Comparison at Two Levels of the Content Treatment of 'Early Algebra' in the Intended Curricula in South Africa and England

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This paper compares the treatment of algebra content in the intended curriculum of the early grades[1] in two countries: England and South Africa. Two levels of analysis are conducted. The first examines the content structure of each curriculum; and the second compares and contrasts the detailed 'learning objectives' in England to the 'assessment standards' in South Africa. The comparison reveals that a curriculum may include algebra by name, but may not deal with it in much substance (as in South Africa) and, a curriculum may include algebra in some substance, while not mentioning it by name (as in England).

Keywords: Early Algebra, Curricular Comparison, South Africa, England

Introduction

In the last few decades, there has been substantial mathematics educational research into precisely what is meant by 'early algebra', and a growing sophistication of its interpretation and use (see, for example: Davis 1985, Bodanskii 1991, Sasman et al 1998, Kaput et al. 1999, Bills et al. 2003, Amerom 2003, and Warren 2004). This has emerged from the identification of a 'cognitive gap' in the transition from arithmetic to algebra (Herscovics et al. 1994), which has resulted in research focusing on how the teaching of arithmetic at primary school level may better inculcate algebraic thinking. There seems, however, to be less published work on how this research has been interpreted and is manifest in particular curriculum frameworks. It therefore seemed opportune to compare particular intended curricula and contrast their treatment of 'early algebra', in order to garner how the research on this topic is being communicated to mathematics teachers in different national contexts. This paper limits its scope to only two countries: England and South Africa.[2] It is hoped that it provides sufficient detail to be of particular interest to mathematics educators from these countries, and simultaneously provides a basis for further comparison for the wider international mathematics education community interested in 'early algebra' or 'curriculum development'.

This paper begins by presenting Kaput's definition of 'early algebra', and explains how this is adapted for the purpose of this paper (2007). This is followed by an analysis of the curriculum policy documents in the two countries: the *Revised National Curriculum Statement for Mathematics (RNCS)* (Department of Education, DOE 2002) in South Africa, and the *Primary Framework for Mathematics* (Department for Children, Schools and Families, DCSF 2009d) in England. These intended curricula are analysed at two levels: the 'macro level' which examines the

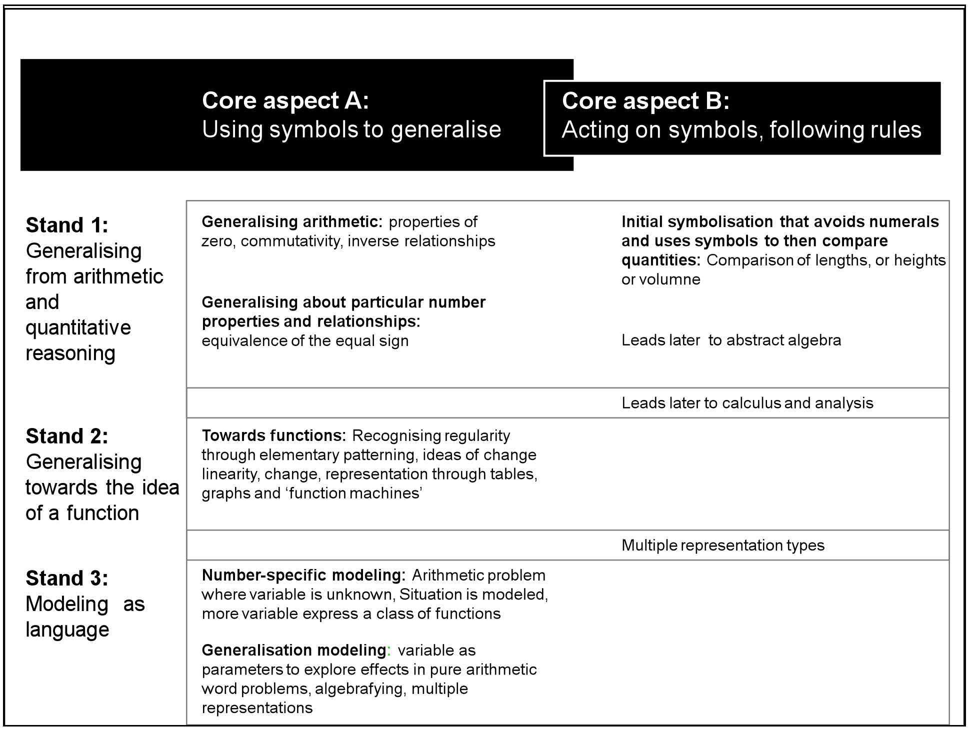
[1] For the purpose of this paper, 'early grades' is defined to be the first four years of compulsory primary education. In England this refers to Reception and Years 1 to 3, which are termed 'Key Stage One'. In South Africa this refers to Grade R and Grades 1 to 3, which are termed 'Foundation Phase'.
[2] These countries were selected as they are the national mathematics education environments with which the author is most familiar having conducted research and taught mathematics in both countries.

content structure of each curriculum; and the 'micro level' which compares and contrasts the detailed 'learning objectives' in England to the 'assessment standards' in South Africa. A third 'meta level' of the published research on early algebra emerging from each country is also of interest, as this informs the other two levels. However this paper is limited to the micro and macro levels, as these are the standard national documents used to communicate the intended curricula to the majority of teachers in the early grades.

