Identifying and developing the mathematical apprehensions of beginning primary school teachers

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In this paper I present a summary of a four year study into the development of mathematical apprehensions in beginning elementary teachers using the Knowledge Quartet as a framework for reflection on, and discussion about, mathematics teaching. The term mathematical apprehension is used as an inclusive term to cover both mathematical content knowledge and conceptions of mathematics teaching. Evidence from three case studies suggest that focused reflection using the Knowledge Quartet facilitated the development of mathematical content knowledge and promoted positive changes in conceptions about mathematics teaching. Experience and working with others in classrooms and schools were also seen to influence development and change in the teachers' apprehensions. However, individual reflection was found to have a mediating role on the influence of these two social factors.

Introduction

The way in which teachers teach mathematics is influenced both by their mathematical content knowledge (Ball, 1988) and by their conceptions about mathematics teaching (Thompson, 1992). The mathematical content knowledge of elementary teachers has been found to be insufficient for teaching (Brown, Cooney and Jones, 1990; OFSTED, 2000). Researchers have also found that elementary teachers often have unhelpful conceptions about mathematics and mathematics teaching (Brown, McNamara, Jones and Hanley, 1999). Initial teacher education courses alone seem unable to produce necessary developments in mathematical content knowledge (Carré and Ernest,1993; Williams, 2008) or to promote sustained positive changes in conceptions of mathematics teaching (Brown *et al*, 1999) in beginning teachers.

The aim of this study was to investigate the effectiveness of a sustained approach to developing the mathematical apprehensions of beginning elementary teachers. The study began with the conjecture that, supported reflection on the mathematical content of teaching might promote developments in mathematical content knowledge and changes in conceptions of mathematics teaching in beginning teachers. The Knowledge Quartet framework (Rowland, 2008) was used to facilitate such reflection both as a means to, and as a measure of, professional development. Three theoretical frameworks underpinned the study.

Theoretical frameworks

Mathematical content knowledge for teaching

A theoretical framework for consideration of teachers' mathematical content knowledge was derived from the seminal work of Shulman (1986; 1987) and from the work of Deborah Ball and colleagues in Michigan. Shulman's three 'knowledge bases' which relate specifically to the content of teaching; subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curriculum knowledge (CK) provided a foundation. The division of SMK into substantive and syntactic knowledge (Schwab, 1978) also informed my research as did

refinements of Shulman's categories by Ball, Thames and Phelps (2008). They identified *common content knowledge* (CCK) and *specialized content knowledge* (SCK) as subdivisions of SMK, and *knowledge of content and learners* (KCL) and *knowledge of content and teaching* (KCT) as subdivisions of PCK. The Michigan group also include the category of *knowledge on the horizon* as an aspect of SMK and *knowledge of the curriculum* as an aspect of PCK. The Table 1 below illustrates the relationship between the categories of Shulman and of the Michigan group and represents the model used in this study as a framework for investigating the teachers' mathematical content knowledge.

Table 1

Mathematical Content Knowledge					
Subject Matter Knowledge (SMK)			Pedagogical Content Knowledge (PCK)		
Common Content Knowledge (CCK)	Specialist Content Knowledge (SCK)	Knowledge o the Horizon	Knowledge o Content and Teaching (KCT)	Knowledge o Content and Learners (KCL)	Knowledge o the Curriculu

Conceptions of mathematics teaching

The models of teachers' conceptions of mathematics teaching identified by Kuhs and Ball (1986) provided the basic framework for this study although other models, such as those of Ernest (1989) and Askew, Brown, Rhodes, Johnson and Wiliam (1997), were drawn on where appropriate. Kuhs and Ball identified four dominant models: *a classroom-focused view;* a *content-focused with an emphasis on performance view*; a *content-focused with an emphasis on conceptual understanding view* and a *learner-focused view*. Ernest (1989) identified six models which were very similar to those of Kuhs and Ball, but included two extra categories combining characteristics from Kuhs and Ball's models. This refinement was found to be generally unnecessary in my research although I drew on it where appropriate. I also drew on the work of Askew *et al* (1997) which identified three orientations in teachers' conceptions about mathematics teaching; a *transmission orientation*, a *connectionist orientation* and a *discovery orientation*.

