in a particular setting, and indeed to define what mathematics "is" in this setting.

We conceptualise, therefore, the classroom interactions as nested within an evolving systems network, in which teacher and students are mutually constituted through the course of their interactions. The notion of close relationship between social processes and developing knowledge draws on the work of post-Vygotskian Cultural Historical Activity Theory (CHAT) (see e.g. Cole (1995), Engeström (1999) etc.), and it is fundamental to Lave and Wenger's (1991) social practice theory which emphasises the notions of "community of practice" and collective knowledge that may emerge within the spaces people share and within which they participate.

Figure 2 provides a useful conceptual schema for identifying the factors that mediate the actions of a community in relation to the object of its activity (in the case of the AS mathematics classroom taken to be the learning of mathematics). The top triangle draws attention to the "instruments" such as texts, notes, examination questions, teacher exposition, and so on that mediate the individual student's learning.

The lower part of the diagram considers the classroom community more widely. Here, how teacher and student roles are constituted giving rise to a division of labour of the learning community and "rules' both implicit and explicit are taken into account. We find, for example, that rules pertaining to examinations and students' performance in these can drive teachers' actions in classrooms and result in offering particular cultural models of what it means to "do mathematics". For example, we note teachers aligning themselves with students in a joint battle with the examinations as teachers talk of "they" in relation to those who set the assessment papers.

We will return to attempt to understand how these particular factors are powerful in helping shape a teacher's narratives after describing the basis of our narrative framework in a little more detail with reference to an example lesson from one of our case studies. The particular lesson to which we refer here should not be considered as typical, as it certainly reflects the individual style of the particular teacher / narrator who is central. However, as you will see it does illustrate aspects of what we might consider a normative script (Wierzbicka, 1999) of lessons at A Level.

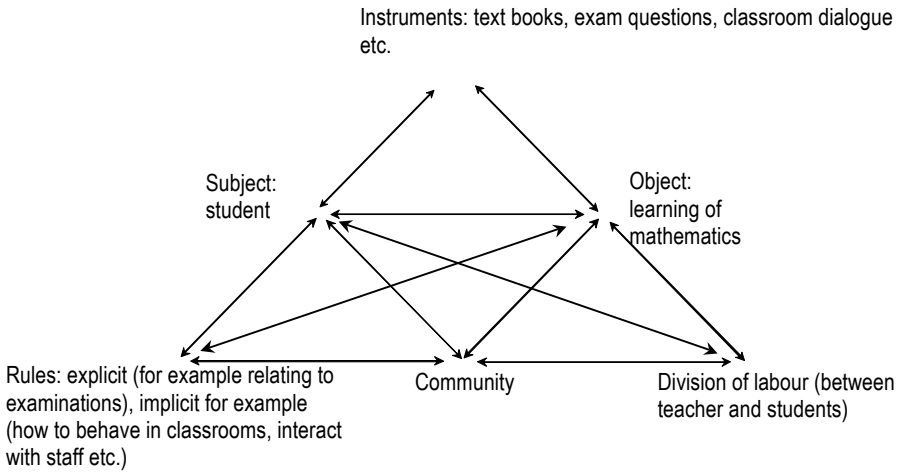

Figure 2: Cultural-historical activity theory schema

The narratives of a lesson

This lesson is in the first term of the GCE AS "Pure" course in a college in the North of England: it focuses on applications of differentiation. In the introductory phase of the lesson the teacher initially drew attention to differentiation as the abstract idea of rate of change of y with respect to x referring to the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$. To illustrate that this might be applied to a "real" situation and that different variables other than x and y might be involved the teacher suggests that at issue might be the rate of change of velocity with respect to time, and went on to ask if anyone knows the "special name" for this particular rate of change as the only question in the first ten minutes of the lesson.

Perhaps the transmission style the teacher employed is encapsulated in his statement at this point of the lesson that, "we just need a couple of definitions before we can move on to what I wanted to look at in detail today."

He drew a non-specific/general curved line and emphasised that the gradient at a specific point is given by the differential of the function introducing appropriate notation $\mathrm{f}(x)$ and $\mathrm{f}'(x)$. At this point he introduced the "new stuff" – the average gradient, or gradient of the chord, between two points (A and B) on the curve, although he did suggest that this idea had been met by the group when differentiation

was first introduced. Here he re-emphasised that the gradient at a point is found using differentiation, $\frac{\mathrm{d}y}{\mathrm{d}x}$, and average gradient between two points is found using the gradient of a chord, and he introduced the notation $\frac{dy}{dx}$.

Following this the lesson moved to a second phase in which the teacher modelled how to answer a problem of a form that the students would practise in the final stage of the lesson. However, at this point of the lesson the teacher introduced a 'social' strand of narrative that from this point interweaves, more or less closely, with the mathematical narrative. This revolved around an imagined world in which the teacher developed a problem situation based on the worms in his garden: as you will see this is not a 'real' context but perhaps is 'realisable' as in the RME approach (see for example, Van den Heuvel-Panhuizen (2001)). As this extract of the transcript of the lesson demonstrates this strand of the teacher's narrative with which he expects the students to engage is not insubstantial.

