Using Realistic Mathematics Education with low to middle attaining pupils in secondary schools

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This paper provides an account of two projects involving the trialling of a new approach to teaching in secondary schools in England. The method being trialled is based on Realistic Mathematics Education (RME), originally developed in the Netherlands. The paper focuses on the underpinning of RME, provides an overview of the associated projects, the research methods and initial findings, and explores emerging issues from the projects.

Introduction

The Smith Report (2004) stimulated a lot of interest in applications of mathematics and supporting learners in becoming mathematically literate. Consequences of the Smith Report included the introduction of GCSE assessment in mathematics which makes greater use of contextual questions and the piloting of a 'twinned pair' of GCSEs, one of which focuses entirely on the application of mathematics. An overriding issue for many mathematics educators is how we support learners in developing application skills as well as learning the normal content associated with mathematics without separating these out and increasing the time allocated to mathematics teaching.

This paper explores the conjecture that it is possible to develop both content knowledge and problem solving skills using an approach based on Realistic Mathematics Education (RME).

Background

Mathematics in England

The mathematics curriculum in England has undergone radical changes in the last fifteen years with the introduction of a variety of forms of formal and informal assessments, emphasis on functional skills and the advice offered to teachers in how they might support learners. A recent study would suggest that despite the investment in mathematics there is little evidence that the standards in mathematics have improved (Hodgen et al. 2009)

Clearly the strategy has been effective in changing some of the patterns of behaviour of teachers and in shifting the emphases on different parts of the mathematics curriculum. However, the work of Anghileri et al. (2002), Brown et al. (2003) and Hodgen et al. (2009) would suggest that there may be grounds to doubt that these changes have been as effective as the government would wish us to believe. Despite apparent short-term improvements as measured by the end of key stage assessments, Smith (2004), Brown (2003) , Anghileri (2002) and Hodgen et al. (2009) all highlight worrying concerns about longer-term conceptual understanding, procedural fluency, and the ability to apply mathematics. Indeed Askew et al. (2010)

argue that, in England, procedural fluency and conceptual understanding are largely seen as mutually exclusive aims.

The Smith Report (2004) suggests the need for "… greater challenges… harder problem solving in non-standard situations, (and) a greater understanding of mathematical interconnectedness …". The report also indicated that the mathematical skills developed by pupils age 16 are not concerned with " the growing mathematical needs of the workplace… mathematical modelling or ... problems set in the real world contexts." Smith also suggested that in comparative terms "England seriously lags behind its European competitors" in terms of the number of pupils achieving an appropriate level 2 qualification. Hodgen et al. (2009), suggest that, while exam passes have risen dramatically in the last 30 years, pupils' underlying understanding of mathematics has changed little.

In summary, we see the above as evidence of a need to explore and develop a practical pedagogy of mathematics education that supports pupils' conceptual understanding, problem-solving skills and the use of these in real world situations.

Mathematics in the Netherlands

The Freudenthal Institute, University of Utrecht was set up in 1971 in response to a perceived need to improve the quality of mathematics teaching in Dutch schools. This led to the development of a research strategy, an approach to teaching and to a theory of mathematics pedagogy called Realistic Mathematics Education (RME). RME uses realistic contexts and a notion of progressive formalisation to help pupils develop mathematically. A strong feature of RME is the simultaneous and integrated development of conceptual and procedural knowledge. Pupils engage with problems and scenarios using common sense/intuitions, collaboration with other pupils, well judged activities and appropriate teacher and textbook interventions. (See Treffers (1991) and Treffers et al. (1999) for further discussion of RME.)

At a surface level, RME resonates strongly with progressive approaches used in England where investigative and problem-solving strategies are utilised and where pupils are encouraged, as a whole class, to discuss their work to resolve important issues. One difficulty with this approach to teaching in England is that pupils may stay with naïve mathematical strategies and are often unwilling to move to more sophisticated strategies and procedures. The need for a teaching and learning trajectory is clear. Through intensive research, trialling and re-evaluating materials and approaches, Dutch mathematics educators have developed a variety of ways of encouraging and supporting pupils' mathematical progress. So, for example, pupils remain in context throughout and stay with a topic for a much longer period of time than would be usual in England.