An approach to developing mathematical content knowledge and changing conceptions of mathematics teaching

My approach was based on the model of professional development through reflection *in* and *on* practice (Schön, 1983). However, the role of reflection was investigated in relation to ideas from socio-cultural theory models of professional development. In social theory knowledge is seen as *situated* in social situations and the development of knowledge as resulting from *enculturation* or socialisation into the professional culture (Cobb, Yackel and Wood, 1991; Lave and Wenger, 1991; Wenger, 1998). I recognised the role of socio-cultural factors in my study and expected teachers' apprehensions, as revealed through observations and discussions of their teaching, would be contingent on the context of their teaching. I also recognised that the individual reflection of participants would be made within the communities of practice of schools, and would reflect the role and relationships of the participants within schools. Jaworski (2007) suggested that *communities of practice* (Wenger, 1998) became *communities of inquiry* when teachers worked collaboratively to reflect on and develop their practice. It was hoped that the participants in my research group would become such a community of inquiry. The rationale for this study was therefore that teachers should

be supported to use reflection on their mathematics teaching within the social contexts of their teaching and also of the research project.

The Knowledge Quartet framework

The KQ framework was developed by mathematics educators at the University of Cambridge from observation and videotaping of mathematics teaching (Rowland, 2008). Analysis of this teaching produced 18 'emergent' codes (Glaser and Strauss, 1967) of situations in which mathematical content knowledge of teachers was made visible, e.g. 'concentration on procedures' and 'making connections between concepts'. These were later classified into four 'superordinate' categories based on associations between the original codes. These categories make up the four dimensions of the Knowledge Quartet; *foundation, transformation, connection* and *contingency*. The *foundation* dimension includes the propositional knowledge of SMK and PCK that teachers draw on in their practice, as well as their beliefs about mathematics and mathematics teaching. *Transformation* encompasses the ways in which a teacher's own knowledge is transformed to make it accessible to learners and *connection* includes issues of sequencing and connectivity as well as complexity and conceptual appropriateness. The final dimension of the KQ, *contingency*, could be described as 'thinking on one's feet' and is concerned with the way teachers respond to unexpected student responses.

The study

The study began with 11 student teachers from the 2004-5 cohort of elementary (5-11 years) postgraduate pre-service teacher education course at the University of Cambridge reducing, as anticipated, to 4 in the fourth and last year of the study. Data came from observation and analysis of teaching using the KQ as well as from post-lesson reflective interviews, group and individual interviews and participant written reflective accounts. Transcripts of interviews and written reflective accounts were all systematically coded using the qualitative data analysis software NVivo 7. A grounded theory approach (Glaser and Strauss, 1967) was used which led to the emergence of a hierarchical organisation of codes into a number of themes.

Case studies were built from the analysis of observed teaching as well as from analysis of interviews and the participants' reflective accounts. Six themes in the development of the participants' mathematics teaching emerged from the NVivo analysis. These were, *beliefs, confidence, subject knowledge, experience, reflection* and *working with others*. The KQ analysis of observed teaching provided a 'spine' for presenting findings in relation to the development of participants' mathematical content knowledge. This was supported by data from interviews and reflective accounts organised under the themes of *subject knowledge* and *confidence*. Findings in relation to changes in conceptions about mathematics teaching drew primarily on data organised under the theme of *beliefs*, and were supported by lesson observation data. Data from the themes of *experience, reflection* and *working with others* gave insight into the influences on developments in the participants' mathematical content knowledge and into influences on changes in their conceptions of mathematics teaching.

Findings

Development of mathematical content knowledge

Looking at the teachers' content knowledge in relation to the *foundation* dimension of the KQ suggested that development in propositional PCK, or *knowledge of content and*

learners, *knowledge of content and teaching and knowledge of curriculum,* were greater than in SMK, or *common content knowledge* (CCK), *specialised content knowledge* (SCK) and *knowledge on the horizon.* Where development in SMK did take place, it was in relation to SCK rather than CCK. Reflection on practice helped the teachers identify areas of their SCK that needed development. Development in these areas was achieved through support from me, attendance at in-service training or through self-study. It is unlikely that such developments would have occurred through reflection alone.

