The relationship between number knowledge and strategy use: what we can learn from the priming paradigm

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Priming methods involve showing a stimulus for a short amount of time (the prime), followed by a second stimulus (the target), which children are asked to perform some operation on. If there is a strong association between the prime and target for a particular child, then the operation on the target will be facilitated by the presence of the prime. This paper describes a project in which priming methods are used to add to our understanding of strategy development for simple addition problems. Children were asked to complete two activities; a priming trial designed to demonstrate priming effects for doubling, and a set of addition problems where participants were asked to explain how they arrived at their answers. Approximately half of the participants used counting strategies (count-on from first, count-on from smallest), while half used non-counting strategies (decomposition, tie or retrieval). Results indicate that a priming effect for doubling relationships but only for the group of children using non-counting strategies. This result could help to explain the relationship between the development of number knowledge and the development of new strategies.

Introduction

There is a well established understanding of the normal course of development of strategy-use when solving single-digit arithmetic problems (e.g. Fuson 1992). Children begin this course of development by counting both addends in an addition problem, often using concrete objects such as fingers to aid the process. The next strategy to appear is the count-one strategy, in which children start with one of the addends, then count on from there to find the answer. The 'min' strategy usually comes next, which involves children choosing to count on from the largest addend. These three strategies all involve children counting in order to arrive at an answer to a problem. At some point, children will begin to be able to solve some simple problems using retrieval – directly accessing answers to problems stored in memory. Although strategies generally appear in this order, children maintain a repertoire of several strategies any any given point during this development, and show a high degree of variability in their application of strategies to problems (Siegler 2007).

Fewer studies have addressed the nature of strategies used to solve problems resulting in answers greater than 10. However, this aspect of the literature is growing due to the recent focus on children's adaptive expertise in selecting amongst strategies (Verschaffel, Torbeyns, De Smedt, Luwel, & Van Dooren 2007). When children are solving addition problems that bridge 10, there are heuristic strategies available that sit between the counting strategies and direct retrieval in terms of efficency. The problem '8 + 7', for example, might be solved by converting the problem to '7 + 7 + 1' – this is often referred to as the 'tie' strategy and is often used in the case of 'near tie' problems where the addends differ by 1. Alternatively, '8 + 7' might be solved by converting to '8 + 2 + 5', if a child knows their number-bonds to 10 (sometimes known as 'ten-friends') – this is often referred to as a 'decomposition' strategy. It is not clear what factors are involved either in stimulating the adoption of new strategies in response to problems or in determining the selection of one strategy over others that are available in relation to a given problem.

The relationship between number knowledge and strategy development

Torbeyns, Verschaffel and Ghesqiere (2005) give a hypothetical example of a child who is able to accurately retrieve the answer to 6+6, but not 7+7 or 8+8. This child would be expected to be more likely to use the tie strategy when solving 6+7 (by transforming the problem to 6+6+1) than when solving 7+8 or 8+9. Torbeyns et al. showed that children in their study differed in the efficiency with which they carried out counting, decomposition and tie strategies, but that children at a range of different ability levels all showed similar levels of adaptivity, generally choosing the strategy that would generate a correct answer most quickly in response to a particular problem. Torbeyns et al. only analysed data from children who were already using either the decomposition or tie strategy – their aim was to study variation in adaptivity related to differences in achievement in mathematics, not to investigate the necessary conditions for the development of these strategies.

Very relevant to the current discussion is the existence of a “tie effect” (LeFevre, Shanahan, & DeStefano 2004), whereby the problem-size effect (the fact that arithmetic problems with larger answers tend to be answered more slowly than those with smaller answers) can generally not be observed for tie problems (where both addends in an addition problem are the same). LeFevre et al. showed that the tie effect is not due to facilitation of encoding (the fact that the same number appears twice means it is more quickly encoded the second time), but is due to calculation and memory access.

It seems reasonable to argue, as do Torbeyn, Verschaffel and Ghesquiere (2005), that good knowledge of doubling relationships (pairings between 6 and 12, 7 and 14 and so on) is required in order for children to begin using the tie strategy. However, this paper aims to go a step further and make the claim that implicit knowledge of doubles is a prerequisite for use of the tie strategy.

Using priming to investigate number knowledge

The first study of number knowledge that employed priming as a method was that of den Heyer and Briand (1986), in which a priming distance effect (PDE) was observed. The PDE is the phenomenon that a reponse to a target stimulus is facilitated by the presentation of a prime that is similar in magnitude to the target. For example, in the lexical decision task used in den Heyer and Briand's study, participants were quicker to respond to 'five' after the prime 'four' than after the prime 'three'. The PDE has been shown to be equally strong in both directions – so the prime '4' facilitates processing of a target '5' as well as it does '3' – and has also been demonstrated in different modes, whereby the prime 'six' facilitates processing of 'seven' or '7', for example (Reynvoet, Brysbaert, & Fias 2002).

