Acknowledging the cultural dimension in research into mathematics teaching and learning

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In this review paper I make a plea for those involved in research into the teaching and learning of mathematics to acknowledge that however their work is framed, it will be located in a culture, not always visible to their readers, that needs making explicit. In order to do this I first examine three key models of culture and their significance for education. Secondly I consider various models of curriculum and ways in which school mathematics is presented. Finally, I summarise some recent comparative research in mathematics teaching showing substantial variation in the ways in which teachers manage the presentation of mathematics in their classrooms. In so doing a plea to researchers is framed: Culture permeates all aspects of educational endeavour and should be acknowledged more explicitly than it is.

Introduction

The purpose of this review paper is to invite researchers in mathematics education to acknowledge, in the theoretical framing and reporting of their work, the manner in which culture pervades all aspects of education in general and mathematics education in particular. Consequently, without wishing to over-assert what I feel is an important point, educational research should not be conducted independently of an acknowledgement of such influences. Having said that, culture is a difficult construct to define, although the consensus seems to be that it is, essentially, a collective manifestation of psychological conditioning and can be construed as a societal analogy to individual memory (Triandis and Suh 2002). Culture embodies the "implicitly or explicitly shared abstract ideas about what is good, right, and desirable in a society" (Schwartz 1999: 25). Culture includes those beliefs, artefacts and practices that history has shown to be effective for the maintenance of a society and its future generations (Hofstede 1980, Triandis and Suh 2002). In summary, cultures are social, historical and behavioural constructions that reflect the "collective mental programming" of their people (Hofstede 1980: 43). Through the educational transmission of their embedded traditions, values, beliefs, knowledge and skills, they ensure their continuing replication. In the following I consider three features of culture and show, in various ways, how they impact on the opportunities to learn students receive. These are various models of culture and how they shape educational systems, models of curricula and their culturally derived expectations, and a selection from the mathematics education literature highlighting the extent to which culture influences classroom practice and related constructs.

Models of culture

A number of researchers have attempted to categorise the ways in which cultures differ, and it is to them that we first turn. Hofstede's well known study initially identified four dimensions of culture. The first, *power distance*, concerns the extent to

which followers accept being led. "A society's power distance level is bred in its families through the extent to which its children are socialised toward obedience or toward initiative" (Hofstede and McCrae (2004: 62). The second, *uncertainty avoidance*, relates to the extent to which "a culture programs its members to feel either uncomfortable or comfortable in unstructured situations" (62). The third is *individualism*, or the "degree to which individuals are integrated into groups". Lastly there are *masculine* as opposed to *feminine* cultures. *Masculine* cultures "strive for maximal distinction between what men are expected to do and what women are expected to do". With respect to education, Hofstede's has shown how his four dimensions of culture can predictably explain variation in educational practice, not least through their impact on differences in the social positions of teachers and students, curriculum relevance, cognitive expectations, and differences in the patterns of participant interactions (Hofstede 1986). Others have exploited Hofstede's dimensions in the framing and interpretation of their work. In a study of 43 countries concerning class size, Cheung and Chan (2008), for example, found that cultural norms associated with power distance and collectivism had more significant impact on decisions concerning class size than economic factors.

Triandis (2001) has proposed an eleven dimensional model of culture, which, by way of brevity, is presented without discussion. Cultures differ in their *complexity*; for example, cities are more complex than villages. Cultures differ in their tightness; *loose* societies tend to be more tolerant of diversity and non-conformist than *tight* cultures. Cultures differ in their perspectives on the individual and collective. For example, tight and simple cultures tend to the collectivist. They differ in their *verticality* and *horizontality*, where the former are more accepting of hierarchical differences than the latter. There are *active* and *passive* cultures. In the former people try to mould their environment to suit them and tend to be more competitive. *Universalist* cultures differ from *particularist* cultures in that any suitably qualified person is eligible for a job. In *diffuse* cultures people are judged holistically while in *specific* cultures they are not. *Ascriptive* cultures judge according to an individual's attributes while *achievement* cultures judge them on their achievement. In *instrumentalist* cultures social activity is subordinated to work, while the opposite is found in *expressive* cultures. Cultures may be *emotionally expression* or *emotionally suppressive*. Lastly, the extent to which norms concerning *physical contact* varies. As with Hofstede's work, such characterisations help us understand the role of culture in framing humans' day-to-day actions and decision making in general and the conduct of education in particular.

