Promoting a Cross-curricular Pedagogy of Risk in Mathematics and Science Classrooms

Ramesh Kapadia and Phillip Kent, with Ralph Levinson, Dave Pratt, and Cristina Yogui

Institute of Education, University of London, WC1H 0AL, United Kingdom

This paper reports on a research project on the teaching of risk with teachers of mathematics and science in Key Stage 4. Software models of socio-scientific issues have been developed and tested to support teachers towards developing a pedagogy of risk in their teaching. Transcripts from a workshop with teachers are used to illustrate some key findings.

Background and Research Aims

This paper reports on the findings of a research project[1] whose aim has been to support and enhance the teaching of risk at Key Stage 4. An innovative and cross-curricular approach has been used, based on modelling socio-scientific issues using new technological tools, designed specifically to enable the consideration of ethical and social issues, whilst building personal value systems alongside quantifiable mathematical models. By interacting more deeply with interdisciplinary knowledge, the aim is that mathematics and science teachers can be empowered to develop meaningful activities around the concept of risk. A key component of the research has been that pairs of teachers (one mathematics, one science) from the same school work together to explore the possibilities of cross-curricular working in their school, and also become involved with the design of materials in the project.

The discourse of risk permeates public policy and, increasingly, the choices people face in everyday life - consider how current medical practice favours informed choices to be made by the patient where previously decisions were taken by the medical expert on behalf of the patient. In response to these societal changes, risk is now recognised as a school topic through its recent incorporation into the mathematics curriculum alongside probability, and it has been part of the science curriculum for some time, particularly through the "How Science Works" Programme of Study. However, we note that science teachers continue to be reluctant to teach risk, not only because of concerns about their own knowledge limitations, but also because they are unsure about appropriate pedagogy for engaging students in the topic. Risk as a science topic is intimately connected with "socio-scientific issues" (such as stem cell-based medicine and climate change) and requires discussion and evaluation of argument, a challenge since evidence shows that science teachers are not always confident in using teaching methods based around discussion (Bryce & Gray 2004). This difficulty is even more a problem for mathematics teachers where "social issues" are unfamiliar ground.

If the current generation of school students are to be sufficiently confident in grasping the idea of risk in varied contexts, a number of obstacles need to be overcome. Mathematics teachers do not feel confident in teaching topics on risk and

need to be able to have the resources and the experience to teach it well; this is also partially true of science teachers though they have more experience of social issues impinging into science lessons. Hence this project is helping teachers with resources and a pedagogy to reflect the new curricula they are teaching, which responds to their present level of knowledge and which is viable to use in the classroom.

The understanding of risk incorporates concepts which go beyond substantive science or probability into affective and social science domains. To address this problem, we have designed (in collaboration with teachers) computer-based modelling tools in which teachers can explore and interrogate their knowledge of risk. The tools are set within decision-making scenarios to help teachers increase their own confidence in teaching about risk, and be able to enthuse their students, particularly with regards to probability.

The idea of microworlds began with Seymour Papert and Logo, as described by Noss and Hoyles (1996). They linked direct manipulation of computational objects with the description of their arrangement in sequences as Logo programs. Their intention was to help students build links between seeing, doing and expressing through the common representational infrastructure of Logo, to investigate how mathematical meanings were structured by the tools available for expressing the relationships under study and to trace how far this mediational process assists in the construction of the mathematical meanings intended by their teachers or curriculum designers. The approach is based on a constructivist philosophy.

The Idea of Risk

The judgement of risk is important in the public understanding of science in modern society (Rees 2006). Many socio-scientific and financial issues in the media can only be rationalised through an appreciation of risk. The confusion about the nature of risk evident in the media is reinforced by its ambiguous use in text books and academic articles. Often, risk is expressed as likelihood, equivalent to probability. We might then ask if the risk of an aeroplane crashing is 1 in 10 000 and the risk of catching measles is also 1 in 10 000, do we wish to equate the two risks? There is the underlying question of how such probabilities are estimated. However, the *impact* of these two events is very different. We adopt the view that risk is best considered to be a combination of both the impact of the hazard (also called the utility or dis-utility) and its likelihood of occurrence. This is the standard concept of risk as used in statistical decision theory (Edwards & Tversky 1967); however we do not go along with the standard view that risk is always a simple arithmetic product of the likelihood and utility values.

