Analysing children's calculations: the role of process and object

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This paper reviews the role of process and object in young children's calculation strategies. By drawing on the Action, Process, Object, Schema (APOS) framework (Dubinsky and McDonald, 2001) children's calculation strategies are analysed. It is suggested that the opportunity for children to reflect on the actions they perform and also to reason about them is important in developing a coherent framework and hence a deep understanding of the calculation strategies they are using.

Introduction

In relation to the current concern to support understanding in mathematics (Williams, 2008) this study examines children's understanding of the calculation strategies that they use. It explores the strategies that children use in single digit addition and subtraction for example (7 + 8; 9 – 7) as they move to the use of single to multi-digit addition and subtraction (for example 5 + 13; 16 – 9).

The children studied were from KS1 classes. Whilst not identified as needing intensive support, they were identified by their teachers as lower attaining. It is recognised that these children are often seen as 'passive' learners. They are often tentative about their own understanding in mathematics (DCSF, 2007) and may not make the expected progress in KS2.

The role of processes and objects

It has been suggested that lower attaining children rely on counting procedures in addition and subtraction (Gray, 1991). Such strategies are seen as primitive and inefficient. A reliance on such strategies could explain the lack of progress in KS2. Whilst it is possible to be successful in calculations up to 20 or so using such strategies, progression to calculations with other 2 digit numbers becomes less successful.

The National Strategies in England have encouraged the use of flexible strategies that provide a more efficient method for calculations. It is now recognised (Gray, 1991, Steffe, 1983) that there is a progression from simple counting strategies ('count-all' and 'count-on' strategies) and the use of commutativity ('counting-on from the larger number') to the use of number facts (additive components) and place value. Initiatives such as the National Strategies in England have proposed the explicit teaching of calculation strategies but limited understanding of the key ideas underlying the progression may result in children becoming passive recipients of the calculation strategies that are presented to them in the classroom (Murphy, 2004). Such flexible strategies require more active ways of using number (Plunket, 1979) and their use goes beyond the counting procedures that lower attaining children often rely on.

Sfard (1991) recognised a reliance on 'count all' where children count out each set (for example 3 + 4 becomes 1,2,3 and 1,2,3,4) and described this as the child's view of numbers as processes, rather than as objects. In this way the 3 has to be counted out. This is distinct from seeing 3 as a cardinal number or object that can be 'counted on' from. Sfard's work looked at the dual nature of processes and objects, where "the ability of seeing a function or a number both as a process and as an object is indispensable for a deep understanding of mathematics…" (p. 5). In order to work with calculations more flexibly children need to see the dual nature. Gray and Tall (1994) have also referred to this as a 'procept' where children see the symbol for an operation as both a process and as a concept.

Dubinsky and McDonald's (2001) work has been used to study undergraduate level mathematics but their theoretical framework Action-Process-Object-Schema (APOS) is drawn on here as a possible way to provide insight into children's calculations as they move from counting procedures to more flexible strategies. As an *action* addition or subtraction are said to be carried out as transformations of objects. Understanding is limited to the performance in each situation. By repeating the action internal reflection is possible and the *process* can be described and the steps reflected on, even without performance. By reasoning about the properties of the processes ideas are treated as *objects*. The eventual aim is that these actions, processes and objects become a coherent frame or *schema*. It could be argued that in order to be flexible in the use of calculation strategies children need to be able to reason about the processes and even to be aware of the overall frame or schema.

In this study examples from a study into children's calculation strategies are presented. The children's strategies are analysed according to their use as actions, processes, objects or schemas. Can the children's use of counting procedures be interpreted as actions or is it possible to identify reflection in relation to a process or reasoning in relation to an object? Are the children also able to see the overall coherent framework or schema? For example a child may use a direct modelling strategy for 13 + 8 by counting out one set of 13 toys and another set of 8 toys. These two sets can then be physically pushed together and the total number of toys counted altogether. It would seem possible to interpret this as an action. However if the child was able to reflect on the action they had taken without performing it again would this be seen as a shift to a process? Would the child also be able to reason why they had used the process? By interviewing children it may be possible to see if the children see a strategy as an action that is performed or whether they are able to reflect on the strategy or even reason it.