With particular regard to education, Schwartz (1999), examined elementary teachers' and undergraduate students' education-related values and, drawing on data from more than 40 countries, identified seven dimensions of culture including conservatism (embeddedness), hierarchy, mastery, autonomy, egalitarian commitment (egalitarianism), and harmony. Thus, with respect to culture and education, for example, a conservative culture which emphasises the "maintenance of the status quo, propriety, and restraint of actions or inclinations that might disrupt the solidary group or the traditional order" would structure educational opportunities very differently from an autonomous culture in which an individual finds "meaning in his or her own uniqueness, who seeks to express his or her own internal attributes (preferences, traits, feelings, motives) and is encouraged to do so" (Schwartz 1999: 24). Importantly, not only for justifying his choice of research subjects but also for highlighting the significance of culture in this discussion, Schwartz (1999:) writes that teachers were chosen for his study because they "play an explicit role in value socialisation", are likely to be "key carriers of culture, and... reflect the mid-range of prevailing value

priorities in most societies".

Culture and curricula

The curriculum, according to the second international mathematics study, comprises intended, implemented and attained forms. However, what is frequently missing from such discussions is the extent to which historical and cultural forces have shaped the development of the curricula, in all its forms, of different countries. A number of researchers have attempted to categorise the manner in which curricula come into being. One example can be found in the work of Holmes and McLean (1989), who summarise four curricular traditions. The first, Essentialism, is linked primarily to English education and refers to the liberal arts curriculum of the late mediaeval English public[1] school, its replication in early models of public education, and its rejection of science and engineering as subjects studied by gentlemen. Cummings (1999: 423) describes the public school as "a boarding school with many rules, mandatory chapel, an emphasis on mind and body, which included daily athletics,... a strong classical thrust in the curriculum, and so forth". The second, Encyclopaedism, is generally linked to post-revolutionary France and draws on Enlightenment principles that education should include all human knowledge, be accessible to all and free from superstition (Cummings 1999). The third, Polytechnicalism, is linked to the Soviet Union and its satellites and incorporates notions of vocationalism into an essentially encyclopaedic model of knowledge located in a socialist moral philosophy (cummings 1999). Lastly, Pragmatism, is linked to the United States and addresses the acquisition of the knowledge and skills necessary for tackling real world problems of a participative democratic. Inevitably, despite their crudity and blurred edges, such models help explain why, for example, why French teachers present an intellectually sophisticated mathematics in comparison to the intellectually simplified mathematics presented by their English colleagues (Jennings and Dunne 1996).

In similar vein, Cummings (1999) writes that all curricula are based on cultural constructions of the ideal person, which, in the light of the discussions above, vary considerably from one culture to another. His analyses led him to conclude that most modern educational systems are derivatives of the traditions of six *core* curricula – the Prussian, Russian, French, English, Japanese and United States. Drawing on a range of factors, he offers a compelling account of not only the development of these six traditions but also the ways in which they have influenced curricula world-wide. In respect of three of these six core nations, Pepin (1999) has argued that the English tradition, with an emphasis on personal morality, has had an anti-rational humanist core privileging intuitive knowledge over the systematic construction of knowledge. With regard to the French tradition she writes that the post-revolutionary principles of *égalité* and *laïcité* facilitated the removal of social inequalities through a common broad curriculum and the expectation, unlike in England, that pastoral and moral issues will be addressed by the family. Lastly, the German humanistic tradition,

[1] As BCME readers will know, the English public school is an elite, usually very old, independent school. They employ competitive entrance examinations and are frequently so popular among the wealthy classes that parents may have to register their desire for their child to attend shortly after the child's birth. They are steeped in tradition and continue, as they have for many centuries, to furnish the higher ranks of the English civil service, the clergy, the judiciary, around half of all students at the leading universities of Cambridge and Oxford, and, of course, a disproportionately high number of members of parliament. Despite the name, they should not be confused with the public school systems of other countries, which, in England, is known as the maintained sector.

drawing on notions of *Bildung*[1], incorporates both encyclopaedic rationalism as well as moralism in its unified promotion of academic and moral education. Such views lead to the equal valuing of academic and practical knowledge as found in the tripartite structure of German schools.