Associated Projects

MiC in the US

In 1991, The University of Wisconsin (UW), funded by the National Science Foundation (USA), in collaboration with the Freudenthal Institute, started to develop a curriculum and pedagogy based on RME (See Romberg and Pedro (1996) for a detailed account of the developmental process and van Reeuwijk (2001) for an account of the care taken in developing one aspect of the scheme.) The first version of Mathematics in Context (MiC), together with comprehensive teacher materials, was published in 1996/7 and has undergone several revisions since then.

MiC in the UK

The Gatsby Foundation agreed to fund Manchester Metropolitan University (MMU) to run a project based around trialling RME (utilising MiC) over a three year period. The Economic and Social Research Council (ESRC) also agreed to fund an examination of how teachers' beliefs and behaviours change as a result of engagement in the project (see Hanley et al. (2007) for an account of the research into the changes in teachers involved in the project).

The project focused on three main issues: developing an understanding of RME in an English context, understanding how learners develop, and supporting teachers to develop practical skills and a deep knowledge of RME.

In terms of pupil development over three years the project team saw evidence that pupils' approach to solving problems changed and that this influenced how they understood the mathematics. More details of this are given below; for other findings of the project see Dickinson and Eade (2005).

MSM in the UK

In 2007, as an extension to the work of the Key Stage 3 project, The Making Sense of Mathematics (MSM) project began. This was aimed at Foundation Level GCSE students (Years 10 & 11) with new resources being produced as a result of collaboration between the Freudenthal Institute and MMU. These resources consist of 11 booklets which together cover the Key Stage 4 Foundation level curriculum. These booklets build upon the experiences gained from the Gatsby project and take account of difficulties highlighted by the Key Stage 3 teachers, such as the need for RME based materials which feature British contexts and are more closely linked to UK national tests.

The MSM project has involved Foundation level classes from 6 schools in the first cohort and 10 schools in the second cohort. MMU has supplied resources to these schools and has provided ongoing support in the form of twilight training sessions and school based observations. Feedback given by the teachers has been used to revise the materials which are currently in their second version.

Key findings from the MSM project to date show influences on both teachers and pupils. For the purposes of this paper, we focus on pupils, and the similarities between KS3 pupils working with MiC and KS4 pupils working with MSM.

Data Analysis

As mentioned above, in both UK based projects (MiC and MSM), data was collected from project pupils and 'control' pupils. In terms of the control, pupils were drawn from parallel classes in the project schools and also from non-project schools whose intake and performance were similar to those in the project. Project and control were matched either in terms of their KS2 SATs scores (for the MiC project) or in terms of GCSE target grades (for MSM).

While data was collected from project and control groups in a number of ways (including national test data and attitudinal questionnaires) the focus for this article is on what we termed 'problem solving'.

In the problem solving test, pupils were given ten minutes to complete each question. Consequently, much more detailed solutions were produced, and it was possible to analyse methods and approaches in addition to whether or not pupils had arrived at a correct answer. Since one of the aims of the project was to construct an account of how pupils develop mathematically, we needed an assessment which would provide data on this. The programme team was also mindful to take account of

Hawthorne related issues (Landsberger 1958) associated with simple tests of content gains and was therefore looking to find evidence of significant differences in pupils' approaches to solving mathematical problems. After substantial trialling, five questions were produced across the attainment targets and all project and control pupils worked on these individually over two lessons. The questions come under the general heading of 'problem-solving' in that they required pupils to 'mathematise' situations which they had not met previously. For further details of the original MiC problem solving tests, see Dickinson and Eade (2005)

The data discussed here are from Year 7 pupils for MiC (sample sizes 100) and Year 10 and 11 pupils for MSM (sample sizes 50). With so many schools and pupils involved, we are confident that the data gathered is reliable.