There is some debate regarding the mechanism underlying the PDE. Some researchers have explained the effect in terms of operations on a 'sub-symbolic' number-line, used in order to compare number information in terms of magnitude. However, there is evidence to suggest that a connectionist approach might generate a more satisfactory explanation. There is evidence, for example, that as well as proximity on the number line, other relationship amongst numbers can give rise to priming effects. Garcia-Orza, Damas-Lopez, Matas and Rodriguez (2009) show that, for adult participants, the prime '2x3' facilitates processing of target '6', using a masked prime protocol. This suggests that, rather than sub-symbolic processing, these priming effects reflect symbolic processing within something like Collins and Loftus' (1975) semantic activation network.

Method

Participants

57 children, from two primary schools, took part in this experiment. They were aged between 7 years, 2 months and 9 years, 11 months. In each school, the Mathematics Coordinator was asked to select those children who were able to reliably solve single-digit addition problems, but did not yet consistently use a retrieval strategy. All of the children who participated in the study had experienced classroom instruction in the use of a range of strategies for solving addition problems, including counting strategies, decomposition and tie.

Instruments and measures

Two tasks were prepared, using the DirectRT psychology experiment software package. Stimuli were presented to participants in the centre of a 17 inch monitor, using a 48 point font. A microphone was used in order to measure verbalisation latency.

Addition problem task

For this task, a set of addition problems was created. All single-digit addition problems with two addends, where the two addends were different and the answer was greater than 10, were included. Participants were asked to respond with an answer to each problem. Following an answer, participants were prompted with the question, “How did you solve the problem x + y?” Problems were presented to participants at random, without replacement. Strategies were coded as being either “count-one”, “min”, “decomposition”, “tie”, “retrieval” or “other – including don't know”. Three practice problems were given before starting the main set of problems. The practice problems were “7 + 3”, “6 + 2” and “4 + 4”.

Priming task

60 prime-target pairs were created. Of these, 15 pairs related to the present study. Primes used were “5”, “6”, “7”, “8” and “9”. The target stimulus in each pair was either the exact double of a prime, or the double +/- 1. The remaining prime-target pairs presented to participants were included in order to ensure that participants could not predict that the purpose of the study was to assess knowledge of doubles, and were constructed as if intended to address participants' knowledge of number bonds and proximity on the number line. Prime-target pairs were presented at random, without replacement.

Timings were as follows: Fixation “*”: 1000ms; Prime stimulus: 200ms; Fixation “*”: 500ms; Target stimulus. Participants were instructed to say into the microphone the second number in each pair (the target), as quickly as possible. The reaction time recorded for each trial was the time it took for the participant to begin reading the target number, following its presentation.

Design

The experiment employed a mixed design, with two independent variables. The first independent variable was the relationship between prime and target in a prime-target pair. There were 3 conditions of this variable; the target was either double the prime minus one, double the prime exactly, or double the prime plus one. The second independent variable was whether or not participants claimed to use the tie strategy at least once whilst completing the addition problem task. The dependent variable was the time that it took to read aloud the target stimulus.

The hypothesis was that reaction time would be least when the target was the exact double of the prime and that this effect would be observed only for those participants who used the tie strategy at least once during the addition problem task.

Procedure

Participants completed the two activities individually, in a quiet room outside of their usual classroom. A laptop computer was used to generate stimuli and record reaction times via a microphone. The experimenter watched the laptop screen during each trial, whilst participants watched a second monitor, synchronised with the laptop.

Half of the participants completed the addition problem task first, followed by the priming task, while half completed the two tasks in the reverse order. The researcher introduced the task, and gave the participant an opportunity to ask questions. Three practice trials were completed, followed by a further opportunity to ask questions. The block of experimental trials for the task were then completed.

Results and Discussion

The addition problem task was used in order to divide participants into two groups. 29 participants reported using the tie strategy on at least one occasion during this task, while 28 participants did not.

A 3 x 2 mixed ANOVA was carried out. The independent variables were prime-target pair (repeated measures: target = 2 x prime – 1; target = 2 x prime; target = 2 x prime + 1) and whether children used the tie strategy at least once during the addition problem solving activity (independent groups). The dependent variable was the median time it took for a participant to begin reading the target stimulus.