The use of the Clinical Interview

Clinical interview refers to a class of interview methods that typically involve an extended dialogue between adult and child (Ginsburg, 1997). This provides an intensive interaction with an individual child and allows for the observation of a child's work with 'concrete' intellectual objects. As a research method it is deliberately non-standardised. It provides an interpretive orientation where the examiner is engaged in interpreting each child's responses. Interviews may start with some common questions but the examiner will react to responses and tailor questions to the individual child. In this way it can provide a way to explore "deep insights into children's thinking" and go "beneath the surface" (Ginsburg, 1997, p.2).

The interview schedule was based on an adaptation of the diagnostic tasks developed by the Shropshire Mathematics Centre (1996) and introduces children to addition and subtraction problems starting with single digit and moving to multi digit problems. The main study is with 72 children aged between 6 and 7 years old from twelve classes. All of the 72 children have been assessed as average to low attainers by their teachers and represent children who use both spontaneous and taught procedures but may not always use these correctly. Four children from one class were selected from the early stages to pilot the analysis.

Results

In this paper results are presented from four children, Helen, Cassie, Bill and Jim who were in a Year 2 class (6 to 7 years old). The class teacher had recently modelled the use of the hundred square for the addition of multi-digit numbers and in particular she had encouraged the children to count on in tens rather than counting on in ones.

In the interviews the children were shown calculations one at a time. They were asked what the answer was and then asked to explain how they had worked the answer out. Observations were made of the children's work and the resources they used were also noted. It was anticipated that the children would use a mix of spontaneous and taught strategies.

Helen

Helen chose to draw figures to help her calculate 6+9 (figure 1) and 13 – 6 (figure 2). It would seem that she was engaged with addition and subtraction as an action and she was able to talk through the steps as she took them. There is little evidence to suggest that she was reflecting on the actions as processes or even reasoning about them as objects.

Figure 1: 6+9

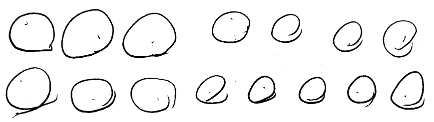
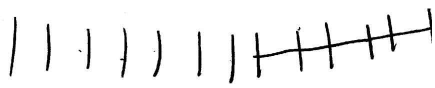


Figure 2: 13 - 6



However Helen's solution to the subtraction problem 7-6 was different:

> H: You have six and then you only need to add one. So if you take away 6 there will only be one left. 'Cos it is the other way round.

In this solution Helen was able to describe the action she had taken and reflect on this as a process without performance. She used her knowledge of counting in ones 'You only need to add one' and she used addition and subtraction as inverse operations. Was she able to reason this or was she using this implicitly? Her use of the next word 'So…' might suggest that there was an element of reasoning about the process as an object.

Cassie

Cassie used her fingers and a hundred square to count on for the addition 5+7:

> C: I can't figure it out.. I think it's 12 or 11. Not sure. My fingers got 12 but I got 11 on the hundred square.
>
> Ex: When you did it with your fingers what did you start with?

C: I started with five and then I went 1,2,3,4,5,6,7 – went on 12 (Counts on again on hundred sq). Oh - I made it.

This could suggest Cassie's use of actions in performing the calculation. When she arrived at two different answers there was a need for her to review the steps she had taken in the actions. Although she did arrive at the correct solution there is little evidence that she reflected on the action as a process. She repeated the performance rather than reflected on it.

Cassie also used a 'count on' procedure to add 7+8 and got the answer 15. She then used the procedure to add 6+9.

C: Mmmm… 15. I got 15 again.

Ex: Did you expect it to be the same?

C: No. I didn't even know it would be the same

Cassie checks her calculation for 7 + 8.

C: Both answers the same.

