Saving Further Mathematics?

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The crisis in science, technology, engineering and mathematics (*the STEM crisis*) has come to the fore in recent years, but problems in mathematics education have persisted for many years. Although there have been many reviews and resulting initiatives none of the attempts to solve the problem has been successful in the long term. This paper reports on research carried out for Mathematics in Education and Industry (MEI) on the impact of the Further Mathematics Network (FMN), which was formed in 2004 to attempt to halt and reverse the large decline in the numbers of students taking further mathematics at A-level. The origin of the FMN in the light of the *STEM* crisis and *Curriculum* 2000 is discussed, as is an analysis of the statistics on the subsequent growth in student numbers. This analysis highlighted two types of schools and colleges; those where numbers had grown substantially in recent years, and those where the institution was able to offer further mathematics despite a small take up. Interviews were conducted with the teacher responsible for Key Stage 5 Mathematics in both these types of institution, and reasons sought for why numbers had grown or how courses could be offered "in house" to a small cohort of students. Factors that emerged included the reputation of the department and institution within its locality, the changes made in 2004 to mathematics specifications by the awarding bodies, flexibility in option block and timetabling arrangements, recognition by students that a qualification in further mathematics is a valuable career asset and support for the students and enthusiasm for mathematics from the teaching staff and the Network. These findings are discussed further in the light of the recent consultation on the future of level 3 Mathematics by QCA.

Introduction and background

There have been concerns for many years about the number of young people coming through the education system who are both suitably qualified and sufficiently motivated to study the subject beyond age 16 and into mathematics related higher education courses and careers. In the government enquiry into the decline in university entrants in science and technology, (Dainton, 1968) the report noted the importance of mathematics to a modern economy. Cockcroft (1982, paragraph 619*)* noted the shortage of good teachers of mathematics but that all pupils should be taught by teachers who are well qualified and enthusiastic. Despite several subsequent initiatives to engage and enthuse young people with mathematics a joint report by the London Mathematical Society, The Royal Statistical Society and the Institute of Mathematics and its Applications (1995) highlighted the persistent problem.

> There is an unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates. There is also a long-standing worry about the number of prospective students in these disciplines.

Smith (2004, paragraph 0.3) in his inquiry into post 14 mathematics education, cites the Roberts Report (2002) that during the 1990s there had been a near 10% decline in the number of candidates taking A-level mathematics to about 60,700 in 1999. Smith highlighted the importance of mathematics both as an intellectual discipline in its own right, but also as underpinning much of the scientific and industrial research essential to a modern economy. Smith identified a perception by young people that mathematics is boring and irrelevant, noting this and many other factors were influential in the decline in numbers. These factors included the perceived poor quality of the teaching and learning experience, the perceived relative difficulty of the subject, the failure of the curriculum to excite interest and provide appropriate motivation and the lack of awareness of the importance of mathematical skills for future career options and advancement.

Porkess (2006, 2-5) in his review of recent changes in the structure and provision of A-level mathematics quoted more startling figures, with numbers taking A-level mathematics falling from about 85000 in 1989 to 66000 in 1993 and then to an all time low of about 50000 in 2002. Porkess highlighted a similar drop in numbers taking further mathematics A-level from about 15000 in 1980 to around 5000 in 2005. In further mathematics the perceived difficulty of the subject by 16 year olds was compounded by a shortage of teachers able to teach it and the disinclination of head teachers and college principals to staff it as a timetabled curriculum subject due to relatively small take up by students. In many schools and colleges further mathematics disappeared from the curriculum. The major revision to all A-level subjects in *Curriculum* 2000, accentuated the problem for further mathematics as part of the philosophy behind *Curriculum* 2000 was a broadening element to a sixth form student's curriculum, whereas to take both mathematics and further mathematics at A-level was seen by many to be narrowing.