There was a significant main effect of prime-target relationship ($F_{2, 84}=4.867$, $p=0.01$). This indicates that participants were significantly quicker to read a target that was exactly double the preceding prime than a target that was 1 greater or 1 less than the exact double of the preceding prime. There was also a significant interaction between prime-target pair and strategy use ($F_{2, 84}=6.879$, $p=0.002$). As can be seen in Figure 1, the effect of variation in prime-target pair on RT is accounted for entirely by the group of children using the tie strategy. There was no significant main effect of strategy use on reaction time.

Participants who used the tie strategy to solve at least one of the set of addition problems were quicker to read a target that was the exact double of the prime than a target that was the exact double of the prime plus or minus 1. The participants in the study had no way of predicting the relationship(s) under investigation. This means that on perceiving the prime stimulus, the double of the prime (amongst other cognitive resources including numbers and concepts) was automatically activated.

When the target stimulus was the exact double of the prime, participants' reading of the target was facilitated due to that number already having been activated.

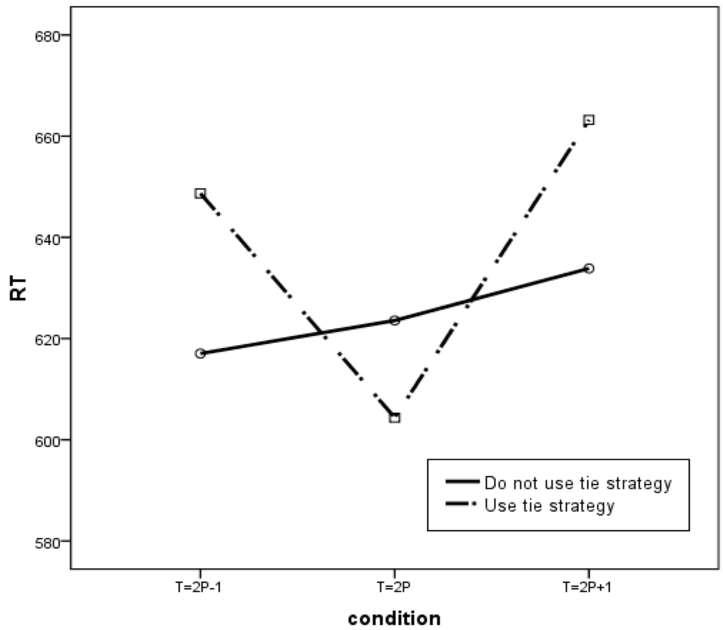

Figure 1: Graph to show effect of prime-target pair type on RT, by strategy use

Thus this study clearly demonstrates the fact that children using the tie strategy have implicit knowledge of the relationship between numbers and their doubles (at least for numbers between 5 and 9). This result contributes substantially to the literature on both children's arithmetic strategy development and its relation to the literature on the nature of children's representation of number.

The findings do not directly identify a causal relationship between the development of knowledge of relationships between numbers and their doubles and the development of the tie strategy. However, of the two possible interpretations (either the implicit knowledge of doubles is a necessary condition for the development of the tie strategy, or children's knowledge of the tie strategy encourages the rapid development of knowledge of doubles) it is intuitively most likely that children must develop a knowledge of the relationships between numbers and their doubles before they can add the tie strategy to their repertoire. This fits fits well within a resource activation framework (Hammer, Elby, Scherr, & Redish 2005). Within this framework, cognitive resources are activated in response to a problem situation. These resources are used in the assembly of ad hoc theory in order to generate a solution.

Further work must be done in order to fully understand the relationship between the development of implicit knowledge and the development of strategies, but some important implications should be considered at this stage. Most importantly, the study calls into question the claim that teachers should be helping children develop ways to select amongst available strategies for solving problems. Torbeyns, Verschaffel and Ghesquiere (2005) found there was no difference in levels of adaptivity (the ability to select the most efficient strategy from a repertoire of available strategies for a given problem) between children across a range of mathematical ability. The present study shows that children do not use the tie strategy if they do not have implicit knowledge of relationships between numbers and their doubles. Taken together, the evidence from these studies shows that the development of new strategies, and the development of the adaptivity necessary to select amongst strategies, occur as a result of the development of associated cognitive resources such as the knowledge of particular types of relations amongst numbers.

These results help to provide an explanation for some effects observed in previous work. For example, Siegler and Stern (1998) observed that a majority of children in a study were using a new strategy for solving a number of problems before they were aware of using it. Within the resource activation framework, the learner's use of a particular procedure (resulting from the automatic activation of relevant cognitive resources) and the learner's representation of that procedure are quite different things. The representation of a particular strategy will always follow that strategy's first use (whether it follows immediately or at some later point).

Conclusion

This study represents an important step in our growing understanding of children's development of mathematical thinking. Its main contribution consists in the argument that implicit knowledge of relationships between numbers and their doubles is a necessary prerequisite for the development of the tie strategy.

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