Although our approach emphasises how subjective values can be marshalled through an objective framework, risk does not have an established and agreed epistemology. Wilensky (1997), in relation to probability, has discussed how such uncertainty about the foundations of a concept can lead to "epistemological anxiety".

Factors other than epistemological anxiety also affect how risk is perceived (Kahneman & Tversky 1979, Campbell 2006). Dying from an aeroplane crash is a concentrated risk in so far as many people simultaneously would suffer the consequences should the accident occur. In comparison, dying from a road accident is a diffuse risk. Fear of flying is not uncommon and can be seen as an example of how concentrated risk is often regarded as worse than diffuse risk, even when, as in this case, the diffuse risk is in fact much greater in terms of actual deaths. Risks that are voluntary or associated with benefit from the observer's point of view may often appear lower, so that risks from alcohol or smoking may be perceived lower than they

really are. The problem in understanding risk is accentuated by a corresponding difficulty in appreciating its mathematical core of probability. The literature contains many studies that demonstrate that, without a secure understanding of (probabilistic) situations, people's reactions are often guided by misleading intuitions, which are rooted in inadequate cognitive biases (Kahneman et al. 1982, Kapadia & Borovcnik 1991, Lecoutre 1992).

Nevertheless, research shows that participation is best enhanced by engaging students with topics about which they are deeply concerned (Ainley & Pratt 2006). Children and adults routinely encounter science in the media: climate change and global warming, smoking and cancer, alcohol consumption and heart disease, nutrition and obesity to name just a few. It is exactly these issues that students, find engaging, so they are an important starting point for increasing involvement (Haste 2004).

Teachers can be assisted to find ways of helping students to recognise the unpredictability of chance at the individual level as well as its apparently contradictory predictability at the aggregated level and to assess the reliability of the source of data on which a judgement is based (Kolstø 2001). They can also be enabled to explain the relationship between deterministic and stochastic thinking (Prodromou & Pratt 2006). There is now an expectation that teachers should respond to this challenge. For example, the *21st Century Science* curriculum scheme[1] (used by about one-third of school students in England) requires that students "can explain why a correlation between a factor and an outcome does not necessarily mean that one causes the other", and "can suggest reasons for given examples of differences between actual and perceived risks" (OCR 2009, Appendix F.5).

There are few materials which have addressed these issues for mathematics teachers. Early work by a Schools Council Project on Statistical Education (1980) developed materials, relating to smoking and other areas where risk can be explored. More recently, the Bowland Maths Project[2] produced several modules for Key Stage 3 called 'How risky is life' and 'Reducing road accidents'. These modules are exploratory and open-ended, but not explicitly cross-curricular. They certainly advance the debate about how to make mathematics engaging but from our perspective they present a limited view of risk: there is a focus on probabilities of hazards to the exclusion of considerations of impact. The modules are clearly developed with the mathematics classroom in mind, and do not explicitly draw students into consideration of ethical and moral values in decision-making.

Our perception of practice in the science classroom (based on literature and discussion with teachers) is that the topic of risk is usually handled with a focus on the "social" dimension of contemporary socio-scientific issues, such as how the popular media mis-represents scientific knowledge and practice for its own ends (for example, the risks involved in children having or not having combined MMR vaccination). What is largely absent is a quantified approach involving numerical probabilistic information or mathematical modelling of risk. We believe that it is essential for teachers and learners to be able to build and explore their own personal models in decision-making situations, which combine quantitative, semi-quantitative and unquantified factors. Hence in our research we have developed scenarios, one of which is described below, which have a strong focus on trying to put numerical data into models, using both probability and impact as dimensions of risk. This also offers the opportunity for mathematics teachers to make probability (which can sometimes

---

[1]   www.21stcenturyscience.org
[2]   www.bowlandmaths.org.uk

be seen as a quite dull topic with artificial situations) more interesting by linking to real world situations and contexts.