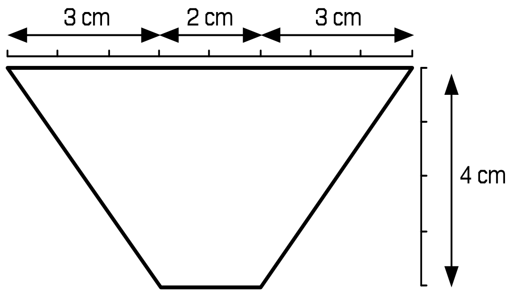
We wish here to focus on two topic areas, area and fractions. We have chosen these firstly because the traditional treatment of these topics at secondary level tends to be procedurally dominated and secondly, because they exemplify nicely the differences between project and control pupils that were noticed across all curriculum areas. We have also focussed on results from Year 7 pupils who were in the lower 40% of the attainment range for KS2 SATs scores) so that results from the two projects can be compared more easily.

Area

Here, the identical question was given to pupils in both projects, with remarkably similar results. The question was

Find the area of the shape shown below.

Show carefully how you worked it out.



If we class 'drawing cm squares and counting', 'splitting into rectangle and triangles', and 'moving a triangle to create a rectangle' as 'sense-making strategies' (as against simply performing a numerical calculation), 74% of MiC and over 80% of MSM pupils attempted to make some sense of the problem (with over 50% achieving the correct answer). The corresponding figures for control pupils were 32% and 37%. Interestingly, control pupils who 'made sense' of the problem tended to get the correct answer; it would seem that for these pupils they can either do the question or they can't, there is no middle ground. In project groups, we see around 25% of pupils who cannot get the correct answer yet are able to make a sensible attempt (for example through counting squares).

Indeed the vast majority of control pupils adopted a purely numerical method, often simply adding, multiplying, or in some way averaging the numbers on display. This is despite the fact that all will have counted squares when initially introduced to area. There is a strong suggestion here that introducing formal mathematical knowledge at too early a stage in a pupils' development will replace any more informal ideas and leave the learner with no knowledge at all once a rule is forgotten (see Anghileri (2002), Hart et al. (1989), and Boaler (1998) for similar conclusions in other mathematical topics).

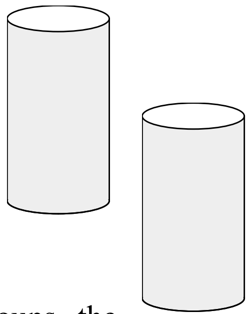
Fractions

In the Key Stage 3 (MiC) project, the following questions were set.

a) Tape is sold in pieces 1/3 of a metre long.

Show how many pieces you can cut from a piece 4 metres long.

b) 3/4 of this glass is full of orange.



1/3 of this glass is full of orange.

Explain whether you can pour the second glass into the first without it overflowing.

In terms of correct responses for lower ability groups, the following results were obtained (n=100)

Question Number	**Project Pupils**	**Control Pupils**
2(a)	42%	7%
2(b)	54%	12%

While the differences here are quite stark a further significant feature, in addition to the proportion of correct answers, was the number of pupils who drew something in order to make progress with the problem. 50% of project pupils attempted to draw something in part (a), compared to only 23% of control pupils. Similarly in part (b), the figures were 74% and 49% respectively.

In the Key Stage 4 project (MSM) a different fractions question was used, as evidence from another trial in a local school had suggested some interesting results.

The question was simply

Find ¼+½

Do you think you have got this right? Explain why.

Given that these pupils would have studied fractions for at least 7 years, the data gathered was somewhat shocking. In terms of pupils getting the correct answer, results were (n=50 in both cases)

	% correct Target grade C	**% correct Target grade D/E**
Project	83%	57%
Control	72%	30%

A striking feature when analysing pupils' work was that in the control groups one only had to look at the work of a small number of pupils to know exactly the method that had been taught to them and the procedure that their teacher wished them to follow. Script after script was identical in how pupils attempted to solve the problem (two of the most common ways are shown in Appendix 2).It then appeared that more able Foundation level pupils remembered the method and got the answer correct, but if they forgot the method (which was often the case for D/E level pupils) they produced something that amounted to mathematical nonsense. In contrast to this, project pupils seemed to have a variety of approaches at their disposal and were able to make sense of the problem.

