Enrichment and engagement in mathematics

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In this paper we examine the notions of engagement and enrichment in mathematics. The Royal Institution of Great Britain (Ri) facilitates the Secondary Mathematics Masterclasses project and has been involved in a QCA project to follow teachers' journeys when developing 'rich tasks' for use in school for whole class teaching, both with the aim of 'Engaging mathematics for all learners'. The Masterclasses were evaluated in 2008 by the CEM centre in Durham, and the Ri conducted case studies for the QCA project on how ideas and methods traditionally reserved for the gifted and talented cohort can be used for a wider range of learners. Drawing on the results of these studies and the research literature, this paper will discuss what is understood by 'enrichment' and 'engagement' in mathematics. In clarifying what we mean by enrichment, we present a structure for enrichment which differentiates between the inputs and the outputs of any enrichment activity, engagement in mathematics being one of the desirable outputs. The findings show that the Ri masterclasses brought about the enrichment outputs we would expect from such activities, with additional outputs for the teachers involved. The Ri's participation in the QCA project has aimed to build on this model in order to maximize such outputs. We explored the impact in teachers' practice when developing rich tasks and the impact on learners. Based on our research, we put forward recommendations for carrying out enrichment activities in mathematics.

Key words: enrichment, engagement

Introduction

The need for mathematical enrichment activities is highlighted by ongoing concerns over students' attitudes towards the subject. Smith (2004) identified a perception by young people that mathematics was boring and irrelevant. He underlined this point by noting the 10% drop in the take-up of A-level mathematics in the 1990s, highlighting possible factors for this decline including the perceived poor quality of the teaching and learning experience, the perceived relative difficulty of the subject, the failure of the curriculum to excite interest and provide appropriate motivation, and the lack of awareness of the importance of mathematical skills for future career options and advancement. Despite a rise in recent years of students taking A-level mathematics, the numbers in 2007/08 were still about 6000 students down on the numbers taking the subject in 1990. In addition, the London Mathematical Society (1995) raised concerns in the past that students entering higher education lacked the necessary ability and skills that might be expected on university mathematics courses. Therefore, it would be hoped that mathematical enrichment programmes tackle these two problems of attitudes towards maths and students' mathematical skills.

Enrichment in mathematics

The question of what is meant by enrichment has been an ongoing question for researchers. Feng (2005) provides a useful summary and quotes Barbe (1960): "an aura of vagueness and confusion seems to surround the term". Feng's own conclusion is that "no overall consensus has yet been reached on the definition and nature of enrichment". The phrases mathematical or analytical thinking and problem solving are also often used when attempting to define the term enrichment. These are discussed by Piggott (2004), and although she creates a model in which these terms are integral to a concept of mathematical enrichment, the difficulty still remains of defining just what is meant by these terms. Despite this difficulty, Feng (2006) put forward four 'paradigmatic positions' on enrichment in order to show the different types of mathematical enrichment activities: (1) Development of mathematical talent, including extending mathematical skills and heightening interest in the subject; (2) Popular contextualisation of the subject, including tackling negative stereotypes and deepening mathematical understanding; (3) Enhancement of mathematical learning processes, including developing learning skills; (4) Outreach to the mathematically underprivileged, including widening student access to mathematics. A point to note here is that these positions on enrichment detail the hoped-for outcomes to enrichment activities only – they do not in themselves define what constitutes enrichment activities. In developing our ideas on what enrichment is in the context of learning mathematics, a definition will be offered at the end of the paper.

Engagement in mathematics

In addition to trying to define 'enrichment', we can also clarify what is meant by engagement in mathematics, which is the overall aim of the enrichment tasks being considered in this paper. Engagement is important primarily because of its relationship with the academic achievement of learners (Peterson and Fennema 1985; Park 2005). Newmann et al. (1992, 12) defined engagement as "students' psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote". Past research (Fredericks et al. 2004; Kong et al. 2003) has highlighted three dimensions to engagement: behavioural, emotional/affective and cognitive. Behavioural engagement is about the active participation in learning activities, emotional engagement is the students' attitudes (e.g. perceived value, interest in) towards the activities, and cognitive engagement can be seen as the 'psychological investment' mentioned previously. Meece et al. (1988) defined cognitive engagement in terms of students' use of metacognitive and self-regulation strategies. Kong et al. (2003) found that approaches to learning (e.g. deep, surface) were closely related to cognitive engagement. Fredericks et al. (2004) included motivational goals and self-regulated learning under cognitive engagement. Looking at Feng's paradigmatic positions on enrichment, we can see that the notion of engagement in fact covers most of these positions – whether it is increasing interest or tackling negative stereotypes through attitudinal engagement, increasing participation in mathematics through behavioural engagement, or it is the enhancement of mathematical learning and development of understanding through cognitive engagement. Therefore, a key outcome from enrichment activities seems to be these different aspects of engagement in mathematics.