Deborah's Dilemma: Some evidence about teachers' thinking in a risk scenario

A software environment has been developed called Deborah's Dilemma. Teachers are invited to advise a fictitious person, Deborah, on whether to have an operation that could cure a spinal condition that is causing her considerable pain. The operation would entail certain hazards with risks that need to be inferred from various sources of information. Choosing not to have the operation would entail a lifestyle choice to manage the ongoing pain resulting from the spinal condition. Two software tools accompany the information about the condition. The first ("Operation Outcomes") is a probability simulator in which users model the possible consequences of having the operation. The likelihoods for various complications (i.e. side effects of the surgery, ranging from minor to serious, such as super bug infection, or death through general anaesthetic) are quoted in differing forms. Ambiguities in the data are deliberately set up in order to provoke discussion and debate; for example a "spine expert in London" and "a surgeon in a regional hospital", give conflicting opinions – who should one trust?

The second tool, the "Painometer", is a less conventional attempt to give a quantified experience of Deborah's pain and how different activities may cause it to increase and decrease, relative to a "tolerable" level. Pain is an experience that defies objective measurement; hence the personal perception of pain is a potentially interesting context for probing people's personal models of risk. Users are required to decide what everyday and leisure activities Deborah should or should not do, and in what amounts, and to infer from the information the effect on Deborah's pain level of those activities (as expressed through the dynamically-varying height of a vertical bar as a "painometer"). The intention is for the tools to promote quantification of risk in a real context, whilst allowing for personal interpretation, and not constrained by formal models of risk such as exist in statistical decision theory, and which are conventionally used in risk assessment.

We now present some data from workshops that we have run with a small number of teachers exploring Deborah's Dilemma. Data were collected in the form of audio recordings of group discussions, and screen capture with audio of teachers working with the software in pairs; also researchers observed and questioned the teachers as they worked. We have assessed the teachers' attempts to compare and weigh up information about the pain against the risks of the operation, using ideas of impact such as minor or major complications, and what these data suggest about the teachers' thinking. Below, some results with one pair of teachers are presented: A is the mathematics teacher and L the science teacher. The transcript is given before the discussion.

Episode 1

- *[The surgical operation]*
- L: It must be pretty bad pain to consider…
- A: If you're going to consider the surgery it's got to be
- A: 'Unwanted after effects' (laughs). So 95 to 98% successful. So that's…is that 95 to 98% of the time the pain is relieved? Or 95 to 98% of the time there's no complications?
- L: That's a good question

- ...
- L: Complication three…oh ok so you don't need to…more risky than what's going on. 'This was helpful…I asked my doctor for a second consultation…' Yeah, major surgery; general anaesthetic is another risk, and the superbug
- A: 'Currently about 0.00025%' 25 in 100,000
- L: See that's what we were thinking; '…developing my lifestyle to support long term management of the pain'. '…Yes, I can live with the pain and I have adjusted my lifestyle successfully'. It's more of a lifestyle operation; it's not something that's going to save her life is it?
- A: Well she'll still be alive but it's got implications right through doesn't it. I suppose if it does deteriorate you set yourself up for…
- L: Well if anything did deteriorate you'd be in a different position but if you lived slightly different then… Do you want to look at the other two options?
- A: One in 1000 for general anaesthetic
- L: That's one in 10,000
- A: One in 4000 for the superbug

An unusual event happens in lines 8-14. A probability of 0.00025% is mistranslated to be 25 in 100,000, or 1 in 4000. This is taken to be the probability of contracting a super-bug. In fact this is far too high for the current prevalence of super-bug infections in UK hospitals. It is even the sort of mistake for which a teacher may admonish a pupil: getting an answer which cannot really be correct. It would mean that the super bug is contracted weekly in hospitals. This illustrates a problem which is known to arise: that people find large numbers and low probabilities very hard to comprehend and deal with. It is likely that, with more time and thought, these teachers would have realised their error. However, it is salutary to remember such issues in the design of materials about risk.

