Working group on trigonometry: meeting 4

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These notes record the discussion at the fourth meeting of this working group. The focus was on the history of trigonometry, and discussing three different approaches to teaching it which have appeared in recent readings.

Keywords: congruence, similarity, astronomy, triangle, trigonometry

Historical development

Leo Rogers had supplied us with a brief overview of the history of trigonometry. Unfortunately he was unable to join us, being occupied with a Working Group on History of Mathematics. There appear to be two strands, the astronomical strand which used circles and arcs as the basic tool to track the position and movement of stars and planets, and the surveying strand which used ratios of sides of right-angled triangles. The sundial uses both.

In an earlier meeting we had circulated the suggested teaching approach of Thompson, Carlson and Silverman (2007) for pre-service teachers. They use arcs of circles in order to present sine etc. not as ratios which have to be imagined, but as measures of arcs which can be seen. We referred to this briefly, and the fact that for small angles one can approximate a circle segment as a triangle. Historically there seems to be a progression from approximating a slim segment as one triangle to bisecting the relevant chord to approximate it as a right-angled triangle on the half-chord.

We thought that the history of trigonometry includes the history of our exploration of their properties.

Two future actions arise from this discussion:

- Learn more about the triangle roots of trig, as illustrated in the *Nine Chapters* to find heights of tall things (Liu Hui); find out what the engineers were doing, as well as the astronomers
- Return to reading the Thompson paper to look for analogies.

Comparing two approaches

We then compared two recent articles on teaching trigonometry (Kemp, 2009; Steer, da Silva and Easton, 2009). Both had appeared in the same issue of *Mathematics Teaching* without editorial comment. We thought it might be helpful to compare them in the light of some of the analysis we have been doing in the working group.

Similarities

- both took an approach which focused on properties of triangles
- both delayed the introduction of technical terms until the relations had been established empirically
- both depended on learners noticing what stays the same when certain features of triangles are varied
- both aimed at complex understanding rather than technical procedures

- both emphasised the significance of similarity
- both mentioned 'SOHCAHTOA', but we were not sure if this mnemonic was being used to communicate to the reader that formalisation and application of the ideas was the endgoal, or whether it had been introduced to learners in that form – the mnemonic itself being the endgoal
- both appeared to have resolution of triangles as the goal, rather than a functional understanding.

Differences

One approach (Steer et al., 2009) depended on dynamic geometry, and was reported a way which emphasised the technology, while the other (Kemp, 2009) used lowtech materials. The paper by Steer et al. described the implementation of ideas developed by Jeremy Burke (2006). It was the third of a series but summarised the overall approach. Burke's ideas had been circulated to this group earlier, and this article omitted relating trigonometric ideas to the unit circle, as he had suggested. We wondered why this might be so, and concluded that the realities of pressures on teaching may have led them to truncate the proposed sequence. Some readers felt that it dwelled in detail on technology use, rather than relegating software to the position of a tool.

Another difference was that, while both used similarity as the central idea, this was approached as a 'constant binary relation between sides of these triangles' rather than through 'preservation of proportion by scaling' being a central idea that makes trigonometry possible. The scaling idea was more prevalent in Steer et al's approach than Kemp's. In the former, similarity arises after consideration of congruence, so the emphasis is on types of sameness, and scaling (multiplication) when triangles share the (a,a,a) characteristic is central. In the latter, similarity arises as a relation among sides of triangles that 'look the same' and multiplication is a choice from four binary operations to find one that is constant.

Neither approach explains why only right-angled triangles are chosen for this exploration, since all sets of similar triangles have common ratios. Use of a unit circle could have made this clear.

Embodiment of trigonometric ideas

Finally we read a passage from Lakoff and Nunez (2000) about how trigonometric ideas can arise from blended fundamental conceptual metaphors which relate to how humans are in the world. They claim that there has to be a metaphor to enable us to relate angles to numbers, for which we have some fundamental understandings. They draw on the idea of the unit circle as the appropriate metaphor, and describe this as a blend of circles in the Euclidean plane with the two-dimensional numerical metaphor of the Cartesian plane, and the angle in the Euclidean plane having two legs that delineate the angle. The final blend gives the familiar diagram of the right angled triangle generated by one rotating leg of the angle and the vertical dropped from it. They show that trig ratios and the functions are both represented and generated by this diagram.

What they offer is logical, in that these ideas are related mathematically in the way they describe, but incompatible with the historical development we had thought about earlier. We want to think more about how 'surveying the earth' and 'measuring the heavens' might have been perceived over centuries, and what role the unit circle might have played. Trigonometrical ideas were used long before Descartes offered the 'metaphorical blend' for number and space. The chronological development does not necessarily match the logical or metaphoric perspective, and does not support the order in which they relate the 'metaphors'.

We observed that what is 'natural' is culturally loaded, since for some cultures distance and direction combined is a fundamental way of seeing the world. Further, some relevant static relations, such as our understanding of near and far objects, are also naturally embodied in our ways of being in the world and also contribute to trigonometric understanding. In summary, we could not understand why it is helpful to see these ideas as metaphors, nor could we agree that these were *the* appropriate metaphors for trigonometric understanding.

Future plans

We shall next meet at the BSRLM meeting in Summer 2010. This is an open group and all are welcome to join. If you would like copies of earlier readings please contact anne.watson@education.ox.ac.uk.

References

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