Analysing Early Algebra in Intended Primary Mathematics Curricula

Kaput provides a detailed framework for considering the various elements of algebra by referring to two core aspects and three strands, as represented in Figure 1.

Figure 1: [1] Kaput's definition of 'early algebra'



The two core aspects are evident in Kaput's explanation that algebraic reasoning is taken to be "symbolisation activities that serve purposive generalisation" (or using symbols to generalise), and simultaneously to be "reasoning with symbolised generalisations" (or acting on symbols, following rules) (Kaput et al. 2007). Kaput explains that it is generally assumed in mathematics curricula that *Core Aspect A* usually precedes *Corey Aspect B*. The first two strands in this categorisation of early algebra consider the two types of generalising that are at the heart of algebraic thinking: generalising arithmetic; and generalising towards the idea of function. The third strand refers to modelling processes where situations are understood and interpreted using algebraic reasoning and language. Kaput explains that there are overlaps between the strands and provides useful illustrative examples relevant to the early grades for each strand, which are summarised in Figure 1.

For the purpose of this paper, 'early algebra' is taken to mean generalising in the early grades of primary school, which is as expressed in Strand 1 and Strand 2 of

[1] Source: Created be author, drawing on explanations provided by Kaput 2007.

Core Aspect A in Kaput's definition.[1] This generalising has two main manifestations: generalising from arithmetic and quantitative reasoning; and generalising towards the idea of a function. As such, one would expect that a primary mathematics curriculum which addresses 'early algebra' should include at least three elements. The first element is generalising arithmetic as the exploration of properties of numbers and operations. The second element is generalising about particular number properties and relationships. This is distinct from the first element, as it relates to properties and relationships for particular numbers and not to properties of numbers and operations in general. It is important to realise that arithmetic approaches which encourage 'partitioning' or 'breaking down' and 'building up' numbers draw on these properties of particular numbers and operations. The third element is generalising towards the idea of a function which includes recognising regularity in elementary patterns, ideas of change including linearity, and representation through tables, graphs and 'function machines'.

Comparison of Early Algebra in the Intended Curricula of England and South Africa

Comparison at the Macro Level: Content Structure of Each Curriculum

In both South Africa and England the mathematics content is expressed in the content focus areas and related assessment criteria in the mathematics curricula. The South African mathematics curriculum statement is structured using the following five learning outcomes: Numbers, Operations and Relationships (LO1); Patterns, Functions and Algebra (LO2); Space and Shape (Geometry) (LO3); Measurement (LO4); and Data Handling (LO5). For England the mathematics learning objectives are organised in relation to seven 'strands of learning' which give a broad overview of mathematics in the primary curriculum. The seven strands of learning in England are Using and Applying Mathematics (Ma1), Counting and Understanding Number (Ma2), Knowing and Using Number Facts (Ma3), Calculating (Ma4), Understanding Shape (Ma5), Measuring (Ma6), and Handling Data (Ma7).[2] Like South Africa, each learning strand in England is used to provide the structure for specifying the learning objectives for each year in the primary school.

In South Africa, learning outcomes one and two are most relevant to early algebra. Algebra could find expression in a variety of contexts including learning outcomes three, four and five, "however the core of the development of learner's knowledge and understanding of algebra will, by the nature of algebra, take place with the first two learning outcomes" (Vermeulen 2007, 21).

For 'numbers, operations and relationships' (LO1) learners are expected to "recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems" (DOE 2002). Although this does not explicitly mention algebra, this learning outcome could be expected to include statements that relate to algebra. For example the

1 The motivation for this is twofold: *Strand 3* and *Core Aspect B* tend to emerge later in the primary curriculum as preparation for algebra in secondary schools and are considered by the author to be less relevant to the early grades; generalising is the core idea of early algebra as relevant to the early grades in primary mathematics curricula. There are other definitions of early algebra but in-depth discussion of these is beyond the scope of this paper.