Evaluation of the Ri Secondary Mathematics Masterclass programme

In 2008, the Royal Institution commissioned the Curriculum, Evaluation and Management (CEM) Centre to carry out the evaluation of their Secondary Mathematics Masterclass programme (Barmby et al. 2008). There were ten specific objectives for the masterclasses, which we categorised under the following areas of impact: (a) Attitudes towards mathematics; (b) Doing mathematics; (c) Participation in masterclasses; (d) CPD opportunities from masterclasses; (e) Facilitation of the masterclasses. The first three objectives again came under the notion of engagement and are hoped for outcomes. An additional outcome of CPD opportunities for teachers was identified, along with the aim of facilitating and developing the masterclasses over time. We describe an example of a masterclass ('The Power of Two') in the Appendix. In order to evaluate the programme therefore, and based on the previous discussions on enrichment and engagement in mathematics, we recognised the importance of differentiating between what can make up enrichment activities (one could say the input) and the desired outcomes from such activities (the output). In order to clarify our view of mathematical enrichment then, especially with regards to the Secondary Masterclasses, we put forward the following structure of the enrichment process:

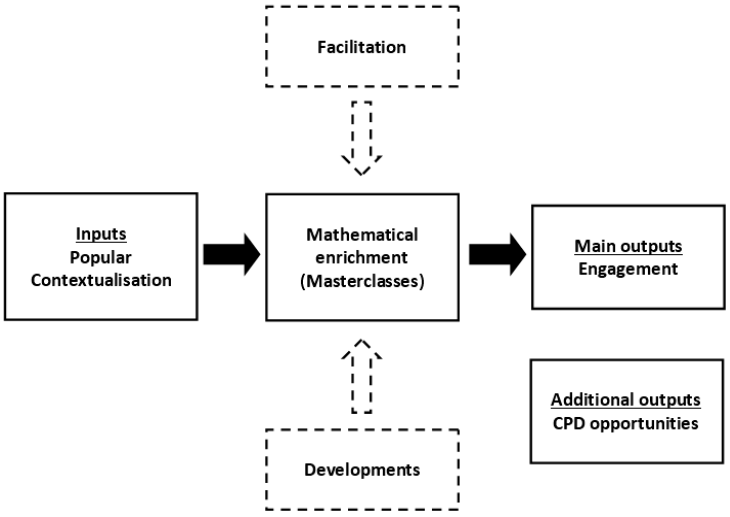

Figure 1: A structure for the mathematical enrichment process

In the model, we specifically split up the inputs and outputs of mathematical enrichment. For example, we have provided an example characteristic of a possible enrichment activity, namely contextualising mathematics within pupils' experience. Looking at the outputs, the evaluation clearly identified particular student outcomes from the Secondary Masterclasses, both from the quantitative results of a student questionnaire, and from qualitative interviews with students carried out during case study visits to masterclasses. The student questionnaire (see Appendix and also Barmby et al. 2008), completed by 917 students, produced the following key findings:

- 64.2% of the students agreed or strongly agreed that their attitude towards mathematics had improved due to the masterclasses;
- 69.6% of the students felt that their ability in mathematics had improved due to the masterclasses.
- 59.3% of the students also agreed that the masterclasses had encouraged them to study maths in the future.

These findings correspond to the three components of engagement identified previously – namely attitude, cognitive (or the result of cognitive engagement) and behavioural. These specific outcomes were also highlighted in the interviews with students:

> "Masterclasses have made me like maths more and now if I've finished, Sir gives me more stuff to do from different books and things."
>
> "When I was in primary, I didn't really get it at all and after coming here I am really good at it."
>
> "Yes, I'm more confident in maths now and I seem to enjoy it because I'd like to go on to a maths career when I'm older."

In addition to these outputs for students, some additional benefits to the masterclasses were identified as well, in particular benefits for teachers attending the masterclasses, especially in terms of professional development and providing materials that they could use in the classroom:

> "There are always plenty of ideas! We always come with a book and write things down and come back to school and try and use some of the things that we have picked up."
>
> "What is really nice is the teachers coming, because they can then take it back to school. What my teachers in [town name] usually do is the two or three that have been, go back and teach the rest of the class. Not all the topic, but bits of the thing."

Having identified the important outputs from the programme, the next step was to identify why the masterclasses were perceived positively, and the evaluation highlighted three factors. The role of the activities was identified by the researchers during their visits to various masterclasses, where practical activities were seen to enthuse students and get them involved in 'doing mathematics'. Activities were also highlighted by students in their interview comments.