Episode 2

The second episode conveys a positive outcome from the exploration of Deborah's dilemma by teachers from different disciplines. A teaches mathematics and is positive about recommending the operation, while L teaches science and has some doubts.

- A: I'd have the operation, give it a go. That's my lifestyle.
- L: If someone said you couldn't play football that would be important? You wouldn't be prepared.
- A: Not only that, it's self-sufficiency as well. You need someone to do your shopping for you, already at 38.
- L: Well she can use a trolley.
- A: No, she can't lift a shopping bag – doing shopping would make you worse. Everything you chose to do for yourself would make you worse, leading to a more serious condition.
- L: If you're recommending it purely on the odds, right...
- A: No, not purely, I'm recommending it possibly against the odds, you know 3... over here [looking at written notes??] complications or failure is 5%.
- L: If you were saying it for yourself, you would be prepared, but say you were making it for her – then you'd be saying, well actually.
- A: Right, we have to come down on one side or the other, we can't just re-

present the evidence, we've got to say 'we recommend you do this', we can't sit on the fence.

- L: Do you think if you were the doctor, if you were someone close to Deborah...
- A: You might have a different opinion.
- L: ...it's going to come down to the relationship with the patient as well. What are we, we're just people looking at statistics, if the computer says no, because at the moment to me the computer is saying no.
- A: 3 in 1000 serious complications.
- L: That's serious, you are 38 your life could-
- A: Yeh!
- L: Because the computer... [laughs] Who are we??
- A: That's the question isn't it?
- L: All these things, is lack of sleep, nausea, even depression
- A: That can ruin your life.
- L: [reads] I will experience high level of ... I've had enough, I've had several years of it It's kind of difficult to see Deborah on paper...
- A: Do you think she should shut up and stop moaning?

A key overall observation about A and L's exploration is that there was a regular switching of position which depended on the information immediately in front of them (the 'availability heuristic), and their shifting estimations of impact. Initial exploration of the "outcomes" tool suggested a likelihood of serious complication arising from the surgery as the order of several cases in a 1000, and they initially talked about this as a "reasonable risk". However as they explored with the Painometer tool they began to favour a non-surgical approach and to re-assess the reasonableness of the surgical risks. Our interpretation is that they were left in an ambiguous position between the two "sets of evidence" and in the end rationalised their position by appealing to the authority of the "specialist in London": L comments:

> I think the fact she's gone to the person who knows more about it, spends his life looking at risk, he says no there are better things out there for you, other options … As a scientist I would go with the specialist spine doctor who knows more about it, I would go with what he says.

There is an interesting – if speculative – positioning here, worthy of more systematic exploration, where L talks "as a scientist" explicitly about authority of a person, whereas at other moments with A leading they look to the evidence coming from "the numbers".

It would seem that A (the mathematics teacher) brings in the probabilities to make the choice; this is mediated by L (the science teacher) in bringing in social issues, placing stress on impact (minor and major complications); A changes his mind, as illustrated in the final recommendation that the pair produce, presented (interestingly) as a personal letter, reproduced below. It should be noted that this happened with one pair of teachers and illustrates the importance of collaboration and discussion between teachers from (different) disciplines. The approach taken by the project has enabled effective cross-curricular working in an aspect of pedagogy which is often absent from classrooms, the discussion of issues beyond the quantitative or scientific evidence.