2 Ma5 in England maps quite neatly to LO3 in South Africa. Similarly Ma6 maps to LO4 , and Ma7 to LO5. Ma1, Ma2, Ma3 and Ma4 in England can be broadly mapped to LO1 and LO2 in South Africa. Ma1 is a cross cutting objective, of which there is no equivalent LO in South Africa.

'Learning Outcome Focus' includes that this learning outcome "develops the learners understanding of how different numbers relate to one another; …how different numbers can be thought about and represented in various ways; and the effect of operating with numbers" (DOE 2002). This clearly shows that there is explicit intention to allow learners to explore the structural nature of arithmetic. Numbers are experienced in how they relate to each other, and can be thought about and represented in various ways, allowing for the exploration of the equivalence of the equal sign. With specific reference to the 'Foundation Phase Focus', the curriculum statement specifies that

> In this phase the number concept of the learner is developed through working with physical objects in order to count collections of objects, *partition and combine quantities*, *skip count* in various ways, solve contextual (word) problems and *build up and break down* numbers. (emphasis added) (DOE 2002)

Again, this appears consistent with the intention of early algebra to allow arithmetic to be seen in general terms: 'partitioning and combining quantities' or 'building up and breaking down numbers' promises the potential of exploring equivalence of numeric expressions; and the use of skip counting hints at the introduction of multiplication as repeated addition, and to the beginning of the concept of number sequences and the concept of a function.

'Patterns, functions and algebra' (LO2) requires learners to "recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills" (DOE 2002). This 'Learning Outcome Focus' provides a detailed indication of the South African interpretation of what is meant by algebra, stating that algebra "can be seen as generalised arithmetic, and can be extended to the study of functions or relationships between variables" (DOE 2002). The description of the 'Foundation Phase Focus' for this learning outcome is that this learning outcome is intended to "lay the foundation for algebra in the Intermediate and Senior Phases" (DOE 2002). This reveals that although algebra is included in the learning outcome description, it does not, in fact, appear to be a major focus at the Foundation Phase.

In England, like in South Africa, early algebra may be found relevant to all of the learning objectives. However the core treatment of early algebra would be expected to be evident in learning objectives Ma1 to Ma4. The curriculum in England does not describe each of its learning objectives in any more detail nor in the same systematic way as the South African curriculum. A selection of 'Guidance Papers' accompany the Primary Framework have some relevance to the mathematics learning objectives. However, the term 'algebra' is not explicitly mentioned in any of the relevant Guidance Papers.[1] Nevertheless the concept of early algebra as generalising is evident in all three of these papers. To illustrate this implicit treatment of early algebra: the *Guidance Paper on Mathematics and the Primary Curriculum* states that "mathematics describes patterns, properties and general concepts" as one of its descriptors of what mathematics is, elaborating that "children's ability to extract the essential properties and generalise from particular cases is a key skill in mathematics" (DCSF 2009b). This is consistent with the notion of early algebra as generalised arithmetic. The *Guidance Paper on Calculating* also describes that children are expected to recognise how operations relate to one another and how the rules and laws of arithmetic are to be used and applied (DCSF 2009a). The *Guidance Paper on Using and Applying Mathematics* elaborates on five theme areas all of which have

[1] See the Guidance Paper on 'Mathematics and Primary Curriculum' on 'Calculation', or on 'Using and Applying Mathematics'.

relevance to early algebra. The following example of an extract from the reasoning theme illustrates this:

> Children need to be taught how to record their thinking and reasoning in mathematics as they describe, replicate and create patterns and explore properties and relationships. (DCSF 2009c)

Describing, replicating, and creating patterns and exploring properties and relationships are central ideas in generalising towards the idea of a function. So, although the electronic curriculum framework of England is rather fragmented and its links to multiple assessment frameworks seem inordinately complex, it does include two notions of early algebra as generalising, but views this as integrated into the treatment of number work more broadly.

In sum, in comparing and contrasting the curricula of the two countries at this macro level, the South African curriculum clearly has a more detailed and explicit focus on early algebra than the England curriculum does. However, when examining the underlying documentation provided in the *Guidance Papers* it is clear that England includes reference to the ideas underpinning early algebra as generalising, although this connection is not made explicit in the curriculum documents.

Comparison at the Micro Level: 'Learning Objectives' in England and 'Assessment Standards' in South Africa

At the micro level there is considerable similarity between the two curricula. Table 1 presents the similarities in the early grades, organised using the three elements that a curriculum which addresses early algebra should include (as outlined above).[1]

Table 1: Similarities in Assessment Standards in South Africa (SA) to Learning Objectives in England (relating to early algebra, in the early grades)

Table 1 shows that there are similarities in the two curricula in relation to all three aspects of early algebra as generalising. There are no assessment standards in the South African curriculum which are relevant to early algebra, which do not find expression, in an equivalent form, in the English curriculum. However, the curriculum in England does include several learning objectives that are relevant to early algebra, which are absent from the South African curriculum, as presented in Table 2.