Again Cassie was relying on actions. The equivalent solutions to the two additions did prompt her to review the steps she had taken but, again, she repeated the performance rather than reflected on it.

Bill

Bill was less reliant on counting strategies for his solution for 10+8.

B: Well I just add 10 and 10 is 20 and I took two away.

Bill was able to draw on known facts and to carry out an adjustment to find the solution. He was able to describe or reflect on the process without performance. However in the account that he gave it was less evident that he was reasoning about the process, he does not explain why he carried this out.

Bill used a counting on strategy for 7+8. He made an error in counting and gave an incorrect response 14. However for the next addition 6+ 9 he responded:

B: 14 again.

Ex: How did you know it was 14 again?

B: 'Cos you lose one from that one and you add that one there.

Although Bill's strategy for 7+8 provided the wrong answer he used the solution to inform the next calculation. He did not have to repeat the action for 6 + 9 but was able to reflect on the action as a process. He was also able to reason why he had used the process ('Cos if you lose one from that one…) and it is possible that he was treating the ideas as objects.

Bill also showed evidence of reflecting on his actions as a process for the addition 8+13. Bill first gave the answer as 23, and then he went on to explain his strategy.

B: Well I just… It's not 23.'

Ex. How did you know it wasn't 23?

B: Cos it's 8, not 10. If it was 23 you would have to have a ten there' (points to the 8)

Ex. So what is it?

B: 21.

Here Bill described the action as a process without performance and was also reasoning the solution.

Jim

Jim used counting strategies for single digit additions. However for 10+8 he asked to try another way using the hundred square as he had been shown by his teacher:

> Jim: You could go back to 8 and then count on 10, then you do down one and you're answer would be 18.

Although not relying on counting strategies Jim's account suggests that he was carrying out an action. He did describe and reflect on the action as a process but there is little evidence that he was reasoning about the process.

His attempt to use this strategy was not always effective. For example 8 + 13:

> Jim: So I go all the way to 13 and then I jumped down one and add on 3. So it will be 26.
>
> Ex. Why were you adding on the 3?
>
> Jim: You go down 10 and you go across 3.
>
> Ex. If you started on 13 and then added ten and then another 3 you would be adding what?
>
> Jim: You would be adding tens and units. That's what we call it.

Here Jim did reflect on the action as a process but he was unable to reason why he had used the action and referred to this as a strategy that he had been taught.

Concluding Remarks

These short accounts have attempted to illustrate how the APOS framework might be used to analyse children's calculation strategies. It is recognised that further studies are needed to refine the use of this framework but even with these short accounts the results show possible insights into children's approach to calculations.

The children were often reliant on counting procedures and on occasions would make errors in their use. However they would often be able to reflect on the action as a process and to describe this without performing it. There was also evidence that children could reason about the processes using properties of the numbers. In these examples it is not the case that the children would consistently work in one area of APOS. Their ability to reflect or reason was often reliant on the problem presented to them. Further studies would be needed to determine if other children also moved within the APOS framework.

These insights also have possible implications for the teaching of calculations. If the aim is to support children's understanding and their use of flexible strategies it would seem that we should be supporting children in working towards the use of objects, in that they can reflect on their actions, and even reason about the processes they use. Cassie's focus on the actions caused her to be surprised regarding the equivalent solutions. In order to support her to reflect more on the process or to reason about the number properties, further work on equivalence using visual images or objects would seem appropriate.

Jim's reflection on his action as a process would initially suggest that he is developing a use of flexible strategies as he uses 'jumps' of ten rather than 'counting on' in ones. He is also able to reflect on and describe the action without performance. However his lack of reasoning would suggest that he does not see this process as an object. If the eventual stage is for the actions, processes and objects to become part of a coherent framework or schema, it would seem important that Jim is supported in being able to reason the use of the jumps of ten so that there is a shift from process to object.

In order for children to become active learners the need to be able to reflect on and reason about the actions and processes they use would seem paramount. Without the opportunity to do this it would not seem possible for children to progress to the development of a coherent framework or schema that would support them in using calculations flexibly.

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