> "So I went into my garden, true story this, and I started digging up some worms. Alright? So I took my fork and I dug up lots of worms and they were all of different sizes so that's quite interesting in the first place. So I thought, well, I wonder if there's any relation between the age of these worms and their length so I collected as many worms as I had time for. For the visual learners amongst you, here's one of them. This is, in fact, it's Japanese. Could be German. Who knows? Ok, this is one of the worms I collected. So I collected till I'd got enough, a decent sample size, right? And I measured these worms, how long they were. I then asked them how old they were. They were quite co-operative. And I plotted how long the worms were at particular ages and, to my surprise, and remember you're not making notes, this is background, to my surprise, when I plotted the age of the worm to its length, all the points roughly lay on what looks to me like a quadratic so, of course, as you yourselves, I got quite excited at that and I thought, well, if I could find the equation of that quadratic, I'm quids in, yeah? I could predict the length of worms at different ages that I didn't have so I got very excited. I also noticed that when the worm wasn't born its age was zero so that was spot on, that fits nicely, so I do know one point that lies on this potential quadratic, quadratic with a negative coefficient of the squared term."

With brief reference to techniques that students had met previously of fitting a quadratic curve to model data such as this the teacher went on to introduce the equation $l = 8t - \frac{1}{2}t^2$ that he claimed to have found for a curve that fits his imaginary data of worm length, l millimeters, at time, t years. The first part of the problem that he set was to find the rate of growth of these worms in the first year of their lives which he immediately translated this applied problem for the students into the more abstract mathematical form of having to calculate "delta l" by "delta t". He proceeded to demonstrate how to find the average gradient by firstly finding l when $t = 1$and then proceeding to calculate the increase in l $\left(7\frac{1}{2} - 0\right)$ divided by the increase in t (ie 1). After brief comment by the teacher that a rate of growth of $7\frac{1}{2}$millimetres per year was, "Quite a lot really" the teacher repeated the procedure carrying out all of the calculations at each stage to find the average rate of growth during the fourth year. In conclusion of this phase of the lesson the teacher "discussed", by asking questions that he answered, the validity of the answers he had found so far:

> "Has the result surprised you or not? 7.5 millimeters per year in the first year, 4.5 millimeters per year in year 3 to year 4. Does that make sense that a worm grows

> really quickly at first and then starts slowing down its growth rate? That seems sensible to me, I think we do the same. Obviously, I'm still growing but…ok."

In what may, due to the shift in the mathematics involved, be considered a third phase of the lesson the teacher posed the question,

> "What is the rate of growth after 3 years? Not, "what is the average rate of growth?" Exactly, on the worm's third birthday - at that instant, what is its rate of growth?"

Again in this phase the teacher modelled how to find an answer by differentiating the function $l = 8t - \frac{1}{2}(t^2)$ and substituting $t = 3$ to give a rate of change of 5 millimetres per year. Again the teacher asked the class to consider the likely validity of this answer by comparing it with the average rates of change he had found for the first and third years of the worm's life.

In a fourth phase of the lesson the teacher posed the question, "How many years before the worm is fully grown?" After suggesting that the students should think about this in terms of the rate of change of the length of the worm a student made the second intervention of the lesson suggesting that this is at a stationary point. Re-interpreting this, the teacher pointed out,

> "In other words, the gradient is zero. When the worm is now fully grown, it's no longer growing so the rate of change of the length with respect to time is zero."

He proceeded to demonstrate that in this case $t = 8$, and once again considered this in the light of the context of the situation, re-introducing a social element of narrative:

> "Now, I hope there aren't any biologists here who are going to tell me that worms don't live to 8 years old. They do in my garden, they wouldn't lie to me. I asked them and they said they were honest about their age. So 8 years is when they stop growing. They're not dead. They just stop growing."

Due to space restrictions we leave the lesson here at a point just before the lesson entered a penultimate phase in which students practised answering questions with a similar form to the example considered by the teacher (without the worms!).

Analysis and further theoretical development

In this particular lesson the teacher demonstrates the use of the two distinct strands of mathematical and "social" narrative and interweaves these: at times using the 'social' narrative to motivate, and at other times ensuring it intersects relatively closely with the mathematics in such a way that engagement with the social requires engagement with the mathematical. For example, consider how the teacher's social and mathematical narratives are closely aligned as he discusses how to find when a worm is fully grown with students thinking about growth mathematically (considering the maximum point of the quadratic function) and socially (the teacher emphasises that in this context the function would suggest that the worms would be shrinking and that "we can't have that for worms".) We suggest that the stories the teacher spins about his garden worms require more attention than just focusing on the 'social': the mathematical is at times intricately interwoven with this. Equally sharing the worm story is also a central element of this teacher's dual goal (engagement with, as well as learning, mathematics) directed action: he wants his students to connect to the worm story in order to learn about the mathematics as well as providing a memorable 'event'.