The teachers' active PCK, as revealed through the *Transformation, Connection* and *Contingency* dimensions of the KQ, was also seen to have developed over the study. The three teachers appeared to focus on different aspects of their *Transformation* knowledge, and this focus was reflected in the apparent developments in their practice. All three teachers made more effective use of demonstrations but this was most apparent in Amy's teaching. A more considered use of representations was a strong feature of Kate's practice and Jess showed particular development in relation to her use of examples.

All the teachers considered making connections to be important for effective teaching, and demonstrated this throughout the study. Focusing on connections developed the teachers' practice in different ways. Amy concentrated on making connections to individual children's understanding and interests, Kate became more likely to make connections between mathematical ideas and Jess increasingly focused on the connections between calculation operations. The ability to act contingently also became a focus for the teachers, and they all increasingly considered this to be integral to effective teaching. Responding to children's needs and ideas became central to Amy's early years practice. Kate became particularly proficient at acting contingently to find alternative representations to address misconceptions or lack of understanding and Jess became more likely to respond to children's errors and to try to understand their mathematical thinking.

Changes in the teachers' conceptions of mathematics teaching

Each of the teachers held complex views of mathematics teaching, incorporating elements from all four of Kuhs and Ball's (1986) views of mathematics teaching. Although the balance of these elements varied, there appeared to be a pattern in the direction of change in the three case studies. The teachers moved towards a focus on developing conceptual understanding and towards a learner-focused view of mathematics teaching. All the teachers had elements of a classroom-focused view at the beginning of the study but this quickly diminished. Throughout the study, the three teachers demonstrated content-focused conceptions of mathematics teaching with differences between them in the balance between an emphasis on performance and an emphasis on conceptual understanding. However, for all three, the direction of change over the four years was towards a greater emphasis on conceptual understanding. Amy demonstrated some emphasis on performance at the beginning but moved to a strong emphasis on conceptual understanding. Kate and Jess increasingly appeared to emphasise conceptual understanding although both retained strong elements of emphasis on performance.

Amy began with a learner-focused view of teaching and this appeared to be strengthened over the study. Kate and Jess also appeared to focus increasingly on the needs of their pupils. However, the way in which they interpreted these needs differed. Amy's interpretation was the most consistent with constructivist ideas inherent in a learner-focused view of mathematics teaching. Kate also increasingly tried to understand children's mathematical thinking but remained focused on helping them achieve pre-determined products or processes. Jess appeared to interpret the needs of her pupils in terms of their 'ability' and the need for them to achieve success in order to develop confidence. Amy increasingly took an inquiry or problem solving approach to her teaching, consistent with a

learner-focused view of mathematics teaching. Kate and Jess also moved towards this approach, but found it more difficult to adopt.

Influences on development and change

Experience of teaching mathematics, and *working with colleagues,* were found to be important influences on the teachers' mathematical apprehensions. However, *reflection* mediated the effects of these two influences. It was reflection on experience which catalysed developments in mathematical content knowledge, and the focus of the teachers' reflection on experience which reinforced or changed their conceptions about mathematics teaching. Working with colleagues was also found to be an important influence in developing mathematical content knowledge and conceptions of mathematics teaching. This influence was quite different for each teacher. Amy's learner-focused view of mathematics teaching was shared by her colleagues and reinforced this view. However, Kate questioned some of the practices in her school and Jess was uncomfortable with the way in which mathematics was taught in her first post. All three teachers reflected on the principles and practices of their schools facilitating *alignment* (Wenger, 1998) for Amy, leading to *critical alignment* (Jaworski, 2006) for Kate and to non-alignment with her first school for Jess.

Reflection using the KQ to focus on the mathematical content of teaching helped the teachers to focus on their own and on their pupils' understanding of mathematical concepts, and this supported developments in their SCK. Their reflection also informed the teachers' thinking about how to transform their own understanding in order to make it accessible to their pupils, and this supported developments in their PCK. Where the teachers focused on the effectiveness of their teaching their KCT was enhanced, and where they focused on the understanding of learners, their KCL was enhanced. The KQ focused the teachers' reflection on the content of the mathematics and on the children's engagement with, and learning of, that content. This supported movement towards conceptions of mathematics teaching that emphasised conceptual understanding, and which were learner focused.

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