It was also interesting that when asked for reasons, project pupils often drew something, talked about 4 quarters making a whole and hence a half being two quarters, or referred to pizzas, cakes, etc. Control pupils invariably referred to a numerical method, often simply describing what they had done. At target grade D/E, over 50% of pupils got the answer 2/6 with the vast majority believing that they were correct and citing the fact that 1+1 =2 and 2+4=6 as their justification for this. No project pupils at this level got the answer 2/6. We see this as further evidence that pupils taught an RME based curriculum are more able to make sense of their mathematics, both in achieving answers, and in reasoning why they feel they are correct. On the other hand, pupils taught formal algorithms have no other mathematical resources to fall back on if the algorithm is not remembered correctly. One control pupil (Year 10 target grade D/E), when faced with ¼ +½ commented tellingly that "I am stuck with this question because I forgot the method".

Why do RME based approaches lead to different results and (in particular) different methods of solution?

Use of contexts

In RME, contexts are used not only to illustrate the applicability and relevance of mathematics in the real world, but also as a source for the learning of mathematics itself. Contexts can be taken from the real world, from fiction or from an area of mathematics that students are already familiar with. It is important that they should be sufficiently real for students to be able to engage with them so that they are solving problems which make sense, but also critical that they reflect the mathematical structures that we want the students to work with. The contexts used are extensively researched and differ significantly from those found in standard UK textbooks.

Students are encouraged to make sense of the context using their experiences, intuitions and common sense. They then stay in context, and remain at a sense-making level, while they develop mathematically. The word 'realistic' is used to emphasise that students are able to imagine the situation.

Experience shows that, through staying connected with the context, students are able to continue to make sense of what they are doing, and do not need to resort to memorising rules and procedures which have no meaning for them. 'Mathematics' and 'context' are not separated – to experience success in one implies success in the other. So, for example, when working with area, the notions of reallocating parts of a shape and visualising arrays are predominant. If a formula or rule emerges, this is referred back to the context for validation. We believe that the benefits of this are clearly seen in the trapezium question analysed earlier-far fewer project pupils appear to view area as simply another numerical procedure.

Use of 'models'

RME provides a different view on how contexts should be chosen, and also on how these can then be used to support mathematical development. The use of 'models' is crucial here (see van den Heuvel-Panhuizen (2003) for a thorough analysis of the use of models under RME).

A model emerges from a context. Initially it may be little more than a representation, for example a picture, suggested by the context. Later, however, these models become more sophisticated mathematical tools such as the number line, ratio tables, etc.

Models bridge the gap between the informal and the formal and so teachers feel less pressure to replace students' informal knowledge with formal procedures. As pupils begin to formalise their mathematics, models and contexts support the process of vertical mathematisation while retaining the 'sense-making' element. In this way, the formal and informal are more likely to 'stay connected' in the minds of the pupils. This is evidenced, for example, by project pupils working with a common denominator in a way which makes sense to them, something which non-project pupils seem unable to do. Models also allow students to work at differing levels of abstraction, so that those who have difficulty with more formal notions can still make progress and will still have strategies for solving problems.

An important part of a student's mathematical development is the recognition that the same model can be used in a variety of situations and to structure solutions to many kinds of problems. As Boaler (1998) suggests, a traditional approach is unlikely to lead to pupils being able to recognise situations which are 'mathematically similar'. The mathematics levels in the 1999 National Curriculum for England characterise students' development by means of the procedures which they are able to perform and the difficulty of the numbers they are using. However, in RME, teachers use a richer range of descriptors to gauge students' progress. This includes observing students' use of models, insights and reflection as well as mathematical landmarks and procedures (see Fosnot and Dolk (2002) for discussion of this in the context of fractions).

Multiple Strategies

One aspect of being 'functional' in mathematics is being able to choose the most appropriate strategy to solve a problem, rather than always relying on one strategy or algorithm. The contexts in RME are chosen to elicit many different strategies and students are constantly encouraged to reflect on these and refine them. Lessons will involve comparison and evaluation of different student strategies; a fundamental tenet of RME is to build sophistication into student-generated procedures rather than for teachers to impose a 'standard method' or algorithm. Evidence of this is clearly seen in the Key Stage 4 data discussed here.

RME encourages the development of more formal methods from students' informal methods. However, it also allows students to continue to be able to use informal methods, where appropriate, rather than relying on being taught a method of solution that fits a certain type of problem.

We feel that there is significant evidence here that average and lower attaining UK pupils can benefit significantly from following an RME based curriculum and that through doing so they will be able to make more sense of their mathematics and hence become more functional in the subject.

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