Of course, the extent to which such underlying principles and traditions are manifested in the written, or intended, presentation of curriculum mathematics may vary from one country to another. By way of illustration, three European perspectives on the teaching of linear equations at the lower secondary level are presented below. Choice, in this respect, was constrained by the availability of curricula in English, while the topic was determined by other work on which I am currently engaged. They are presented alphabetically and, in respect of linear equations, verbatim.

The Finnish national curriculum[2] for grades 6-9 asserts that students, by the end of grade 8, "will know how to… solve a first degree equation".

The Flemish mathematics curriculum[3], expects students in the first grade of secondary education to "solve equations of the first grade with one unknown and simple problems which can be converted to such an equations". During the second grade they will "solve equations of the first and second degree in one unknown and problems which can be converted into such equations".

The Hungarian curriculum[4] for grades 5-8 (upper primary) writes that in year 5 students should "solve simple equations of the first degree by deduction, breaking down, checking by substitution along with simple problems expressed verbally". In year 6 they should "solve simple equations of the first degree and one variable with freely selected method". By year 7 they should "solve simple equations of the first degree by deduction and the balance principle. Interpret texts and solve verbally expressed problems. Solve equations of the first degree and one variable by the graphical method". Lastly, by year 8 students should "solve deductively equations of the first degree in relation to the base set and solution set. Analyse texts and translate them into the language of mathematics. Solve verbally expressed mathematical problems"

In these three examples can be seen very different perspectives on a core topic of the lower secondary curriculum. The Finnish specification is loose and offers little in terms of how content should be interpreted and presented at any specified moment in the four year cycle. The Flemish appears more tightly specified although the shift from one year to the next, in respect of linear equations, seems vague. The main difference lies in the explicit expectation of problems to be translated into equations for solving. Lastly, the Hungarian curriculum offers a tightly specified progression over a four year period with methods and problem solving, including word problems, increasingly exploited. While not wishing to over-speculate, it is interesting to compare such specifications with three of Hofstede's (1986) dimensions. When compared with other developed European nations, he presents Finland as low power distance, low uncertainty avoidance and a strong tendency towards the feminine. This contrasts with Belgium (not just Flanders) as not only substantially higher than Finland on power distance and uncertainty avoidance but also among the most

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1 According to Prange (2004) Bildung is a broad concept that does not translate easily, but is "something noble and undeniably good... Bildung is much better than mere education, or Erziehung, to give the German word. It is associated with liberty and human dignity, whereas education is associated with teaching skills and morals" (Prange 2004: 502).

2 See http://www.oph.fi/english/publications/2009/national_core_curricula

3 See http://www.ond.vlaanderen.be/dvo/english/

4 See http://www.okm.gov.hu/letolt/nemzet/kerettanterv36.doc

masculine of the developed European nations. Unfortunately, Hungary is not represented in Hofstede's analyses as they derived from data collected before the countries of Eastern and Central Europe joined the capitalist world. However, there is some evidence here of the ways in which cultural norms and characteristic patterns of social behaviour find a voice in the curricular presentation of mathematics.

Culture and mathematics teaching

As indicated above, teachers are proxies for an educational system's values and there is growing evidence that mathematics teachers in one country behave in ways that identify them more closely with teachers in their own country than teachers elsewhere. Much of this research draws on the perception that "teaching and learning are cultural activities (which)... often have a routineness about them that ensures a degree of consistency and predictability. Lessons are the daily routine of teaching and learning and are often organised in a certain way that is commonly accepted in each culture" (Kawanaka, 1999: 91). This sense of routine predictability has been variously described as the *traditions of classroom mathematics* (Cobb et al, 1992), the *cultural script* (Stigler and Hiebert, 1999), *lesson signatures* (Hiebert et al, 2003) or the *characteristic pedagogical flow* of a lesson (Schmidt et al, 1996). The latter embodies the pedagogical strategies which, through repeated enactment, are typical of a country's lessons, routine, and beneath the consciousness of most teachers (Cogan and Schmidt 1999). Explanations for such patterns draw on beliefs that cultures "shape the classroom processes and teaching practices within countries, as well as how students, parents and teachers perceive them" (Knipping 2003: 282), to the extent that many of the processes of teaching are so "deep in the background of the schooling process ... so taken-for-granted… as to be beneath mention" (Hufton and Elliott, 2000: 117).