Table 2: Learning Objectives in England where there are no equivalent Assessment Standards in SA

Aspect of Early Algebra	Example	ENGLAND	SOUTH AFRICA
Generalising arithmetic as the exploration of properties and number operations	Commutative, associative, and distributive properties	Ma4 Year 1: Relate addition to counting on; *recognise that addition can be done in any order* (emphasis added)	Does not make commutative property of addition explicit.
	Inverse relationships	Ma3 Year 2: Understand that halving is the inverse of doubling	Does not make explicit that halving is the inverse of doubling
		Ma3 Year 3: Use knowledge of number operations and corresponding inverses, including doubling and halving to	Does not make explicit use of inverse relationships for calculations

[1] See the last paragraph of the section on Analysing 'Early Algebra' in the Primary Mathematics Curricula

Aspect of Early Algebra	Example	ENGLAND	SOUTH AFRICA
		estimate and check calculations. Ma4 Year 2: Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences.	Does not make explicit that addition is inverse of subtraction
	Properties of multiples, including odd and even numbers	Ma2 Year 2: Recognise odd and even numbers Ma3 Year 2: Recognise multiples of 2,5 and 10	Does not consider properties of multiples (for odd and even numbers, or other multiples)
Generalising about particular number properties and relationship	Using equivalence of '=' sign to find an unknown in an equation	Ma4 Year 2: Calculate the value of an unknown in a number sentence (e.g. □+2=6; 30-□=24)	Does not make explicit the equivalence of the equal sign, and its use beyond an instruction to compute

Table 2 demonstrates that the English curriculum makes the commutative property of addition explicit, while the South African curriculum does not. The English curriculum introduces the idea of an inverse and is explicit about both doubling and halving and addition and subtraction being inverse relationships. Also learners in England are expected to find an unknown in a simple linear equation. This shows that at the micro level of learning objectives and assessment standards, the English curriculum addresses early algebra concepts in more detail.

The South African curriculum includes examples of algebra as generalised arithmetic, but does not make this explicit through identifying general properties of numbers and relationships in arithmetic. Although doubling and halving are introduced, this is not explicitly taught as an example of an inverse relationship. There is no exploration of odd and even numbers, or properties of other multiples. Although multiplication is introduced as repeated addition, this is not done to explore the underlying general properties of association and commutativity.

Conclusion

This curriculum comparison focuses on how a particular conceptualisation of early algebra (an adaptation of Kaput's definition) is manifest in the intended curricula in the early grades in two countries. The analysis reveals that, the South African intended curriculum makes clear commitments to the teaching and learning of algebra, but it does not deal with it in any substance, and that, in contrast, the English intended curriculum includes more algebra content, although does not address 'early algebra' explicitly.

At the macro level of analysis, the content structure of the South African curriculum includes an explicit focus on early algebra from the first year of primary school as one of the five learning outcomes focuses on 'patterns, functions and algebra'. However at the first phase of primary school in South Africa the treatment of algebra is primarily a stepping stone towards algebra for subsequent phases in primary school. There appears to have been little reflection and exploration of how early algebra may manifest in the Foundation Phase. There is little focus on generalising, as having intrinsic mathematical value for this age group. England in contrast, does not include a specific content focus on early algebra in its primary mathematics curriculum. However, some of its *Guideline Papers* refer to the ideas underpinning early algebra as generalised arithmetic, even though this connection is not made explicit. Making this connection explicit and enabling teachers to see how the curriculum in England supports a growing body of research work around the world on the topic of early algebra would be useful.

At the micro level of analysis, the level of learning objectives and assessment standards, clearly shows that the English curriculum addresses early algebra in more detail. While the South African curriculum includes some examples which can be recognised as having the potential for exploring generalised arithmetic, it does not expect the underlying algebraic reasoning to be explicitly recognised. Several examples of generalised arithmetic relevant for this age group are included in the curriculum in England but are absent from the South African curriculum.

This paper is not intended to advocate for a change in the curriculum framework in South Africa. Nor is it intended to advocate for yet another assessment framework in England. Rather, this paper strives to highlight the need for more detailed guidance for South African teachers on the treatment of early algebra, and the need for England to simplify and streamline the supporting documentation for its intended curriculum, which should make the treatment of early algebra explicit. It is hoped that this paper allows for reflection on how intended curricular may be used to better communicate research in early algebra, at an appropriate level of detail, to teachers in the early grades.

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