In this climate of a declining number of students taking further mathemtics, in 2000 the curriculum development organsiation, Mathematics in Education and Industry (MEI) initiated its *Enabling Access to to Further Mathematics Project*. The aim of the Project was to make further mathematics available to all sixth form students who could benefit from studying it irrespective of whether their school or college offered it in its curriculum. The Project also aimed to raise the general awareness of further mathematics as an AS-level and A-level subject amongst sixth form students as many might have been unaware of its existance let alone the benefits it had to offer to any student planning to study a mathematics related course at university. The Project began with four centres in England, where students could meet with each other and a personal tutor to discuss their progress in self-supported study using text books and web based resources provided by MEI. These four centres would evolve to become the 46 centres of the Further Mathematics Network (FMN).

The Smith Report (2004, paragraph 4.38) was instrumental in bringing about the FMN.

> Following the revision of the GCE criteria for Mathematics following the Curriculum 2000 debacle, many respondents are in no doubt that A-level mathematics has been made easier for the very best candidates. In terms of the potentially most able mathematics students, the Inquiry believes that far too few able candidates are entered for AS or A-level Further Mathematics because their school or college do not have sufficient resources to provide these courses …. University departments in all subjects identified as vulnerable in the Roberts report would benefit greatly if more candidates were qualified at this level.

The response by the DfES (2004, 43) initiated the formation of the FMN.

> To encourage the increased take up of Further Mathematics we will also develop proposals to replicate and expand the current MEI Project with a view to establishing a Further Mathematics Centre in each of the 47 local Learning Skills Council areas.

The setting up of the FMN by MEI can be seen as part of the response to what had become to be called the *STEM* crisis, where similar concerns to those in mathematics had been raised in science, technology and engineering as well. The *STEM* crisis and the responses to it were highlighted in the DfES/DTI report (2006), and again by Sainsbury (2007) in his review.

The Centre for Evaluation and Monitoring at Durham University was commissioned by MEI, to monitor the development of the FMN and to evaluate its impact. Part of the evaluation was to analyse the data on the take up and achievement in A-level and AS-level further mathematics during the development of the FMN.

Data analysis 2003-04 to 2007-08

The baseline year for the data analysis was taken as 2004 as that was the year in which the FMN formally started work, with funding provided by the DfES. The data on A and AS-level entries was derived from the National Pupil Database for Key Stage 5, made available by the DCSF. The figures in Tables 1 and 2 below represent the number of candidates that were awarded a grade, subdivided by gender.

Table 1 Entries in A-level further mathematics

A-level 2004			A-level 2006			A-level 2008		
female	male	total	female	male	total	female	male	total
1418	3711	5129	1924	4598	6522	2605	5918	8523
28%	72%		30%	70%		31%	69%	
Percentage increase on 2004			36%	24%	27%	84%	59%	66%

Table 2 Entries in AS-level further mathematics

AS-level 2004			AS-level 2006			AS-level 2008		
female	male	total	female	male	total	female	male	total
897	1689	2586	1542	2533	4075	2021	3463	5485
35%	65%		38%	62%		37%	63%	
Percentage increase on 2004			72%	50%	58%	125%	105%	112%

It can be seen that growth in numbers has been substantial in both the A-level and the AS-level entries, the latter more than doubling over this 5-year period. A substantial number of the total entries achieved were at grade A, this being 58% for the A-level and 39% for the AS-level in 2008, with similar figures for the previous years. It is notable that there are considerably more male entries than female entries; the male entry has constituted about 70% of the total entry for A-level and about 65% of the AS-level entry over the last few years. However, in terms of growth in numbers since 2004, it can be seen that proportionally more girls are being attracted into the subject than boys. Further analysis of the data showed that the growth in numbers was substantially in the state sector compared to the independent sector. Schools and colleges in the state sector accounted for 74% of the growth in A-level numbers between 2004 and 2008 with a corresponding figure of 90% for the AS-level. The growth in numbers represents a substantial influence of the work of the FMN.