> "I just like puzzles, sometimes in maths you just get told stuff or you just have to work out things, whereas in those ones you're actually making things and doing things that are imaginative."
>
> "You learn yourself through experimenting rather than just being told something."
>
> "It is different methods and stuff and we put into practice Pythagoras' theory and type of stuff. You cover it at school, but here you actually do it."

In addition to the activities being practical, the evaluation highlighted that the activities were different to what students were doing in school.

> "I think it is definitely more outside the box than addition and subtraction!"
>
> "I like maths, but I don't enjoy it very much in school at the moment … I think it is just in school, you do not get much chance to learn something new, most of you are just going over the same things."
>
> "Before this I didn't think paper folding was maths, so it's made me see that maths is more than just numbers and algebra. I have seen the wider side of maths."

Finally, the fact that the masterclasses had shown students the usefulness and importance of mathematics was highlighted.

> "I think they are good because they show you how maths is used in the real world. Used to solve problems."

Case studies from the QCA engaging activities project

The evaluation of the mathematics masterclasses above identified both benefits for students and teachers participating in the classes. In the project 'Engaging mathematics for all' led by the then QCA (now QCDA), one of the authors looked into how masterclass-type activities could be used in the classroom for a wider range of learners. We worked with two institutions; a very high-achieving girls' state school and an all-boys technology college (perceived by the staff as a difficult school). We gathered written and oral statements, some of them on video, from both pupils and teachers. Other information came from observations carried out by one of the authors. We attempted to isolate here the characteristics of the developed tasks (inputs), the impact on learners (outputs) and teachers (additional outputs). The work on the case studies drew our attention to issues that require further research.

Characteristics of the developed tasks (inputs)

Teachers from both schools wished their pupils to think more deeply about what is learnt; make not just connections across the curriculum but between different areas of mathematics; and expand their comfort zone. Both welcomed hands-on activities; in the girls' school the majority of the pupils could feel out of their depth when asked to put their knowledge into practice. The boys' school identified kinaesthetic tasks as a way to engage their pupils and use the sense of achievement that comes from 'making something' as a way of creating a positive experience associated to mathematics (Le Roux and Santos 2009). Pupils worked on the mathematics underlying fashion design and architecture, connecting geometry with algebra. They studied representations of 3D objects in 2D; Picks' Theorem; perimeter and area through the story of Queen Dido of Carthage; stability of shapes; paper folding dodecahedra and buckyballs; links between fractions, algebra and paper folding through folding 'perfect thirds'. Some visited central London's architectural icons and interviewed a team of architects and designers. In some activities older pupils that had previously been taught the activity helped as classroom session leaders and mentors. For details see Le Roux and Santos (2009), Santos (2009), Teachers' TV (2009) and Ri (2009). The accompanying videos 'Cutting the cloth to fit' and 'The Gherkin shapes up' can be found on the QCDA's curriculum website http://curriculum.qcda.gov.uk.

Impact on learners (outputs)

Some impacts were observed by the teachers and by the researcher, whereas others were verbalised by the pupils involved. Although the collected accounts are few, the information gathered motivates a future research study. During the activities, pupils were on task, showed commitment, interest and effort, had a positive attitude in class and behaved well. The pupils engaged in conversations related to the work proposed; in lessons that were led by pupils, their response was positive with commitment and respect from both the mentees and the mentors. It was observed that the pupils actually did mathematics. The teachers summarised it as a 'general buzz around the mathematics department', that for the duration of the case study pupils manifested improved attitudes towards mathematics inside and outside the classroom, such as positive attitude while working, pride in their work, had fun and built confidence in their own abilities to tackle the tasks. Pupils felt challenged, made connections within mathematics and across subjects, claimed to think more deeply about the concepts involved, were able to improve their practical skills within the context of

mathematics. In the corridors several pupils asked the teachers about the mathematical paper folding work on display at the school entrance. Teachers from other subjects got interested in the project as a consequence of the displays. The younger pupils benefitted from being taught by their older peers as they felt more comfortable to ask questions. The older students explaining mathematics to younger ones benefited from having to think deeper about the concepts involved.

Impact on teachers (additional outputs)

The process of development, trialling and tailoring such activities in schools resulted in professional development for the teachers. From their reflections and from observations made by the researcher, the following specific benefits were identified: expansion of one's comfort zone; coping with being challenged; understanding what it is to be a learner; feeling excited about mathematics and the teaching of it; feeling energised and motivated to change practice to incorporate more rich-tasks in everyday teaching; learning and doing mathematics; understanding the value of collaborative work as a teacher; building positive experiences to refer to in future. This work subsequently raises questions about engagement in teachers and not just the pupils. Can teachers provide their pupils with an engaging experience if they are not having regular engaging experiences in the subject? What constitutes engaging experiences for teachers and how do these translate into the classroom? These are issues to examine in future research activities.