In this regard, there is a growing body of research highlighting substantial national variation not only in the ways in which teachers act out their roles but also in the resources available to them. For example Kaiser et al. (2006) have offered persuasive summaries of the distinguishing characteristics of English, French, German and Japanese mathematics teaching, particularly in respect of proof and the structural properties of mathematics. The two TIMSS video studies have examined a range of teacher practices in Australia, the Czech Republic, Hong Kong SAR, the Netherlands, Switzerland, the United States and Japan. Huegener et al (2009) have focused on differences in the ways teachers present the theorem of Pythagoras in Germany and Switzerland, while Santagata (2005) has highlighted substantial differences in the ways in teachers handle students' mathematical errors in Italy and the US. Campbell and Kyriakides (2000) and Haggarty and Pepin (2002) have shown how school texts reflect differences in systemic expectations and traditions. An et al. 2004) have identified culturally located differences in teachers' mathematical content knowledge, while Correa et al. (2008) have done the same for teachers' mathematics-related beliefs. Finally, although it is acknowledged that the scope of this paper presents an extensive review, it is important to acknowledge the Learner's Perspective Study. In this respect, Clarke and his many colleagues have contributed much to our collective understanding of differences in the ways in which mathematics teachers around the world construe their roles (Clarke et al 2006a, 2006b).

With regard to my own and my colleagues' work, we have examined the ways in which teachers present mathematics to students in the age range 10-14 in England, Flanders, Hungary and Spain. The episodes of videotaped lessons were coded against a framework developed iteratively and collaboratively over the course of a year

(Andrews 2007a), where an episode was that part of a lesson in which the teacher's observable didactic intention remained constant. In terms of teachers' observable learning objectives, the project found (Andrews 2009a) that teachers in all four countries privileged the development of both conceptual procedural knowledge in high and comparable proportions. The major variation lay in the other outcomes. For example, Hungarian teachers placed a higher emphasis on the structural properties (links within and between topics), mathematical efficiency (comparing solutions strategies for elegance and efficiency), problem solving and reasoning than elsewhere. In similar vein, English teachers were rarely seen to encourage structural links or efficiency. Such differences reflect not only differing curricular expectations but long standing mathematics teaching norms. The same study (Andrews 2009b) found, in relation to the observable didactic strategies employed by teachers, that teachers explain regularly and in comparable proportions irrespective of country. However, all other strategies distinguished between the didactical practices of project teachers. For example, Flemish teachers were exploited explicit motivational strategies in smaller proportions than elsewhere, while Spanish teachers employed them in over half of all observed episodes. English teachers very rarely questioned (used higher order questions) while Hungarian teachers did so constantly. Spanish teacher coached - offered hints and suggestions to facilitate their students' successful completion of given tasks - in more than three quarters of observed episodes, while teachers elsewhere did so in equal and significantly smaller proportions. Hungarian teachers invited students to share publicly their solution strategies in almost every episode while teachers elsewhere did so consistently at around the 60 per cent level.

Of course, cultural emphases do not end with learning outcomes and didactic strategies. In a comparison of Hungarian and English mathematics teachers' beliefs Andrews and Hatch (2000) found that while English teachers valued the systematic decorating of their classrooms with examples of students' work or mathematical posters, such practices were alien to Hungarian teachers who tended to work within classrooms with, essentially, bare walls. In a second study Andrews (2007b) found English teachers espousing collective beliefs about the value of school mathematics lying in its applicability to a world beyond school, while Hungarian teachers articulated a collective belief whereby the value of mathematics lay within mathematics itself. In short, the ways in which teachers conceptualise and present mathematics to their learners, the environments they create and their beliefs as to the nature and importance of the subject highlight well that mathematics teaching, in all aspects, is a cultural activity that differs significantly from one country to another.

Final appeal

In conclusion, my appeal to colleagues researching any aspect of mathematics teaching and learning is that they make explicit in the reporting of their work the cultural context in which it was undertaken. Too frequently research is reported with no indication, other than the authors' designation, of a study's location. Moreover, and I know I am equally to blame in this regard, writers assume when synthesising the literature, that generalities derived from a study undertaken in one cultural context are generalisable to another. That is, when constructing the theoretical frames within which we conduct our research, too frequently we ignore the cultural implications embedded in the studies we analyse. Such assumptions, that literature can be synthesised independently of context, may lead to poorly designed and reported studies.

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