As may be apparent from this extract here this particular lesson had relatively little variation in pedagogic practice, with the teacher in the main choosing to use a predominant transmission style for long stretches with the only interruption being a substantial period in which students practised the techniques that the teacher had modelled. This resulted in a long period of passive activity for the students followed by a period in which they were more actively engaged but on the whole working individually: the result was little or no sociability throughout the lesson. To explore the students' experiences further a CHAT analysis can be captured by the schema of fig.2. This draws attention to key mediational issues regarding informal rules such as those that have developed to determine how students are expected to be passively engaged (!) and the resulting division of labour.

In our CHAT analyses of different lessons, therefore, we note that different teachers operationalise their actions by delivering their own unique interweaving of mathematical and social strands in their narratives. On the one hand the mathematical strand is driven by the mathematical argument that the teacher wants to present and reflects the way in which the teacher understands how mathematical ideas and processes familiar to his or her students may be (re-) introduced and interconnected to develop new (to the students) mathematics. Whilst on the other hand, the social strand contains references to 'why' as the teacher draws on a range of experiences, practices and discourse with which he or she attempts to motivate and engage his or her students in learning.

For any individual teacher their uniquely constructed narratives are dependent to a large extent on their knowledge (content knowledge, pedagogic content knowledge, and knowledge of students), beliefs and the environment (school, department, programme) in which they operate. We suggest that the social system in which teachers operate, therefore, gives an overall feel or *texture* that whilst all-pervasive is perhaps consequently less visible than the narrative strands we have discussed thus far. So far we have not referred explicitly to this but merely hinted at how aspects of teachers' narratives give a flavour of the world in which they are situated. CHAT draws our attention particularly to the implicit and explicit rules of the college / classroom activity system that help develop this texture and we suggest that these combine and are pervasive across all aspects of the teacher's narrative. Consequently this texture, we suggest, encapsulates the teacher's operationalisation of cultural expectations which we consider as normative and particular to different educational phases in relation to studying mathematics. Particularly important in this regard we draw attention to the key elements of cultural expectations in relation to phase specific epistemologies and didactical contracts. To unwrap this a little we suggest that in the A Level mathematics classrooms that we explored there is a normative epistemology that persists across individual teachers and which reflects curriculum specifications and the way these are mediated by texts and assessment. This in turn pervades teacher's narratives and builds lesson by lesson, week by week and term by term to enculturate students into what it means to be a mathematics practitioner in this particular setting. For example, we note the highly procedural nature of student's working that stems from text books that "bite sizes" mathematics into manageable rules and procedures that we suggest inhibits deep understanding. In our observations of student working the procedural approach was stark but perhaps not surprising if students' long term contact with mathematical narratives reflects the type of transmissionist and disconnected teaching that our project found.

We therefore suggest that teacher / student classroom interactions can be analysed using a frame that considers mathematical and social strands of narrative that is textured by normative expectations regarding teaching, learning and values. Our

own use of such narrative analysis of classroom pedagogy in combination with CHAT suggests the mathematics education community and policy makers should widen the debate to include further deliberation about how a range of mediating factors interact to determine the mathematical experiences of learners.

As our project demonstrates teacher's narratives are immensely powerful in shaping learners' identities in relation to mathematics and consequently their likelihood of continued engagement with the subject.

Acknowledgements This research was funded by the ESRC Teaching and Learning Programme under the theme of Widening participation in HE (RES 139-25-0241). We would like to thank them for their support.

References

Brousseau, G. 1997. *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Cole, M. 1995. 'Socio-cultural-historical psychology, some general remarks and a proposal for a new kind of cultural-genetic methodology', In *Sociocultural Studies of Mind*, ed. J.,V. Wertsch, P. del Rio and A. Alvarez, 187–214. Cambridge, Massachusetts: Cambridge University Press.

Bruner, J. 1996. *The Culture of Education*. Cambridge Massachussets: Harvard University Press.

Engström, Y. 1999. Innovative learning in work teams: analyzing cycles of knowledge creation in practice. In Y. Engström, R.Miettinen & R.Punamäki (eds), *Perspectives on activity theory* (pp.377-404). Cambridge, UK: Cambridge University Press.

Lave, J. & Wenger, E. 1991. *Situated learning: legitimate peripheral participation*. New York: Cambridge University Press.

Leont'ev, A.N. 1981. 'Primaeval Collective Hunt', in *Problems of the 'Development of*
Mind, 210–213. Moscow: Progress Publishers.

Ricoeur, P. 1984. *Time and Narrative, vol.1* Chicago: University of Chicago Press.

Van den Heuvel-Panhuizen, M. 2001. Realistic Mathematics Education as Work in Progress, in Common Sense in Mathematics Education. In *Proceedings of The Netherlands and Taiwan Conference on Mathematics Education*, ed. Lin F.L. Taipei, Taiwan.

Wierzbicka, A. 1999. German "cultural scripts": Public signs as a key to social attitudes and cultural values. *Discourse and Society* 9(2): 241-282.