Discussion of the findings and implications for enrichment activities

In looking at the literature on enrichment in mathematics, we identified the fact that the term 'enrichment' was actually difficult to define. What we did identify was that possible definitions were based on the hoped-for outcomes from enrichment activities. In fact, when we also examined the notion of 'engagement' in mathematics, what we found was that this largely described the outcomes we were looking for from enrichment. Therefore, based on our discussions in this paper, we propose a rather simplified definition of enrichment as simply activities that bring about engagement in mathematics. In this context then, we need to be clear about what we mean by engagement, and we have used the definition provided by the research (Fredericks et al. 2004; Kong et al. 2003) which includes the three strands of behavioural, emotional/affective and cognitive engagement. When we are considering enrichment activities then, we need to look for these three outcomes. Research has suggested that the first two components – behavioural and emotional/affective – may in fact be quite closely associated, with a person's attitudes possibly predicting behaviour (Crano and Prislin 2006). However, we need to insure that the third component of cognitive engagement is present as well. We want students to 'do mathematics', to take part in mathematical thinking, to develop their knowledge and understanding of the subject, as well as simply taking part in activities and enjoying or being interested in them.

In fact, from the two enrichment programmes that we have examined in this paper, we can see this broad coverage of the components of engagement from the enrichment activities. For example, in the engaging activities projects, although teachers did talk about 'hands-on' activities and kinaesthetic tasks, and also positive experiences, they also spoke about their wish for students to think more deeply about the mathematics. From the secondary mathematics masterclasses, we identified the importance of the activities themselves, rather than say the presenters of the masterclasses, or the facilities available to the students, as being the important factor

in whether students were engaged in mathematics. Although the novelty and the usefulness of the mathematics were also positive issues that were raised by the students, in themselves, these would only perhaps impact on the affective and behavioural aspects of engagement. However, students also reported their cognitive engagement in the masterclasses, how the sessions promoted active thinking, 'making them think' and 'keeping their brain going', as they were encouraged to find solutions to problems and be engaged in harder mental work.

> I don't feel like a master either! If I was a master I would get all the problems straight off! But it does make me think. It doesn't make me think I am amazing, it makes me think how I can solve this.

Therefore, when designing enrichment activities, it is important that all these components of engagement are catered for.

This recommendation for enrichment brings us back to the view of enrichment put forward by Piggott (2004) where problem solving and mathematical thinking was integral to the enrichment process. Indeed, we emphasise once again that clarity is required in ensuring that cognitive engagement is an outcome of the enrichment process, whether that is achieved through problem solving activities or other open-ended activities. We end with the following quote from Leone Burton (1984, 9):

> "Mathematics is used to solve useful problems; it can be played with in a creative way to see what can be discovered; it is the basis on which amusing puzzles can be invented; it has a great power to inform. But they are not the best reason. The greatest value of this approach is in the effect it has in the classroom."

In engaging students through enrichment activities in mathematics, the possible benefit is the impact on the learning and understanding of mathematics, over and above the enjoyment and participation of students.

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Appendix - Questionnaire

The questionnaire on the Masterclasses for pupils contained the following items:

How I feel about the Masterclasses
(five point scale of strongly agree, agree, neither agree nor disagree, disagree, strongly disagree)
Example questions from the 20 items in this section included:

1. I enjoyed the classes
2. The masterclasses showed me the importance of mathematics
3. Following the masterclasses, I now expect to do better in maths courses that I take
4. have enjoyed the social side of the masterclasses
5. learnt a lot of mathematics from the classes
6. The masterclasses have encouraged me to study maths in the future

Quality of the masterclasses
(five point scale of excellent, very good, good, poor, very poor)

1. Quality of the presenters and their presentations
2. Quality of the activities you carried out during classes
3. Quality of the facilities where the classes were held

Possible impact of the masterclasses
(five point scale of strongly agree, agree, neither agree nor disagree, disagree, strongly disagree)

1. My attitude towards mathematics has improved due to the masterclasses
2. My ability in mathematics has improved due to the masterclasses

Appendix - 'The Power of Two'

A mathematics masterclass is a two-and-a-half hour interactive session. The Power of Two is inspired in the Josephus problem: given a circle of *n* people where every alternate living person is killed in succession, where should one stand in order to be the last person to be killed and hence to survive? Through a mixture of pattern-recognition, problem solving and group work, the challenge provides an informal and accessible introduction to number systems (in particular, place value and binary representation) and to the notion and importance of proof.