> Dear Deborah, our recommendation is that you do not have the operation and further investigate pain relief and other options for support and managing your lifestyle. We have looked at the probabilities of failure and complications during surgery and whilst the likelihood of severe complications is around 0.4% - possibly quite low – we are unsure as to the exact chance of success as the study quoted by your first doctor referred to arm pain only and we are inclined to take the opinion of the spine specialist that managing your condition is the best course of action. We feel the probabilities are too high against you if the surgery is not entirely necessary. This is obviously balanced against how your condition is affecting your daily life and how often the pain is above your tolerance level. If the condition worsens then surgery is not a prohibitively dangerous option in that case.

Conclusions

The research is continuing with a wider evaluation of our decision-making scenarios with mathematics and science teachers, and advisers and educational researchers. However, a number of interesting ideas have already emerged.

Deborah's Dilemma encourages teachers to introduce quantitative and numerical elements via a decision problem. It also encourages them to recognise and introduce personal and social elements into decision-making through ideas of impact and utility of different choices. The data suggests that the cross-curricular dialogue approach does produce a rich exploration for both, and leads to views about risk being shared. This is certainly important for mathematics learning as social issues rarely feature in mathematics classrooms and yet are obviously important in studying uncertainty.

The risk context also offers a fresh perspective on research into perceptions of uncertainly. For example, it has been known for some time that people find it hard to manipulate and use very low (or very high) probabilities (Kapadia & Borovcnik 1991). This research has found some evidence of this problem (cf. episode 1) (confirming the need for more study), which may need to be taken into account in the pedagogy of teaching risk, particularly as the spectrum of risks commonly discussed range over many orders of magnitude.

References

Ainley, J., D. Pratt and A. Hansen. 2006. Connecting engagement and focus in pedagogic task design. *British Educational Research Journal* 32(1): 21-36.

Bryce, T. and D. Gray. 2004. Tough acts to follow: the challenges to science teachers presented by biotechnological progress. *International Journal of Science Education* 26(6): 717-733.

Campbell, S. 2006. Risk and the Subjectivity of Preference. *Journal of Risk Research* 9(3): 225-242.

Edwards, W. and A. Tversky, eds. 1967. *Decision making: Selected readings* Harmondsworth: Penguin Books.

Haste, H. 2004. *Science in my future, Nestle Social Research Programme report No. 1.* Accessed January 8th, 2010 from http://www.spreckley.co.uk/nestle/reports.htm.

Kahneman, D. and A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* 47(2): 263-292.

Kahneman, D., P. Slovic and A. Tversky. 1982. *Judgment under uncertainty: Heuristics and biases*. Cambridge: Cambridge University Press.

Kapadia, R. and M. Borovcnik, eds. 1991. *Chance encounters: Probability in education.* Dordrecht: Kluwer Academic.

Kolstø, S. D. 2001. 'To trust or not to trust, . . .' - Pupils' ways of judging information encountered in a socio-scientific issue. *International Journal of Science Education* 23(9): 877-901.

Lecoutre, M. P. 1992. Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics* 23: 589-593.

Noss, R., and C. Hoyles. 1996. *Windows on mathematical meanings: Learning cultures and computers*. Dordrecht: Kluwer Academic.

OCR. 2009. *GCSE in Science A Twenty First Century Science suite specification*. Cambridge: Oxford Cambridge and RSA Examinations.

Prodromou, T. and D. Pratt. 2006. The role of causality in the coordination of two perspectives on distribution within a virtual simulation. *Statistics Education Research Journal* 5(2): 69-88.

Rees, M. 2006. Address of the President, Lord Rees of Ludlow Kt FRS, given at the Anniversary Meeting on 30 November 2006. *Notes & Records of the Royal Society* (22 January 2007) 61(1): 75-83. (Accessed January 4th, 2009 from http://rsnr.royalsocietypublishing.org/content/61/1/75.full)

Schools Council Project on Statistical Education. 1980. *Statistics in your world*. Slough: Foulsham.

Wilensky, U. 1997. What is normal anyway? Therapy for epistemological anxiety. *Educational Studies in Mathematics* 33: 171-202.

Corresponding author: R. Kapadia@ioe.ac.uk