The FMN was set up to address the problem of students who could benefit from taking further mathematics but who could not access tuition. The FMN also

offered support to institutions with a small number of students but also encouraged student take up of the subject in general. In the light of this we analysed cohort sizes from 2004 to 2008 to assess the impact of the FMN at institution level. This analysis is summarised in Table 3 and Table 4.

Table 3 Cohort sizes in A-level further mathematics

A-level 2004			A-level 2006			A-level 2008		
Cohort size	Schools Colleges	Students	Cohort size	Schools Colleges	Students	Cohort size	Schools Colleges	Students
	1088	5129		1267	6522		1482	8523
1	21%	5%	1	20%	4%	1	21%	4%
5 or less	74%	42%	5 or less	69%	35%	5 or less	66%	29%
10 or less	92%	69%	10 or less	90%	64%	10 or less	87%	58%
15 or less	97%	83%	15 or less	95%	77%	15 or less	93%	70%
30 or less	100%	100%	30 or less	99%	93%	30 or less	99%	91%

Table 4 Cohort sizes in AS-level further mathematics

AS-level 2004			AS-level 2006			AS-level 2008		
Cohort size	Schools Colleges	Students	Cohort size	Schools Colleges	Students	Cohort size	Schools Colleges	Students
	933	2585		1109	4075		1319	5485
1	45%	16%	1	34%	9%	1	34%	8%
5 or less	89%	63%	5 or less	83%	49%	5 or less	78%	40%
10 or less	98%	87%	10 or less	93%	71%	10 or less	93%	66%
15 or less	99%	93%	15 or less	98%	85%	15 or less	97%	79%
30 or less	100%	99%	30 or less	100%	96%	30 or less	99%	90%

The data on changes in cohort sizes reflects the growth in student numbers over the time period as the cohorts have in general got larger. However many students are still studying in cohorts of 5 students or less but it notable that the number of institutions offering further mathematics has also shown a substantial increase.

The growth in entries as seen in Tables 1 and 2 and the cohort sizes as seen in Tables 3 and 4 raised two research questions. Firstly were there particular institutions where there had been a large growth in numbers in either A-level or AS-level entries, and if so could the factors that influenced this growth be identified. Secondly, where institutions had small entries but were teaching further mathematics "in house", how was this being achieved. Telephone interviews were conducted, with the teacher responsible for Key Stage 5 Mathematics, usually the Head of Department (HoD), to investigate these questions.

Interviews with Heads of Department on growth in student numbers

In May 2008 approaches were made to the HoD at 40 institutions where growth had been identified between 2005 and 2007, asking to interview them. The criterion for selection was that an institution had entered 5 or fewer students in 2005 and 10 or more in 2007 at either A-level or AS-level in further mathematics. This criterion was met by 32 institutions at A-level and by 39 at AS-level. The sample drawn from these institutions included comprehensive schools, state grammar schools, post 16 colleges and independent schools. The criterion chosen was arbitrary in the sense that other criteria would have identified other institutions, but was considered adequate to gather HoD's views on why growth had occurred in their school or college. The response rate was 9/20 for the A-level (45%) and 13 /20 for the AS-level (65%). HoDs were asked to comment on both A-level and AS-level. The five main factors identified as leading to growth were as follows.

1 The reputation of the school and department in the locality.

If a department has built up a reputation for providing good quality teaching that leads to good results at A-level, it attracts students. It was apparent that once further mathematics became established, it attracted students. Some HoDs had started by offering the subject as extra curriculum using a post 4:00 pm slot or a sports afternoon to make time available for lessons. With growing interest and success, senior management had eventually allowed further mathematics onto the timetable. The commitment of the HoD was notable here, particularly those who had been appointment fairly recently, in wanting to offer further mathematics to their students.

2 The changes made in 2004 to the mathematics specifications

Many HoDs noted that further mathematics had become accessible to a much greater range of students due to the changes in the specifications in mathematics modules introduced in 2004. Some departments are now able to offer a 9 module course in mathematics, in which students aim for both A-level mathematics and AS-level further mathematics within one class over a two year period. In some schools and colleges where numbers made it possible, there was considerable flexibility in that students were able to continue to the full further mathematics A-level if they wished to or drop back to just mainstream mathematics. Most HoDs noted that the 2004 changes had done much to dispel the image of further mathematics as difficult and demanding and thus only for the most able. In this context some HoDs raised fears about changes that may be made again to the modular structure in 2012 in that changes could result in similar problems to those that followed *Curriculum* 2000, and numbers in A-level mathematics and further mathematics could plummet again just as they are becoming re-established and growing steadily.

3 Changes to timetable and option block structure

Most post-16 institutions which offer A-levels operate a timetable based on option blocks. In some schools and colleges the popularity of mathematics has grown to where it appeared in several option blocks, and if numbers warranted it, further mathematics would also be an option with an intention that those students who study both would take at least 12 modules. Again the flexibility of the modular structure of mathematics and further mathematics was stressed in meeting particular student's needs. Some HoDs had struggled to get further mathematics onto the timetable, and it was apparent that the attitude of the senior management to further mathematics was essential to this. Often, where numbers had grown and senior management gave full support to the development of further mathematics, a member of the team was a mathematician or scientist. Senior management in general endorsed further mathematics once it was seen to be bringing success to the school or college, as seen in the examination results.

4 Recognition that a qualification in further mathematics is a career asset

Many HoDs spoke of a growing enthusiasm for mathematics in their institution, which is probably related to enrichment type activities. It was reported that many students went on to study mathematics related degree courses in higher education, such as engineering, computer science and economics. Students were coming to see the value of studying further mathematics, at least to AS-level, as being an asset on their application forms, and also of being of benefit to their studies once their degree

course had begun. Some HoDs had introduced further mathematics because they had students who were asking to study it.

5 Support and a general enthusiasm from the teaching staff for mathematics.

Many of the HoDs noted they were fortunate in that their staff were largely well qualified in mathematics. Most staff were involved in supporting students in some way. For A-level study, this often involved a "drop-in clinic", or a specified time when staff would be available for help with any problems. Some HoDs had introduced mentoring systems, in which older students helped younger ones. Nearly all HoDs spoke enthusiastically of the involvement of their staff and students in the UKMT Challenges, at junior, intermediate and senior level and face-to-face team competitions. Some HoDs spoke of their aim to make mathematics both challenging and fun. Thus from Year 7 onwards, they would have extra-curricula activities like puzzle solving, or they would extend the mathematics curriculum through for example, GCSE statistics or Free Standing Mathematics Qualifications (FSMQ) in Year 11. Opinion was split as to whether it was a beneficial idea to introduce the AS-level mathematics core work in Year 11. Some thought this gave students an insight into sixth form study and a helping start, whilst others thought it premature and better to pursue other aspects of mathematics. However, all departments were active in encouraging suitable Year 11 students to continue their study of mathematics after GCSE, often involving talks from external speakers on the importance of mathematics and the career opportunities a qualification in mathematics can bring.

No HoD put the development of further mathematics down specifically to the FMN. However, most commented that they thought the Network was important and they valued it, noting it was doing a good job in raising the profile of mathematics.

Interviews with Heads of Mathematics Departments concerning "in-house" teaching of further mathematics to small cohorts of students

Towards the end of 2008, the FMN identified schools and colleges in the Network where a decision had been taken to teach at least some of the further mathematics modules themselves in 2008/09 despite having relatively small numbers of students, rather than using FMN tutors. In total, 52 such schools and colleges were identified of which a sample was contacted for interview. Interviews were conducted during January 2009 with the teacher responsible for Key Stage 5 Mathematics in 33 of these institutions, covering a wide range of types of school and colleges and most areas of England. The main points arising from the interviews are as follows.

1 Managing provision in further mathematics

Most HoDs had persuaded their senior management that further mathematics should be in the sixth form option blocks and given timetabled time. However, this was rarely a full allocation of teaching time. Typically well established subjects would have 5 hours, whereas further mathematics would be allowed 2 or 3 hours. This was often said to be adequate for a small group of able and committed students; in some institutions the time was supplemented with voluntary extra time. Some senior managers supported further mathematics as it brought kudos to their institution and as such, was a good marketing tool in both attracting and retaining able students.

There is an apparent difficulty for HoDs as to when students actually begin their study of further mathematics. For some it is only offered as an AS-level course

in Year 13, whereas others "parallel run it" with the main mathematics course in both year groups. These are the extremes of many flexible arrangements that allow students to go as far in mathematics as they want to. The main problem encountered is the pre-requisite knowledge required for some of the further mathematics, particularly in the pure mathematics but it was said able students can cope with this. Some believed only high calibre students could successfully begin further mathematics at the start of Year 12. It was noted that for students who wish to take extra modules beyond examination requirements, funding is not available to cover any teaching.

2 Involvement with the Further Mathematics Network

All these institutions were registered with the FMN and were to some extent in contact with their local Centre Manager, even if this meant just receiving information through e mail. Some HoDs had appreciated the help and advice they had received from the local Centre Manager in initiating further mathematics, some saying they had found it invaluable. Many made use of the MEI online resources and encouraged their students to attend revision and enrichment events. Students who had attended such events had largely found them enjoyable and useful; they particularly liked meeting with students from other institutions. However, other commitments often prevented attendance. The timing of these events was raised by many as an issue and some noted small local events might be preferable to large regional events.

3 Mathematics in the institution as a whole

In general the HoDs considered the attitude of pupils and students in their institution to be positive towards mathematics particularly amongst the more able. In all schools there was some sort of enrichment in the mathematics programmes for all year groups. Involvement in the UKMT Maths Challenge and Team competitions was common and also in some more local competitions. Pupils clearly enjoyed taking part of in these. Many schools had "maths days" or similar in which pupils would solve puzzles or be given "real world" problems. External visits, or visitors to the school, were occasionally organised, with *Maths Inspiration* and *Murderous Maths* being mention by several HoDs. Pupils in Year 11 who had shown an aptitude for mathematics were encouraged to consider taking further mathematics. All schools and colleges had at least some members of the department who are well qualified in mathematics and able to teach mathematics at A-level and at least some of the further mathematics modules. Many noted the need for Continuing Professional Development (CPD) in this respect, particularly for younger members of staff, and noted the need for further development of online provision of CPD.

Discussion

It was clear from the interviews that the FMN has provided far more that just support for students. Enrichment events and revision sessions provided by the FMN on most A-level mathematics modules, especially the pure core, are valued. Enrichment events are aimed at inspiring both sixth form and younger students, particularly in Years 10 and 11, from whom the next cohorts of AS and A-level students of mathematics and further mathematics will come. These events often take place in the local university letting young students experience this environment for the first time and also to see a presentation by a professional mathematician. If the numbers taking mathematics and *STEM* related courses at A-level and in higher education are to grow then it is

essential to engender interest and enthusiasm for mathematics prior to age 16. The FMN enrichment events contribute substantially to that aim.

It was also clear from the interviews that one of the main reasons for attracting students to further mathematics was the flexibility of the current arrangement in which some modules can count towards either mathematics or further mathematics. Many institutions have used creative timetabling in which Year 12 and 13 students can study some modules together. If this flexibility were to be removed as was proposed in the QCA consultation on A-level provision in 2009, then there could be dire consequences for the number of students taking further mathematics. It is notable that some professional bodies, such as The Royal Statistical Society and MEI, wrote a position paper in response to the consultation, proposing there be no change. Many of the issues involved have been discussed in the Institute of Mathematics and its Applications publication, *Mathematics Today*. (2009). At the time of writing the outcome of the QCA consultation is not known, but the research reported here shows the post 2004 specifications have been instrumental in creating the growth in student numbers and it would seem sensible to leave the current arrangements alone. Any substantial change could have a detrimental affect on future student numbers.

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